KAPSABET HIGH SCHOOL

SECTION I (50 marks): Answer all questions in this section

1. Use logarithm tables to evaluate $\sqrt[3]{\frac{0.4239 \times 149.6}{\log 6}}$ (4 marks)

No	standard form	log
0.4239	4.239×10^{-1}	$\bar{1}.6272$ +
149.6	1.496×10^{2}	2.1750 M1
		1.9022 −
log 6 = 0.7782	7.782×10^{-1}	$\bar{1}.8911$ M1
		$\frac{1.9111}{3}$
4.335	4.335×10^{0}	0.6370 M1

$= 4.335$ A1

2. Solve the equation $6^{2x+1} = 2^{3x+1}$ (3 marks)

$(2x+1)\log 6 = (3x+1)\log 2$

$\frac{2x+1}{3x+1} = \frac{\log 2}{\log 6}$ M1

$\frac{2x+1}{3x+1} = 0.3869$

$2x+1 = (3x+1)(0.3869)$ M1

$2x+1 = 1.1607x + 0.3869$

$2x - 1.1607x = 0.3869 - 1$

$0.8393x = -0.6131$

$x = -0.7305$ A1

3. Kevin truncated 0.00627 to 3 decimals and 487.74 to 3 significant figures. Calculate his percentage error in calculating product of numbers in truncated values to 1 decimal places. (3marks)

Actual $= 0.00627 \times 487.74$

$= 3.0581298$

Truncated $= 0.006 \times 488$ M1

$= 2.928$

% error $= \frac{\text{Actual} - \text{Truncated}}{\text{Actual}}$

$\frac{3.0581298 - 2.928}{3.0581298}$ M1

$= \frac{0.1301298}{3.0581298} \times 100$

$= 4.255$

$= 4.3\%$ 1dp A1

4. A new laptop depreciates at 8% per annum in the first year and 12% per year in the second year. If its value at the end of the second year was sh121,440. Calculate the original value of the laptop. (3marks)

let the original value be P

$A = P\left(1 - \frac{r}{100}\right)^n$

$P\left(1 - \frac{8}{100}\right)^1$

$A = P\left(\frac{92}{100}\right)$ M1

$= 0.92P$

2nd year

$A = P\left(1 - \frac{r}{100}\right)^n$

$0.92P\left(1 - \frac{12}{100}\right)^1$

$0.92P(0.88)$

$A = 0.8096P$

$0.8096P = 121{,}440$ M1

$P = \frac{121440}{0.8096}$

$P = $ shs. 150,000 A1

5. Rationalize the denominator and simplify (3 marks)

$$\frac{\sqrt{3} + 2\sqrt{5}}{\sqrt{5} - \sqrt{3}}$$

$\frac{(\sqrt{3} + 2\sqrt{5})(\sqrt{5} + \sqrt{3})}{(\sqrt{5} - \sqrt{3})(\sqrt{5} + \sqrt{3})}$

Numerator

$\sqrt{3}(\sqrt{5} + \sqrt{3}) + 2\sqrt{5}(\sqrt{5} + \sqrt{3})$

$\sqrt{15} + 3 + 10 + 2\sqrt{15}$ M1

$13 + 3\sqrt{15}$

Denominator

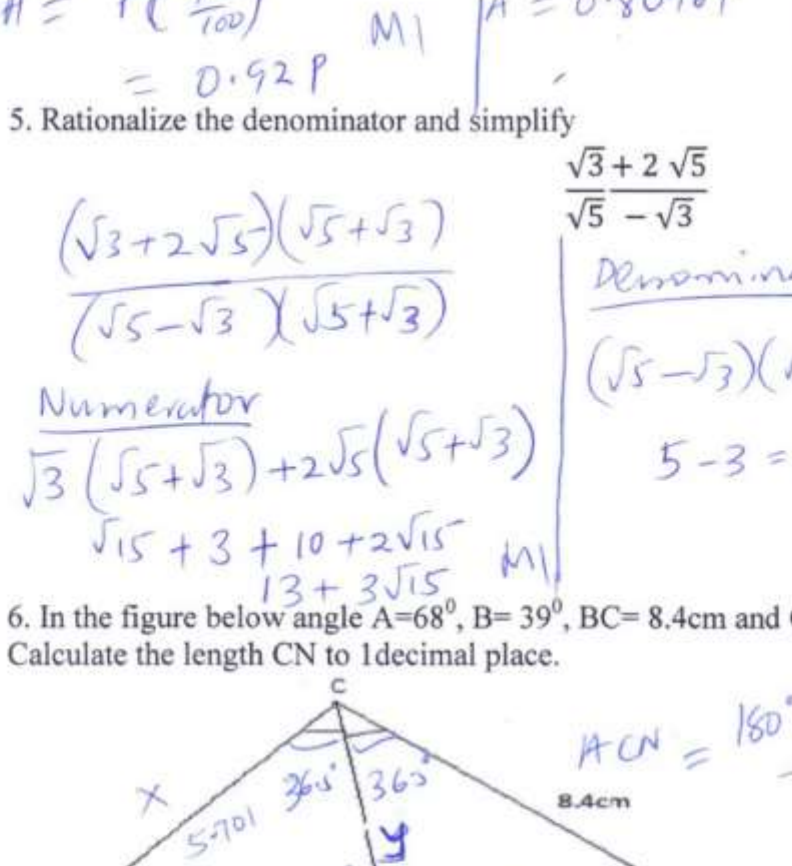
$(\sqrt{5} - \sqrt{3})(\sqrt{5} + \sqrt{3})$

$5 - 3 = 2$ M1

$\frac{13 + 3\sqrt{15}}{2}$

$6.5 + 1.5\sqrt{15}$ A1

6. In the figure below angle A=68^0, B= 39^0, BC= 8.4cm and CN is the bisector of angle ACB. Calculate the length CN to 1decimal place. (3 marks)



$ACN = \frac{180° - (68 + 39)}{2} = 36.5°$

$\frac{8.4}{\sin 68°} = \frac{x}{\sin 39°}$

$x = \frac{8.4}{\sin 68} \times \sin 39° = 5.701$

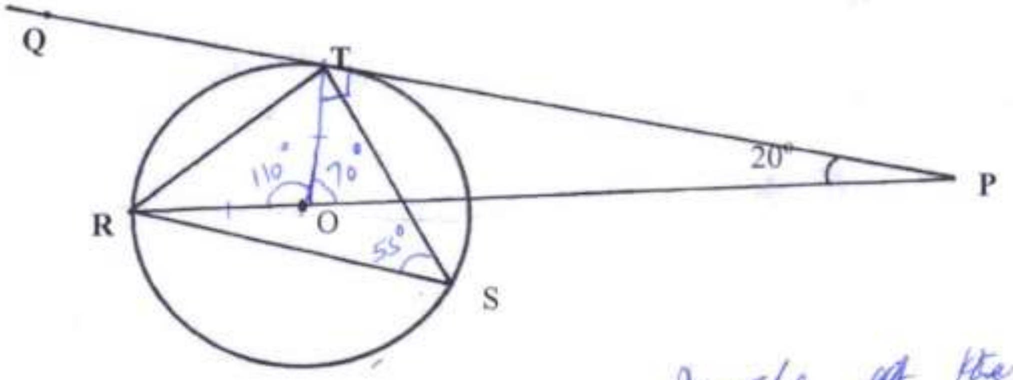
$\frac{CN}{\sin 68} = \frac{5.701}{\sin 75.5}$

$CN = \frac{5.701}{\sin 75.5} \times \sin 68°$

$CN = 5.459$

$CN = 5.5$ (1 dp) cm

7. In the figure below R, T and S are points on a circle centre O. PQ is a tangent to the circle at T, POR is a straight line and $\angle$ QPR =20^o. Find the size of $\angle$ RST (3marks)



$RST = 55^o$

Angle at the subtended by a chord at the centre is twice what it subtends at the circumference

8. Use binomial expansion to find the value of $(1.02)^5$ correct to 3 decimal place. (4 marks)

$(1+x)^5 = 1^5(x)^0 + 1^4(x)^1 + 1^3(x)^2 + 1^2(x)^3 + (1)^1(x)^4 + (1)^0(x)^5$

$x = 0.02$ $\quad 1 + x + x^2 + x^3 + x^4 + x^5$

$1 \quad 5 \quad 10 \quad 10 \quad 5 \quad 1$

$1 + 5x + 10x^2 + 10x^3 + 5x^4 + x^5$ M1

$1 + 5(0.02) + 10(0.02)^2 + 10(0.02)^3 + 5(0.02)^4 + (0.02)^5$ M1

$1 + 0.1 + 0.004 + 0.00008 + 0.0000008 + 0.0000000032$ M1

$= 1.1040808 \Rightarrow 1.104$ 3 dp A1

9. Make x the subject of the equation (3 marks)

$$\frac{t}{s} = \frac{b}{\sqrt{x-4}}$$

$\left(\frac{t}{s}\right)^2 = \left(\frac{b}{\sqrt{x-4}}\right)^2$ M1

$\frac{t^2}{s^2} = \frac{b^2}{(x-4)}$

$t^2(x-4) = s^2b^2$

$t^2x - 4t^2 = s^2b^2$

$t^2x = s^2b^2 + 4t^2$ M1

$x = \frac{s^2b^2 + 4t^2}{t^2}$ A1

10. The equation of the circle is given by $x^2 + y^2 + 8x - 2y - 1 = 0$. Determine the radius and the centre of the circle. (3marks)

$x^2 + 8x + \boxed{16} + y^2 - 2y + \boxed{1} = 1 + 16 + 1$ M1

$(x+4)^2 + (y-1)^2 = 18$ M1

Centre $(-4, 1)$ radius $= \sqrt{18} = 4.243$ units A1

or radius $\sqrt{18} = 3\sqrt{2}$

11. Given that the minor arc of a circle subtends an angle of 140^0 at the centre of a circle of radii 3.5cm. Calculate the area of the major segment correct to 4 significant figures (3 marks)

$360° - 140° = 220°$

M1

$\frac{220}{360} \times \frac{22}{7} \times 3.5^2 = 23.52778$ M1

$= 23.53\,cm^2$ A1

12. Given that the matrix $\begin{pmatrix} x & -3 \\ 0 & x-1 \end{pmatrix}$ is a singular matrix, find the values of x. (3marks)

$x(x-1) - 0 = 0$

$x^2 - 1 = 0$ M1

$x^2 = 1$

$x = \sqrt{1}$ M1

$x = \pm 1$

$x = 1$ or $x = -1$ A1

13. The mass of a mixture A of peas and millet is 72 kg. The ratio of peas to millet is 3:5 respectively;

(a)Find the mass of millet in the mixture. (1mark)

Total mass $= 72$kg

peas : millet

3 : 5

a) millet $= \frac{5}{8} \times 72 = 45$kg B1

(b)A second mixture of B of peas and millet of mass 98 kg is mixed with A. The final ratio of peas to millet is 8:9 respectively. Find the ratio of peas to millet in B (2marks)

a

A + B mixture $= 72 + 98 = 170$kg

Mass of peas in A $= \frac{3}{8} \times 72 = 27$kg.

A and B 8:9 of 170kg

peas $= \frac{8}{17} \times 170 = 80$kg

millet $\frac{9}{17} \times 170 = 90$kg

In mixture

B; millet $= 90 - 45$

$= 45$kg

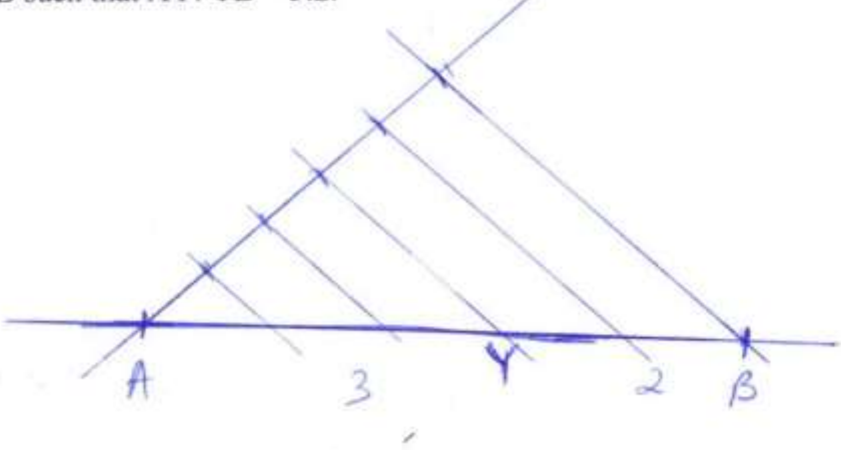
In mixture B peas

$80 - 27$

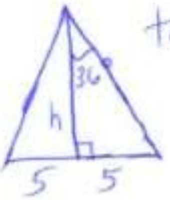
$= 53$

Ratio $= 53 : 45$

14. Draw a line AB= 8cm long. Divide the line proportionally into5 equal parts. Locate a point Y on the line AB such that AY: YB = 3:2. (3 marks)



15. A solid prism is made of a pentagonal cross section of sides 10cm. If the prism is 30cm long calculate area of the cross section hence the volume of the prism (3 marks)



$\tan 36° = \frac{5}{h}$

$h = \frac{5}{\tan 36°}$

$h = 6.882$

Cross sectional area

$5\left(\frac{1}{2} \times 6.882 \times 10\right) = 172.05\ cm^2$

Volume

Cross sectional area x length

$V = 172.05 \times 30$

$= 5161.5 cm^3$

16. Given that X = 2i + j -2k, y = -3i + 4j – k and z= 5i + 3j + 2k and that p= 3x – y + 2z, find the magnitude of vector p to 3 significant figures. (3marks)

$P = 3\begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix} - 1\begin{pmatrix} -3 \\ 4 \\ -1 \end{pmatrix} + 2\begin{pmatrix} 5 \\ 3 \\ 2 \end{pmatrix}$ M1

$\begin{pmatrix} 6 \\ 3 \\ -6 \end{pmatrix} + \begin{pmatrix} 3 \\ -4 \\ 1 \end{pmatrix} + \begin{pmatrix} 10 \\ 6 \\ 4 \end{pmatrix} = \begin{pmatrix} 19 \\ 5 \\ -1 \end{pmatrix}$

$|P| = \sqrt{(19)^2 + (5)^2 + (-1)^2}$ M1

$= \sqrt{387}$

$= 19.7$ A1

SECTION II (50 Marks) Answer any five questions in this section

17. The masses in kilograms of patients who attended a clinic on a certain day were recorded as:

38 52 46 48 60 59 62 73 49 54 49 41 57 58 69 72 60 58 42 41
79 62 58 67 54 60 65 61 48 47 69 59 70 52 63 58 59 49 51 44
67 49 51 58 54 59 39 59 54 52

a) starting with class 35-39, make a frequency distribution table for the data indicating the class and frequency. (3 marks)

Class	Tally	Freq.	x	fx	cf
35-39	II	2	37	74	2
40-44	IIII	4	42	168	6
45-49	~~IIII~~ III	8	47	376	14
50-54	~~IIII~~ IIII	9	52	468	23
55-59	~~IIII~~ ~~IIII~~ I	11	57	627	34
60-64	~~IIII~~ II	7	62	434	41
65-69	~~IIII~~	5	67	335	46
70-74	III	3	72	216	49
75-79	I	1	77	77	50
		$\Sigma f = 50$		$\Sigma fx = 2775$	

b) state the modal class (1 mark)

55-59

c) Calculate the mean mass (3 marks)

$$\text{Mean} = \Sigma fx = \frac{2775}{50}$$

$$= \underline{\underline{55.5}}$$

d) Calculate the median mass (3 marks)

$$54.5 + \frac{2}{11} \times 5 = \underline{\underline{55.41}}$$

18. The income tax rates of a certain year were as shown below:

Monthly taxable income in Ksh	Tax rate in %
0-9680	10
9681-18800	15
8801-27920	20
27921- 37040	25
37041 and above	30

In that year, Sayao monthly earnings were as follows; basic salary Ksh. 30 000, house allowance Ksh.15 000, and medical allowance of Ksh 3,500. He is entitled to a monthly tax relief of Ksh. 1056.

a) Calculate Sayao's taxable income (2 marks)

$30,000 + 15,000 + 3500 = \underline{48,500}$

b) Calculate his P.A.Y.E (5 marks)

$9680 \times \frac{10}{100} = 968$

$9120 \times \frac{15}{100} = 1368$

$9120 \times \frac{20}{100} = 1824$

$9120 \times \frac{25}{100} = 2280$

$11460 \times \frac{30}{100} = 3438$

Gross tax = 9878

P.A.Y.E = 9878 − 1056

= Ksh. 8822 p.m

c) A part from P.A.Y.E, other deductions is education insurance policy Ksh. 1500 and Ksh 2500 as cooperative shares. Find his net income at end of the month. (3 marks)

Total deductions = 8822 + 2500 + 1500

= 12,822.

Net income = 48,500 − 12822 = $\underline{35,678}$ Ksh.

19. A Quantity P varies partly as the square of m and partly as n. When p= 3.8, m = 2 and n = -3, When p = - 0.2, m = 3 and n= 2.

a) Find

i) The equation that connects p, m and n (4marks)

$P = xm^2 + yn$

$3.8 = 4x - 3y$

$-0.2 = 9x + 2y$

$7.6 = 8x - 6y$

$-0.6 = 27x + 6y$ +

$7 = 35x$

$x = \frac{7}{35} = \frac{1}{5} = 0.2$

$3.8 = 0.8 - 3y$

$3 = -3y$

$y = -1$

$P = 0.2m^2 - n$

ii) The value of p when m = 10 and n = 4 (1mark)

$P = 0.2 \times 100 - 4$

$= 20 - 4$

$= 16$

b) Express m in terms of p and n (2marks)

$P = 0.2m^2 - n$

$0.2m^2 = P + n$

$m^2 = \frac{P+n}{0.2}$

$M = \sqrt{\frac{P+n}{0.2}}$

$M = \pm\sqrt{\frac{P+n}{0.2}}$

c) If P and n are each increased by 10%, find the percentage increase in m correct to 2 decimal place. (3marks)

$M_0 = \sqrt{\frac{P+n}{0.2}} = D$

$M_1 = \sqrt{\frac{1.1P + 1.1n}{0.2}}$

$M_1 = \sqrt{\frac{1.1(P+n)}{0.2}}$

$M_0 = 2.236\sqrt{7}$

$= 5.916$

$M_1 = 2.236\sqrt{1.1(P+n)}$

let $P = 5$ and $n = 2$

$M_1 = 2.236\sqrt{7(1.1)}$

$= 6.2046$

% increase

$= \frac{6.2046 - 5.916}{5.916} \times 100$

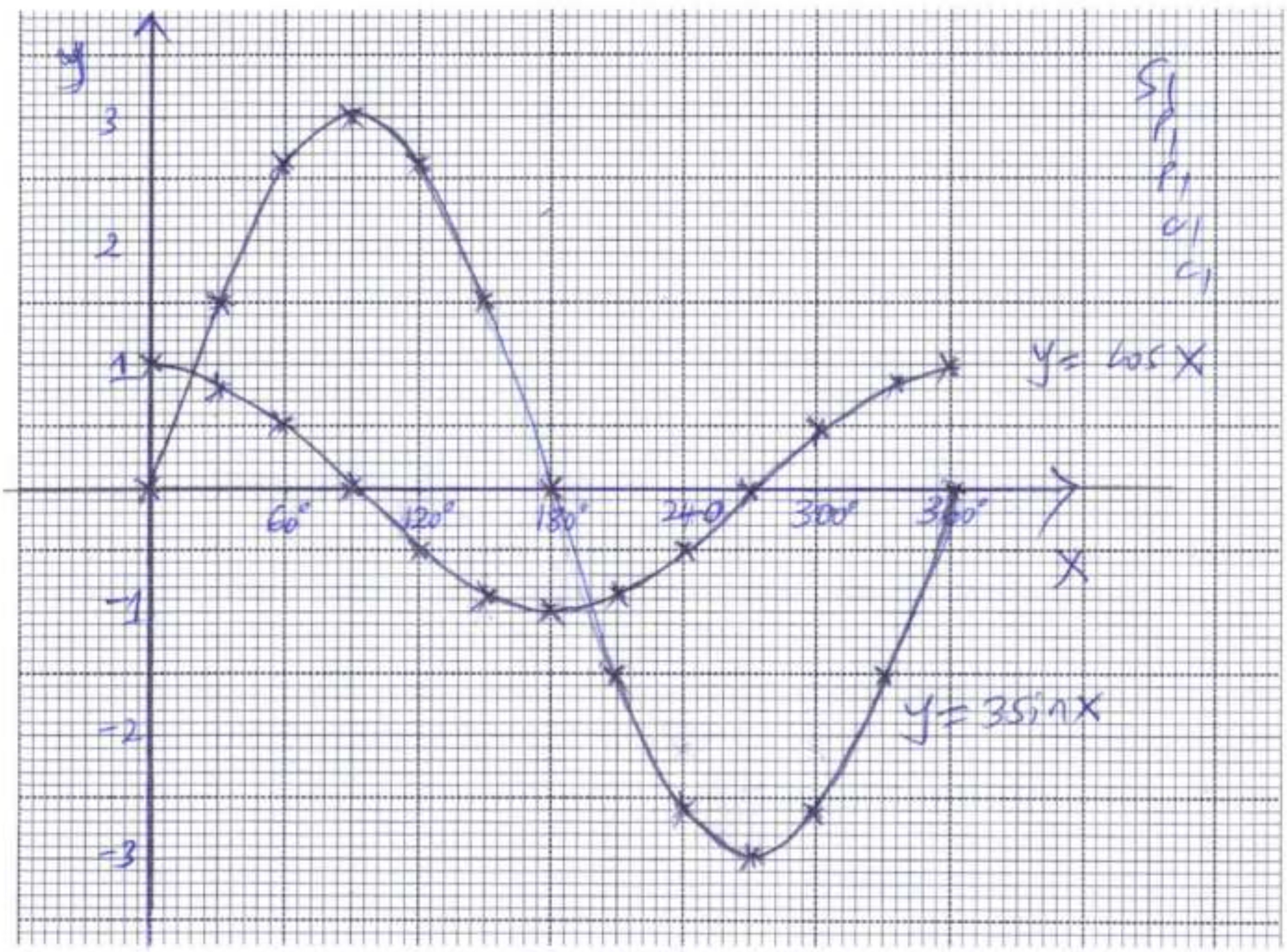
4.8782

$= 4.88\%$ (2dp)

20.a) Complete the table below by filling in the blank spaces (2 marks B1 B1

x	0^0	30^0	60^0	90^0	120^0	150^0	180^0	210^0	240^0	270^0	300^0	330^0	360^0
y=Cos x	1.00	0.87	0.50	0.00	-0.50	-0.87	-1.00	-0.87	-0.50	0.00	0.50	0.87	1.00
y=3sinx	0.00	1.50	2.60	3.00	2.60	1.50	0.00	-1.50	-2.60	3.00	-2.60	-1.50	0.00

b) using the scale 1cm to represent 30^0 on the x-axis and 2 cm to represent 1 unit on the vertical axis, draw on the graphs of cosx and 3sinx (5 marks)



c) use your graph to solve the equation cosx =3sinx (2 marks)

$x = 18^0, 198^0$ B1 B1

d) What is the difference in the values of y=cosx and y=3sinx at x=120^0 (1 mark)

$2.60 - -0.5 = 3.1$ B1

21. The 5[th] term of an AP is 16 and the 12[th] term is 37.
Find;

i) The first term and the common difference (3 marks)

$T_n = a+(n-1)d$

$T_5 \Rightarrow a+4d=16$ M1

$T_{12} \Rightarrow a+11d=37$

$7d = 21$ A1

$d = 3$

$a+4(3)=16$

$a+12=16$

$\underline{a=4}$ B1

ii) The sum of the first 21 terms (2 marks)

$S_{21} = \frac{21}{2}\left[2(4)+20(3)\right]$ M1

$= \underline{714}$ A1

b) The second, fourth and the seventh term of an AP are the first 3 consecutive terms of a GP. If the common difference of the AP is 2.
Find:

i) The common ratio of the GP (3 marks)

$a+d,\ a+3d,\ a+6d$

$a+2,\ a+6,\ a+12$

$\frac{a+6}{a+2} = \frac{a+12}{a+6}$

$(a+6)^2 = (a+2)(a+12)$

$a^2+12a+36 = a^2+14a+24$

$12 = 2a$

$a = 6$

common ratio

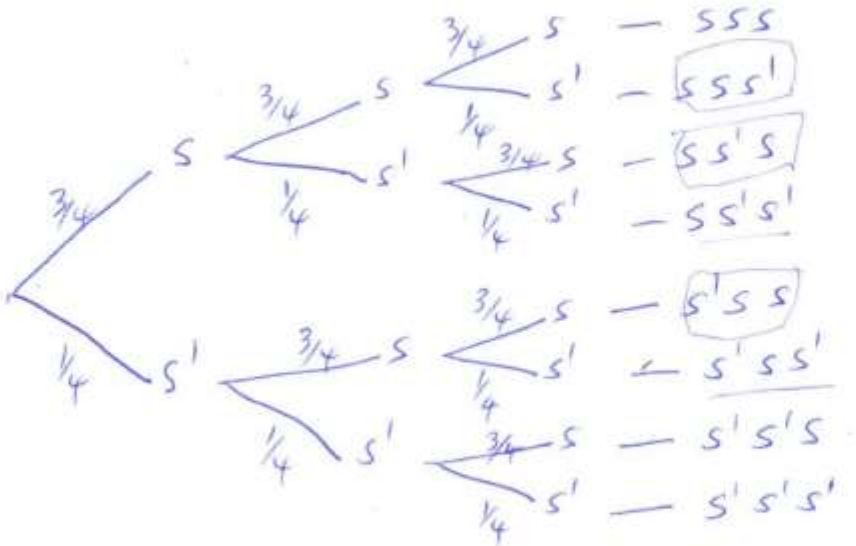
$r = \frac{12}{8} = \underline{1\tfrac{1}{2}}$

ii) The sum of the first 8 terms of the GP (2 marks)

$S_8 = \frac{8\left(\left(\frac{3}{2}\right)^8-1\right)}{1\frac{1}{2}-1}$ M1 $= \frac{197.03}{\frac{1}{2}} = \underline{394.063}$ A1

22. In driving to work, John has to pass through three sets of traffic lights. The probability that he will have to stop at any of the lights is ¾

(a) Draw a tree diagram to represent the above information. (2 marks)



(b) Using the diagram, determine the probability that on any one journey, he will have to stop at:

(i) All the three sets. (2 marks)

$$P(SSS) = \tfrac{3}{4} \times \tfrac{3}{4} \times \tfrac{3}{4} = \underline{\underline{\tfrac{27}{64}}}$$

(ii) Only one of the sets (2 marks)

$$P(SS'S') \text{ or } P(S'SS') \text{ or } P(S'S'S)$$

$$\left(\tfrac{3}{4} \times \tfrac{1}{4} \times \tfrac{1}{4}\right) + \left(\tfrac{1}{4} \times \tfrac{3}{4} \times \tfrac{1}{4}\right) + \left(\tfrac{1}{4} \times \tfrac{1}{4} \times \tfrac{3}{4}\right) = \tfrac{3}{64} \times 3 = \underline{\underline{\tfrac{9}{64}}}$$

(iii) Only two of the sets (2 marks)

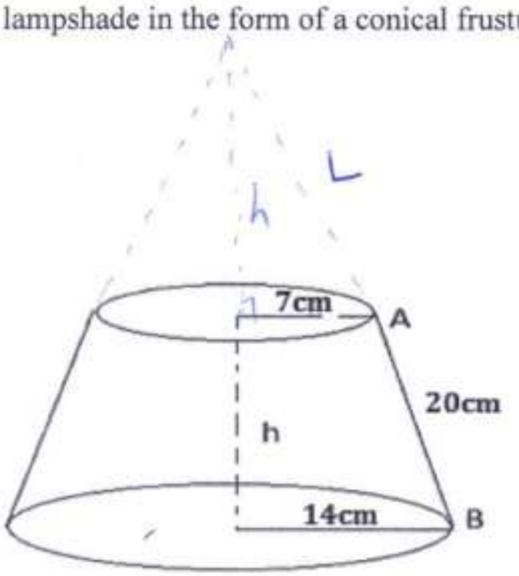
$$P(SSS') \text{ or } P(SS'S) \text{ or } P(S'SS)$$

$$\left(\tfrac{3}{4} \times \tfrac{3}{4} \times \tfrac{1}{4}\right) + \left(\tfrac{3}{4} \times \tfrac{1}{4} \times \tfrac{3}{4}\right) + \left(\tfrac{1}{4} \times \tfrac{3}{4} \times \tfrac{3}{4}\right) = \left(\tfrac{9}{64}\right)3 = \underline{\underline{\tfrac{27}{64}}}$$

(iv) None of the sets. (2 marks)

$$P(S'S'S') = \tfrac{1}{4} \times \tfrac{1}{4} \times \tfrac{1}{4} = \underline{\underline{\tfrac{1}{64}}}$$

23. The figure below shows a lampshade in the form of a conical frustum



The top and bottom radii are 7cm and 14cm respectively. The slant height AB is 20cm.
Calculate:

a) The slant height of the original cone correct to two decimal places (2 marks)

$\frac{L+20}{L} = \frac{14}{7}$ | $14L = 7L + 140$, $7L = 140$, $L = 20$ | slant height $= 20 + 20 = \underline{40cm}$

b) The height h, of the lampshade (2 marks)

$40^2 = H^2 + 14^2$

$1600 = H^2 + 196$

$1600 - 196 = H^2$

$H^2 = 1404$

$H = 37.47$

$20^2 = x^2 + 49$

$400 - 49 = x^2$

$x = 18.73$

$h = \underline{18.74cm}$

$37.47 - 18.73 \Rightarrow \underline{18.74}$

c) The curved surface area of the lampshade (3 marks)

$\pi RL - \pi rl$

$\frac{22}{7} \times 14 \times 40 - \frac{22}{7} \times 7 \times 20$

$1760 - 440 = 1320cm^2$

d) The volume of the lampshade correct to 4 significant figures (3 marks)

$\frac{1}{3}\pi R^2 H - \frac{1}{3}\pi r^2 h$

$\frac{1}{3} \times \frac{22}{7} \times 14^2 \times 37.47 - \frac{1}{3} \times \frac{22}{7} \times 7^2 \times 18.73 =$

$= 7693.84 - 961.47 = \underline{6732.37 cm^3}$

24. Gary bought 5 tins of plums and 3 tins of peaches from a supermarket for Ksh.75, while Mike bought 3 tins of plums and 5 tins of peaches for Ksh.77

a) Set up the simultaneous equations which represent the given information (2 marks)

let plums be x and peaches be y

$5x + 3y = 75$

$3x + 5y = 77$

b) Write down the matrix equation (2 marks)

$$\begin{pmatrix} 5 & 3 \\ 3 & 5 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 75 \\ 77 \end{pmatrix}$$

c) Using the matrix method, find the cost of

i) 4 tins of plums (5 marks)

Inverse of $\begin{pmatrix} 5 & 3 \\ 3 & 5 \end{pmatrix}$

$$\frac{1}{16}\begin{pmatrix} 5 & -3 \\ -3 & 5 \end{pmatrix} = \begin{pmatrix} \frac{5}{16} & \frac{-3}{16} \\ \frac{-3}{16} & \frac{5}{16} \end{pmatrix}$$

$$\begin{pmatrix} \frac{5}{16} & \frac{-3}{16} \\ \frac{-3}{16} & \frac{5}{16} \end{pmatrix}\begin{pmatrix} 5 & 3 \\ 3 & 5 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \frac{5}{16} & \frac{-3}{16} \\ \frac{-3}{16} & \frac{5}{16} \end{pmatrix}\begin{pmatrix} 75 \\ 77 \end{pmatrix}$$

$A^{-1}A = I$

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \frac{5}{16} \times 75 - \frac{3}{16} \times 77 \\ -\frac{3}{16} \times 75 + \frac{5}{16} \times 77 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 9 \\ 10 \end{pmatrix}$$

ii) 2 tins of peaches (2 marks)

$IA = A$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 9 \\ 10 \end{pmatrix} \quad x = 9, \; y = 10$$

2 tins of peaches

$10 \times 2 = \underline{\underline{\text{Ksh. } 20}}$

4 tins of plums $= 9 \times 4 = \underline{\underline{\text{Ksh. } 36}}$