GATUNDU SOUTH FORM FOUR EVALUATION EXAMINATION 2018

MATHEMATICS PAPER 2 ALT. A.

MARKING SCHEME

SECTION I

1. $\frac{6}{\sqrt{5}+\sqrt{3}} - \frac{4}{\sqrt{5}-\sqrt{3}}$

$= \frac{6(\sqrt{5}-\sqrt{3}) - 4(\sqrt{5}+\sqrt{3})}{(\sqrt{5}+\sqrt{3})(\sqrt{5}-\sqrt{3})}$ m1

$= \frac{6\sqrt{5}-6\sqrt{3}-4\sqrt{5}-4\sqrt{3}}{5-\sqrt{15}+\sqrt{15}-3}$

$= \frac{2\sqrt{5}-10\sqrt{3}}{5-3}$ m1

$= \frac{2\sqrt{5}-10\sqrt{3}}{2} = \sqrt{5}-5\sqrt{3}$ A1

2. $3(3-x) < 5x-9 \leq 2x+8$

$3(3-x) < 5x-9$ m1

$9-3x < 5x-9$

$9+9 < 5x+3x$

$\frac{18}{8} < \frac{8x}{8}$

$2\frac{1}{4} < x.$

$5x-9 \leq 2x+8$ m1

$5x-2x \leq 9+8$

$\frac{3x}{3} \leq \frac{17}{3}$

$x \leq 5\frac{2}{3}$

$2\frac{1}{4} < x \leq 5\frac{2}{3}$ m1

Integral values are 3, 4, 5 A1

3.	$\log\left(\frac{15^2}{x}\right) = \log 5(x-4)$ $\frac{15^2}{x} = 5x - 20$ $225 = 5x^2 - 20x$ $5x^2 - 20x - 225 = 0$. $x^2 - 4x - 45 = 0$ $(x-9)(x+5) = 0$ $x = 9$ or $x = -5$ So $x = 9$	M1 M1 A1
4.	$P(W) = \frac{5}{10}$, $P(B) = \frac{3}{9}$, $P(G) = \frac{2}{8}$. $= \frac{5}{10} \times \frac{3}{9} \times \frac{2}{8}$ $= \frac{1}{24}$	 M1 A1
5.	$= (2x)^4 - 4(2x)^3(\sqrt{3}) + 6(2x)^2(\sqrt{3})^2 - 4(2x)^1(\sqrt{3})^3 + (\sqrt{3})^4$ $= 16x^4 - 32\sqrt{3}x^3 + 72x^2 - 24\sqrt{3}x + 9$	M1 M1 A1
6.	$a - b = \sqrt{b^2 + c^2}$ $(a-b)^2 = b^2 + c^2$ $a^2 - 2ab + b^2 = b^2 + c^2$ $a^2 - 2ab + b^2 - b^2 = c^2$ $a^2 - 2ab = c^2$ $\pm\sqrt{a^2 - 2ab} = c$	 M1 M1 A1
7.	Amount borrowed (P) = $28000 - 8000$ $= 20000$ New principal (A) $= 15 \times 2000$ $=$ sh. 30000	 M1

$30000 = 20000\left(1+\frac{r}{100}\right)^{15}$

$\frac{30000}{20000} = \left(1+\frac{r}{100}\right)^{15}$ M1

$1.5 = \left(1+\frac{r}{100}\right)^{15}$.

$1.02774 = 1+\frac{r}{100}$

$r = 2.74\%$. A1

8. $\frac{2}{2}\sin\left(x-\frac{\pi}{6}\right)^c = -\frac{\sqrt{3}}{2}$

$\sin\left(x-\frac{\pi}{6}\right)^c = -\frac{\sqrt{3}}{2}$ M1

3rd, 4th quad. $\frac{\pi}{6} = 30°$

$\sin^{-1}\frac{\sqrt{3}}{2} = 60°$

So $x - 30° = 240°\left(\frac{4}{3}\pi^c\right)$

$x = 270°\left(\frac{3}{2}\pi^c\right)$ M1

& $x - 30° = 300°\left(\frac{5}{3}\pi^c\right)$

$x = 300° + 30° = 330°\left(\frac{11}{6}\pi^c\right)$

Ans $x = \frac{3}{2}\pi^c = 1\frac{1}{2}\pi^c$ and $\frac{11}{6}\pi^c = 1\frac{5}{6}\pi^c$ A1

9. $\text{Det} = (4\times2) - (5\times1)$

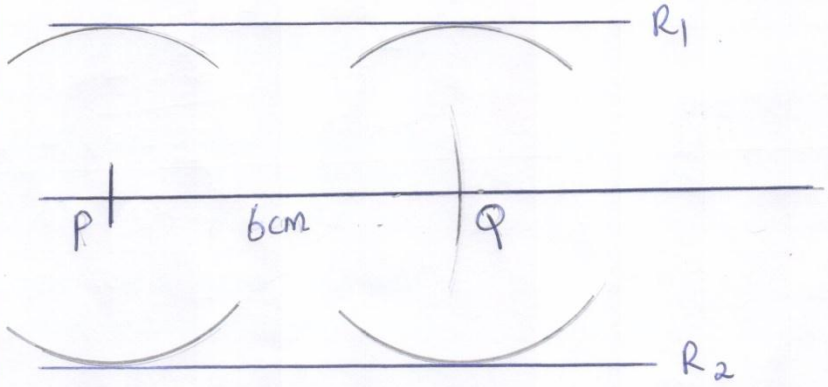
$= 8 - 5 = 3$. m1

Area of image $= 3 \times 7$ m1

$= 21\,cm^2$. A1

10. $A = \frac{1}{2}\times6\times h = 9$

$h = 3\,cm$ m1



B1 for R1

B1 for R2

11a) $CD \times DE = AD \times DF$.

$4x = 5 \times 3.6$.

$x = \frac{5 \times 3.6}{4} = 4.5$ — B1

b) $CB \times BE = AB^2$.

$16 \times 7.5 = AB^2$ — M1

$120 = AB^2$

$10.95\text{cm} = AB$ — A1

12a) $12.5 + 0.05 = 12.55$

$9.23 + 0.005 = 9.225$ — M1

Maximum difference $= 12.55 - 9.225$

$= 3.325$ m. — A1

b) A.E in difference $= 0.05 + 0.005$

$= 0.055$.

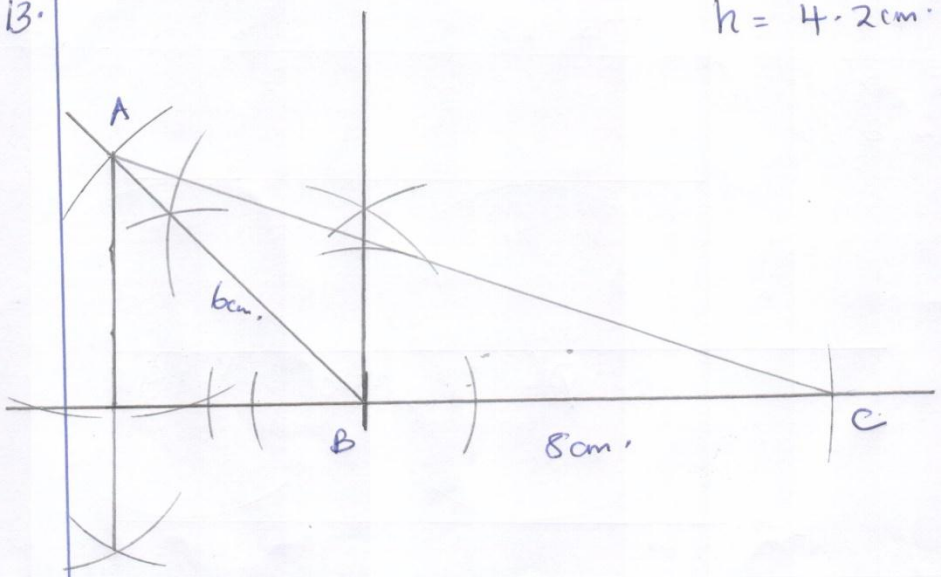
Actual difference $= 12.5 - 9.23$

$= 3.27$.

% error $= \frac{0.055}{3.27} \times 100$ — M1

$= 1.682\%$. — A1

13. $h = 4.2$cm. — B1



135° — B1

Triangle — B1

Dropping perpendicular — B1

14. Mean $= \frac{45+52+54+65+57+57+62+66}{8} = \frac{448}{8} = 56$

d	-11	-4	-2	-1	1	1	6	10
d^2	121	16	4	1	1	1	36	100

M1

Variance $= \frac{\Sigma d^2}{n} = \frac{280}{8} = 35$ M1

S.D $= \sqrt{35} = 5.9$ A1

15a) $(0-3)^2 + (0-4)^2 = 25$

$-3^2 + -4^2 = 25$

$9 + 16 = 25$

$25 = 25$

B1 Showing

b) $(x-3)^2 + (0-4)^2 = 25$ M1

$x^2 - 6x + 9 + 16 = 25$

$x^2 - 6x = 0$

$x(x-6) = 0$.

$x = 0$ or $x - 6 = 0$

$x = 6$.

$(6, 0)$ A1

16. $P_1 = KQR^2$

$P_2 = 1.5 \times 0.8^2 KQR^2$

$P_2 = 0.96 KQR^2$ B1

$\frac{P_2 - P_1}{P_2} \times 100$

$\frac{0.96KQR^2 - KQR^2}{KQR^2} \times 100$ M1

$= \frac{0.96 - 1}{1} \times 100$

$= -4\%$.

P decreases by 4%. A1

SECTION II

17a) $180° - (90° + 72°) = 18° = \angle YXW$ — B1
$\angle XYW = 90°$ (subtended by diameter. — B1
Sum of angles in a triangle XYW

b) $\angle YZX = 72° = YXZ$ (angles subtended by chord) — B1
$180° - (72° + 72°) = 36° = \angle XYZ$ — B1

c) $\angle OYZ = 18°$ (base angles of isoceles triangle) — B1 B1

d) $\angle WTZ = 180° - (36° + 18°) = 126°$ — B1
Angle sum of a triangle — B1

e) $\angle WZV = 54°$ — B1
Angles in alternate segments. — B1

18 a) $a = 2, \quad n = 8, \quad S_n = 156.$

i) $S_n = \frac{n}{2}(2a + (n-1)d)$ — M1

$156 = \frac{8}{2}(2 \times 2 + (8-1)d).$

$156 = 4(4 + 7d)$

$156 = 16 + 28d.$

$156 - 16 = 28d$

$140 = 28d$

$5 = d$ — A1

(ii) $416 = \frac{n}{2}(2 \times 2 + (n-1) \times 5)$ — M1

$416 = \frac{n}{2}(4 + 5n - 5)$

$416 \times 2 = n(5n - 1)$

$832 = 5n^2 - n.$

$0 = 5n^2 - n - 832.$

$$n = -\frac{(-1) \pm \sqrt{(-1)^2 - 4(5)832}}{2 \times 5}$$

$$= \frac{1 \pm \sqrt{16641}}{10}$$

$$= \frac{1 \pm 129}{10} \quad \text{So } n = 13 \text{ or } -12.8$$

So $n = 13$. A1

b) i) $a+2d,\ a+4d,\ a+7d$.

$$\frac{a+4d}{a+2d} = \frac{a+7d}{a+4d}.$$ M1

$$(a+4d)(a+4d) = (a+2d)(a+7d)$$ M1

$$a^2 + 8ad + 16d^2 = a^2 + 9ad + 14d^2$$

$$2d^2 = ad.$$ M1

$$2d. = a$$

$$2 \times 3 = a$$

$$6 = a$$ A1

ii) $n = 9,\ a = a + 2d = 12$.

$$S_n = \frac{a(r^n - 1)}{r - 1} = \frac{12(1.5^9 - 1)}{1.5 - 1}$$ M1

$$= \frac{449.3}{0.5}$$

$$= 898.6$$ A1

19.

a)

θ	0°	30°	60°	90°	120°	150°	180°	210°	240°	270°	300°	330°	360°
$2\sin\theta$	0.00	1.00	1.73	2.00	1.73	1.00	0.00	−1.00	−1.73	−2.00	−1.73	−1.00	0.00
$2\sin(2\theta+60^\circ)$	1.73	1.73	0.0	−1.73	−1.73	0.00	1.73	1.73	0.00	−1.73	−1.73	0.00	1.73

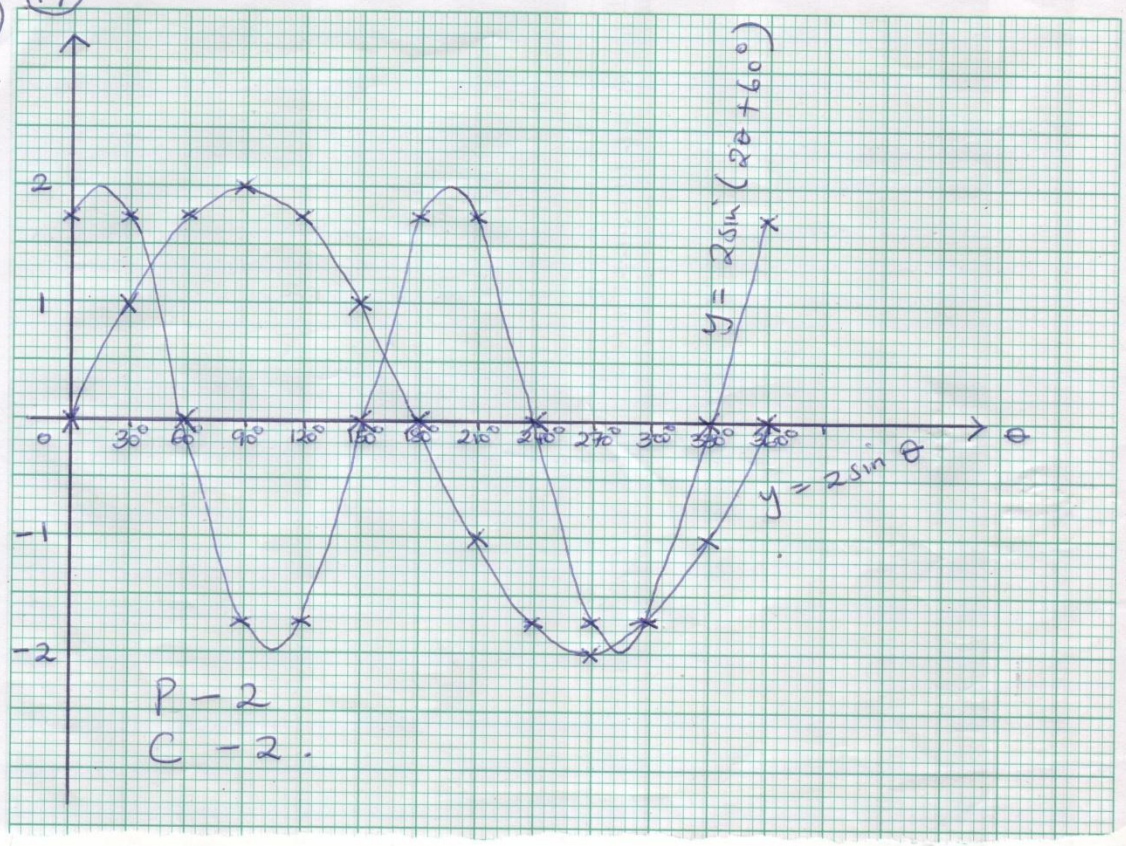
B_2

c) Period of $y = 2\sin(2\theta + 60^\circ) = 180^\circ$ B1

i) Amplitude $= 2$ B1

ii) $\theta = 40^\circ, 160^\circ, 280^\circ, 300^\circ$ B1 for all correct

b) (19)


2
1
0
-1
-2
30°
60°
90°
120°
150°
180°
210°
240°
270°
300°
330°
360°
θ
y = 2sin θ
y = 2sin (2θ + 60°)

P – 2

C – 2.

d)	Translation vector $= \begin{pmatrix} -60^\circ \\ 0 \end{pmatrix}$	B1
20a (i)	B(33°S, 129° W	B1
(ii)	Dist $= \frac{180^\circ}{360^\circ} \times 2 \times \frac{22}{7} \times 6370$	M1
	$= 20020$ Km.	A1
b)	$45^\circ - 32 = 13^\circ$	B1
	Dist $= \frac{13^\circ}{360^\circ} \times 2 \times \frac{22}{7} \times 6370$	M1
	$= 1445.87$	
	$= 1446$ Km.	A1
c)	$D = S \times T$	
	$= 700 \times 3 = 2100$ Km.	B1
	$2100 = \frac{\theta}{360^\circ} \times 2 \times \frac{22}{7} \times 6370.$	M1
	$\frac{2100 \times 360^\circ \times 7}{2 \times 22 \times 6370} = \theta$	M1
	$18.88^\circ = \theta$	
	$70^\circ - 18.88^\circ = 51.12^\circ$	
	Q(51.12°N, 32°E)	A1
21a (i)	$\frac{1}{6} + \frac{1}{8} = \frac{7}{24}$	M1
	$\frac{7}{24} \times 3 = \frac{7}{8}$	A1
ii)	$\frac{1}{8} - \frac{1}{4} = -\frac{1}{8}$ (out)	M1
	In 1 hr empty $\frac{1}{8}$.	M1
	? " $\frac{7}{8}$.	
	7 hours to empty	A1

b) (i) $\frac{35m+80B}{m+B} = 66$ — M1

$66m + 66B = 35m + 80B$

$66m - 35m = 80B - 66B$ — M1

$31m = 14B$

$\frac{m}{B} = \frac{14}{31}$

$m : B = 14 : 31$ — A1

(ii) $= \frac{31}{45} \times 225$ — m1

$= 155\ kg.$ — A1.

22. a) i) Taxable income $= 26000 + \left(\frac{15}{100} \times 26000 - 800\right)$ — M1

$= sh.\ 29100$ — A1

ii) $\frac{10}{100} \times 9680 = 968$

$\frac{15}{100} \times 9120 = 1368$ — M1

$\frac{20}{100} \times 9120 = 1824.$

$29100 - 27920 = 1180.$ — m1

$\frac{25}{100} \times 1180 = 295.$

Total tax $= 968 + 1368 + 1824 + 295$ — M1

$= sh\ 3782$

Tax paid $= 3782 - 1162$ — A1

$= sh.\ 2620.$

iii) Net salary $= 29100 - (2620 + 800)$ — B1

$= sh.\ 25680$

b) $16 \div 4 = 4\% = r$, $1\frac{1}{2} \times 4 = 6 = n$.

$P = 10,000$

$A = P\left(1 + \frac{r}{100}\right)^n$ — M1

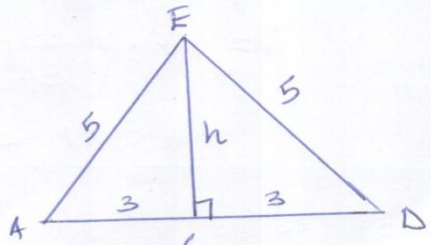
$= 10\,000\left(1 + \frac{4}{100}\right)^6$. — M1

$= 10\,000(1.04)^6$

$= 12\,653.19$

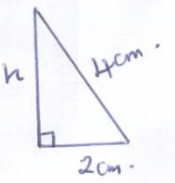
$= Sh.12653$ — A1

23. a)



$h^2 = 5^2 - 3^2$

$= 4cm$. — M1



$h^2 = 4^2 - 2^2$

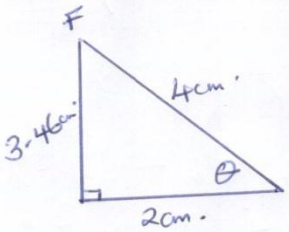
$= 12$

$h = \sqrt{12}$

$= 3.4641$

$= 3.46cm$. — A1

b)i)

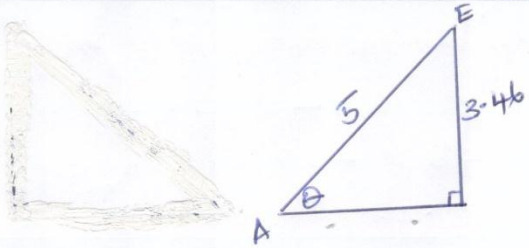


$\sin\theta = \frac{O}{H} = \frac{3.46}{4} = 0.865$ — B1

$\sin^{-1} 0.865 = 59.88°$

$= 60°$. — A1

ii)

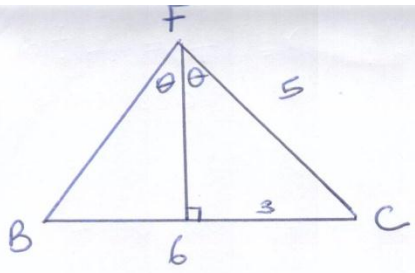


$\sin\theta = \frac{3.46}{5} = 0.692$ — M1

$\sin^{-1} 0.692 = 43.788°$ — M1

$= 43.8°$. — A1

iii)



$\sin\theta = \frac{O}{H} = \frac{3}{5} = 0.6$ M1

$\sin^{-1} 0.6 = 38.87$ M1

$=$ A1

$38.87 \times 2 = 77.74$

$= 77.7^\circ$

24. a) $\frac{dy}{dx} = 3ax^2 + b$ M1

At $x = 1$, $3a(1)^2 + b = -5$

$3a + b = -5$

$y = ax^3 + bx$.

$1 = a(1)^3 + b(1)$

$1 = a + b$.

$3a + b = -5$ — ①

$a + b = 1$ — ②

M1 both equations.

$a = 1 - b$.

$3(1-b) + b = -5$.

$3 - 3b + b = -5$

$-2b = -8$.

$\frac{-2b}{-2} = \frac{-8}{-2}$

$b = 4$ A1 both values.

$a + 4 = 1$

$a = 1 - 4 = -3$.

b) i) $V = \frac{1}{2}(8)^2 - 3(8) + 7$

$= 15 \text{ m/s}$ B1

ii) A minimum velocity, $a = 0$.

$\frac{dv}{dt} = a = t - 3$ M1

$t - 3 = 0$

$t = 3$

$V = \frac{1}{2}(3)^2 + 3(3) + 7 = 2.5 \text{ m/s}$ A1

iii) $S = \frac{1}{6}t^3 - \frac{3}{2}t^2 + 7t$ M1

When $t = 2$

$S = \frac{1}{6}(2)^3 - \frac{3}{2}(2)^2 + 7(2)$ M1

$= 9\frac{1}{3} \text{ m}$

When $t = 3$

$S = \frac{1}{6}(3)^3 - \frac{3}{2}(3)^2 + 7(3)$ M1

$= 12 \text{ m}$

$D = 12 - 9\frac{1}{3} = 2\frac{2}{3} \text{ m}$ A1