Name....Marking Scheme.................................

121/2

Mathematics

Paper 2

2 ½ Hours June 2022

KASSU JET EXAMINATIONS

Kenya Certificate of Secondary Education (K.C.S.E)

INSTRUCTIONS TO CANDIDATES

- Write your name and Admission number in the spaces provided at the top of this page.
- This paper consists of two sections: Section I and Section II.
- Answer ALL questions in section 1 and ONLY FIVE questions from section II
- All answers and workings must be written on the question paper in the spaces provided below each question.
- Show all the steps in your calculation, giving your answer at each stage in the spaces below each question.
- Non – Programmable silent electronic calculators and KNEC mathematical tables may be used, except where stated otherwise.

FOR EXAMINERS USE ONLY

SECTION I

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	TOTAL

SECTION II

17	18	19	20	21	22	23	24	TOTAL

GRAND TOTAL

SECTION I. *Answer all the questions* (50 marks)

1. A student spends $\frac{3}{8}$ of his time playing basketball, $\frac{1}{4}$ of the remaining in playing table tennis and $\frac{3}{4}$ of the remaining time playing volleyball. The rest is spent on reading novels. What fraction of the time is spent on reading novels. (3 mks)

$\frac{3}{8} \Rightarrow$ B.ball

$\frac{1}{4} \times \frac{5}{8}$

$= \frac{5}{32}$ M1

$\frac{3}{8} + \frac{5}{32}$

$= \frac{17}{32}$ M1

Remaining

$\frac{15}{32}$

$\frac{3}{4} \times \frac{15}{32} = \frac{45}{128}$

$\frac{17}{32} + \frac{45}{128} = \frac{113}{128}$

$1 - \frac{113}{128}$

$= \frac{15}{128}$ A1

2. Simplify; $\frac{\sqrt{5}-1}{\sqrt{5}+1}$. (3 marks)

$\frac{\sqrt{5}-1}{\sqrt{5}+1} \times \frac{\sqrt{5}+1}{\sqrt{5}+1}$ M1

Numerator

$6-2\sqrt{5}$ M1

Denominator

$5-1$

$=4$

$\frac{6-2\sqrt{5}}{4} = \frac{3}{2} - \frac{1}{2}\sqrt{5}$ A1

or

$\Rightarrow 1.5 - 0.5\sqrt{5}$

3. Solve the equation $2\log 3 + \log(x-2) = 2\log x$ (3marks)

$\log\{3^2(x-2)\} = \log x^2$ M1

$9(x-2) = x^2$

$x^2 - 9x + 18 = 0$ M1

$(x-6)(x-3) = 0$

$\left.\begin{array}{l} x = 6 \\ x = 3 \end{array}\right\}$ A1

4. The base and perpendicular height of a triangle measured to the nearest millimetre are 15.0 cm and 9.5 cm respectively. Find

(a) The absolute error in calculating the area of the triangle (1 mark)

Min prod	Actual	Max Product
14.95	15.0	15.05
9.45	9.5	9.55
141.2775	142.5	143.7275

A.E

$= \frac{143.7275 - 141.2775}{2}$

$= 1.225$ A1

(b) The percentage error in the area, giving the answer to 1 decimal place. (2mks

$\frac{1.225}{142.5} \times 100$ M1

$= 0.859$

$= 0.9\%$ A1

5. Find the value of θ, given that; $\frac{1}{2}sin\theta = 0.35$ for $0^0 \leq \theta \leq 360^0$

(3 mks)

$\sin\theta = 0.70$ M1

$\theta = \sin^{-1} 0.70$

$\theta = 44.43^0$ A1

$180 - 44.43$

$= 135.57$ A1

6. Make Q the subject of formula $P = \sqrt{\frac{Q^2}{Q^2-1}}$ 3marks

$P^2 = \left(\frac{Q^2}{Q^2-1}\right)$ M1

$P^2Q^2 - P^2 = Q^2$

$P^2Q^2 - Q^2 = P^2$ M1

$Q^2(P^2-1) = P^2$

$Q^2 = \frac{P^2}{P^2-1}$

$Q = \pm\sqrt{\frac{P^2}{P^2-1}}$ A1

7. The coordinates of the end points of a diameter of a circle are $A(2,4)$ and $B(-2,6)$. Find the equation of the circle in the form $ax^2 + by^2 + cx + dy + e = 0$ (4marks)

Centre

$\left(\frac{2+-4}{2}, \frac{4+6}{2}\right)$

$(0,5)$ B1

Radius =

$\sqrt{(0-2)^2+(5-4)^2}$

$r=\sqrt{5}$. B1

M1

$(x-0)^2+(y-5)^2=(\sqrt{5})^2$

$x^2+y^2-10y+25=5$

A1

$x^2+y^2-10y+20=0$

8. Kimani wants to buy a TV on hire purchase. It has a cash price of Ksh.30,000. He makes a down payment of Ksh.9,000 and 12 monthly instalments of ksh. 2,200 each. Calculate the rate of compound interest charged per month.(Give your answer to 1 dp). (3 mks)

$P = 30,000 - 9,000$

$= 21,000$

$A = 12 \times 2,200$

$= 26,400$

M1

$26,400 = 21,000\left(1+\frac{r}{100}\right)^{12}$

$1.257 = \left(1+\frac{r}{100}\right)^{12}$

$\sqrt[12]{1.257} = 1.0193$ M1

$1.0193 = 1 + \frac{r}{100}$

$r = 1.93$

$r = 1.9$ A1

9. Expand $(3 + 3x)^6$ in ascending powers of x. Hence use the expansion up to the 3rd term, to find the value of $(3.03)^6$ correct to 2 decimal places. (3mks)

$(3+3x)^6$

M1

$729 + 4356x + 10935x^2 + 14580x^3 + 10935x^4 + 4356x^5 + 729x^6$

$3+3x = 3.03$

$x = 0.01$ M1

$729 + 4356x + 10935x^2 \Rightarrow 729 + 4356(0.01) + 10935(0.01)^2$

$= 773.6535$ A1

$\simeq 773.65$ (2dp)

10. The following are ages of students in a class 7,9,8,9,11,12,10 9,8,6,7,10,11,12,6,9,7, and 11.

a). Complete the frequency distribution table below (1mark)

Ages x	6	7	8	9	10	11	12
No of students	2	3	2	4	2	3	2
$(x-\bar{x})$	-3	-2	-1	0	1	2	3
$(x-\bar{x})^2$	9	4	1	0	1	4	9

B1

$\bar{x} = \frac{162}{18}$

$= 9.$

b).Calculate the standard deviation of their ages in five years' time. (2mks)

$V = \frac{\Sigma d^2}{N}$

$= \frac{28}{18}$ M1

$= 1.\dot{5}$

$S.d = 1.247$ A1

11. Find the possible values of x given that $\begin{pmatrix} x+8 & 8 \\ 6 & x \end{pmatrix}$ is a **singular** matrix. (3 mks)

$x^2 + 8x - 48 = 0$ M1

$x^2 + 12x - 4x - 48 = 0$

$x(x+12) - 4(x+12) = 0$ M1

$(x+12)(x-4) = 0$

$x = -12$

$x = 4$ A1

12. Evaluate using the logarithm table; (4 mks)

$\left(\frac{\log 9.814}{4.283\, x(0.009478)^2}\right)^{-\frac{1}{2}}$ $= \left(\frac{4.283 \times (0.009478)^2}{0.9931}\right)^{\frac{1}{2}}$ M1

NO	Log.
4.283	0.6317. M1 → All logs Correct.
0.009478	$\bar{3}.9767$
	$\times 2$
	$\bar{5}.9534 +$
	0.6317
	$\bar{4}.5851$
0.9931	$\bar{1}.9969 -$
	$\bar{4}.5882$

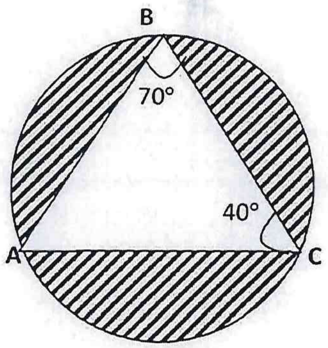
$\frac{\bar{4}.5882}{2}$ M1

$= \bar{2}.2941$ Antilog

A1

$= 0.01969$

13. The figure below is that of a circumcircle of the triangle ABC. The radius of the circle is $5cm$. Given that $\angle ABC = 70^0$ and $\angle ACB = 40^0$. Calculate the area of ΔABC. (3 mks)



$BC = AC = a$

$\frac{a}{\sin 70^0} = 2 \times 5$

$a = 10 \times \sin 70$ M1

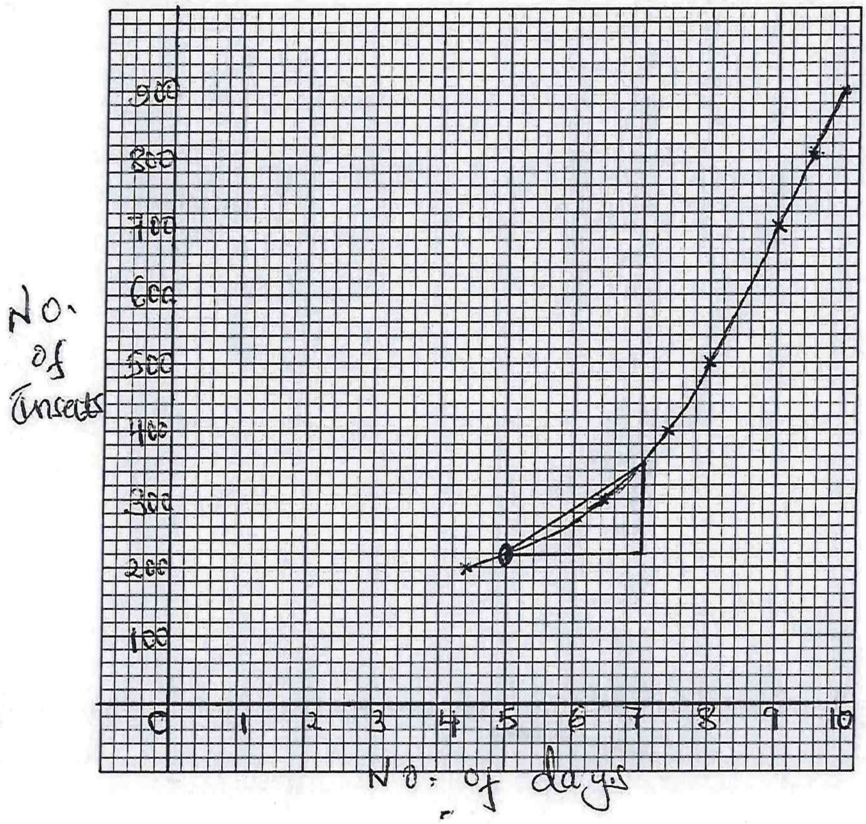
$A = \frac{1}{2} \times (10 \sin 70)^2 \times \sin 40^0$ M1

$A = 28.38\ cm^2$ A1

14. The table below shows the number of insects and corresponding number of days in breeding.

Number of insects	200	300	400	500	600	700	800	900
days	4.4	6.4	7.4	8.0	8.5	9.0	9.5	10

a). On the grid provided, draw the graph of number of insects against the number of days. (1mark)



b). Determine the rate of breeding between 5[th] and 7[th] day. (2 marks)

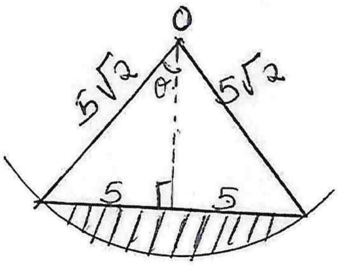
$5^{th} \Rightarrow 220$

$7^{th} \Rightarrow 350$

$\frac{350-220}{7-5}$ M1

$= 65 \pm 1$ A1

15. Calculate the area of the minor segment of a circle of radius $5\sqrt{2}cm$, cut off by a chord of length $10cm$ (3marks)



$\sin\theta = \frac{5}{5\sqrt{2}}$

$\sin\theta = \frac{1}{\sqrt{2}}$

$\theta = 45^0$ M1

$\frac{90}{360} \times 3.142 \times (5\sqrt{2})^2$ M1

$= 39.275$

$\frac{1}{2}(5\sqrt{2})^2 \times \sin 90^0$

$= 25$

$39.275 - 25$

$= 14.275$ A1

16. A quantity P varies partly as the cube of **Q** and partly varies inversely as the square of **Q**. when **Q** = 2, **P** = 108 and when **Q** = 3, **P** = 259. Find the value of **P** when **Q** = 6. (3mks)

$P = Q^3 k + \frac{n}{Q^2}$

$108 = 8k + \frac{n}{4}$

$259 = 27k + n/9$

M1 $432 = 32k + n$

$2331 = 243k + n$

$-1899 = -211k$

$k = 9$. M1

$108 = 8(9) + \frac{n}{4}$

$n = 144$

$P = 6^3(9) + \frac{144}{36}$

$P = 1948$ A1

SECTION II: *Answer any 5 questions from this section.* (50 marks)

17. Income rates for income earned were charged as shown in the table alongside:

A civil servant earns a monthly salary of *Ksh.* 27,000. He was also given a house allowance of *h*12,000 , transport allowance *Ksh.* 1,800 and medical allowance *Ksh.* 2,000. He is entitled to a family relief of Kshs. 1040 per month.

Income in Ksh. pm	**Rate in Ksh. Per Shs. 20**
1 – 8400	2
8401 – 18,000	3
18,001 – 30,000	4
30,000 – 36,000	5
36,000 – 48,000	6
48,001 *and above*	7

Determine:

a) (i) His taxable income per month in Ksh. (2 mks) M1

$27,000 + 12,000 + 1800 + 2,000$

$= 42\,800$ A1

(ii) His net tax. (6 mks)

M1 $\frac{8400}{20} \times 2$	$= 840$	Total M1
$\frac{9600}{20} \times 3$	$= 1440$	$840 + 1440 + 2400 + 1500 + 2040$
M1 $\frac{12{,}000}{20} \times 4$	$= 2400$	$= 8{,}220$ M1
$\frac{6{,}000}{20} \times 5$	$= 1500$	$8220 - 1040$
M1 $\frac{6800}{20} \times 6$	$= 2040$	$= 7180$ A1

b) In addition, the following deductions were made

NHIF	shs. 430
Loan repayment	Kshs. 6500
Bank shares	Kshs. 1000.

Calculate his net pay per month. (2 mks)

Total deductions

$= 7930 + 7180$ M1

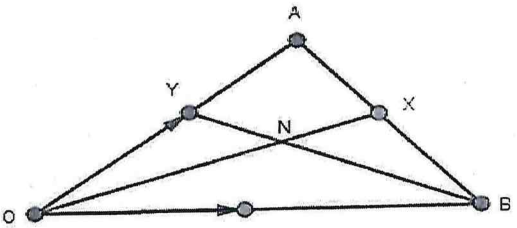
$= 15110$

$42800 - 15110$

$= 27{,}690$ A1

18. a). In the figure below, $OY:YA = 1:3$, $AX:XB = 1:2$, $\mathbf{OA} = \boldsymbol{a}$ and $\mathbf{OB} = \boldsymbol{b}$. n is the point of intersection of BY and OX.

Determine;



i. **OX** (2 marks)

$\vec{OX} = \vec{OA} + \vec{AX}$

$= \underset{\sim}{a} + \frac{1}{3}\vec{AB}$ M1

$\vec{AB} = -\underset{\sim}{a} + \underset{\sim}{b}$.

$\vec{OX} = \underset{\sim}{a} + \frac{1}{3}(-\underset{\sim}{a} + \underset{\sim}{b})$

A1

$\vec{OX} = \frac{2}{3}\underset{\sim}{a} + \frac{1}{3}\underset{\sim}{b}$.

ii. **BY** (1 marks)

$\vec{BY} = \frac{1}{4}\underset{\sim}{a} - \underset{\sim}{b}$. A1

b) Given that $\boldsymbol{BN} = m\boldsymbol{BY}$ and $\boldsymbol{ON} = n\boldsymbol{OX}$, express **ON** in two ways in terms of a, b, m and n (3marks)

$\vec{BN} = m(\frac{1}{4}\underset{\sim}{a} - \underset{\sim}{b})$ M1

$= \frac{1}{4}m\underset{\sim}{a} - m\underset{\sim}{b}$.

$\vec{ON} = \vec{OB} + \vec{BN}$

$= \underset{\sim}{b} - m\underset{\sim}{b} + \frac{1}{4}m\underset{\sim}{a}$ A1

$\vec{ON} = n(\frac{2}{3}\underset{\sim}{a} + \frac{1}{3}\underset{\sim}{b})$

$= \frac{2}{3}n\underset{\sim}{a} + \frac{1}{3}n\underset{\sim}{b}$. A1

c) Find the values of m and n (4 marks)

M1

$(1-m)\underset{\sim}{b} + \frac{1}{4}m\underset{\sim}{a} = \frac{2}{3}n\underset{\sim}{a} + \frac{1}{3}n\underset{\sim}{b}$.

$\frac{1}{4}m = \frac{2}{3}n$.

$m = \frac{8}{3}n$ --(i)

M1 $1 - m = \frac{1}{3}n$

$3 - 3m = n$.

$3 - 3(\frac{8}{3}n) = n$

$3 = 9n$.

$n = \frac{1}{3}$. A1

$m = \frac{8}{3}(\frac{1}{3})$

$m = \frac{8}{9}$ A1

19. (a) In a geometrical progression the sum of the second and third term is 12 and the sum of the third and fourth terms is -36. Find the first term and the common ratio. (4marks)

$ar + ar^2 = 12$ M1

$ar^2 + ar^3 = -36$

$ar(1+r) = 12$

$ar^2(1+r) = -36$

$\frac{ar^2}{ar} = \frac{-36}{12}$ M1

$r = -3.$ A1

From $ar + ar^2 = 12$

$-3a + 9a = 12$

$a = 2$ A1

(b) In an arithmetic progression the 12^{th} term is 25 and the 7^{th} term is three times the second term, find;

i) The first term and the common difference (4marks)

M1 $\begin{cases} a + 11d = 25 \\ (a+d)3 = a + 6d \end{cases}$

M1 $\begin{cases} a + 11d = 25 \\ 2a = 3d \end{cases}$

$a = \frac{3}{2}d$ - (i)

$\frac{3}{2}d + 11d = 25$

$3d + 22d = 50$

$25d = 50$

$d = 2$ A1.

$a = \frac{3}{2}(2)$

$a = 3$ A1

ii) The sum of the first 10 terms of the arithmetic progression. (2marks)

$S_n = \frac{n}{2}(2a + (n-1)d)$

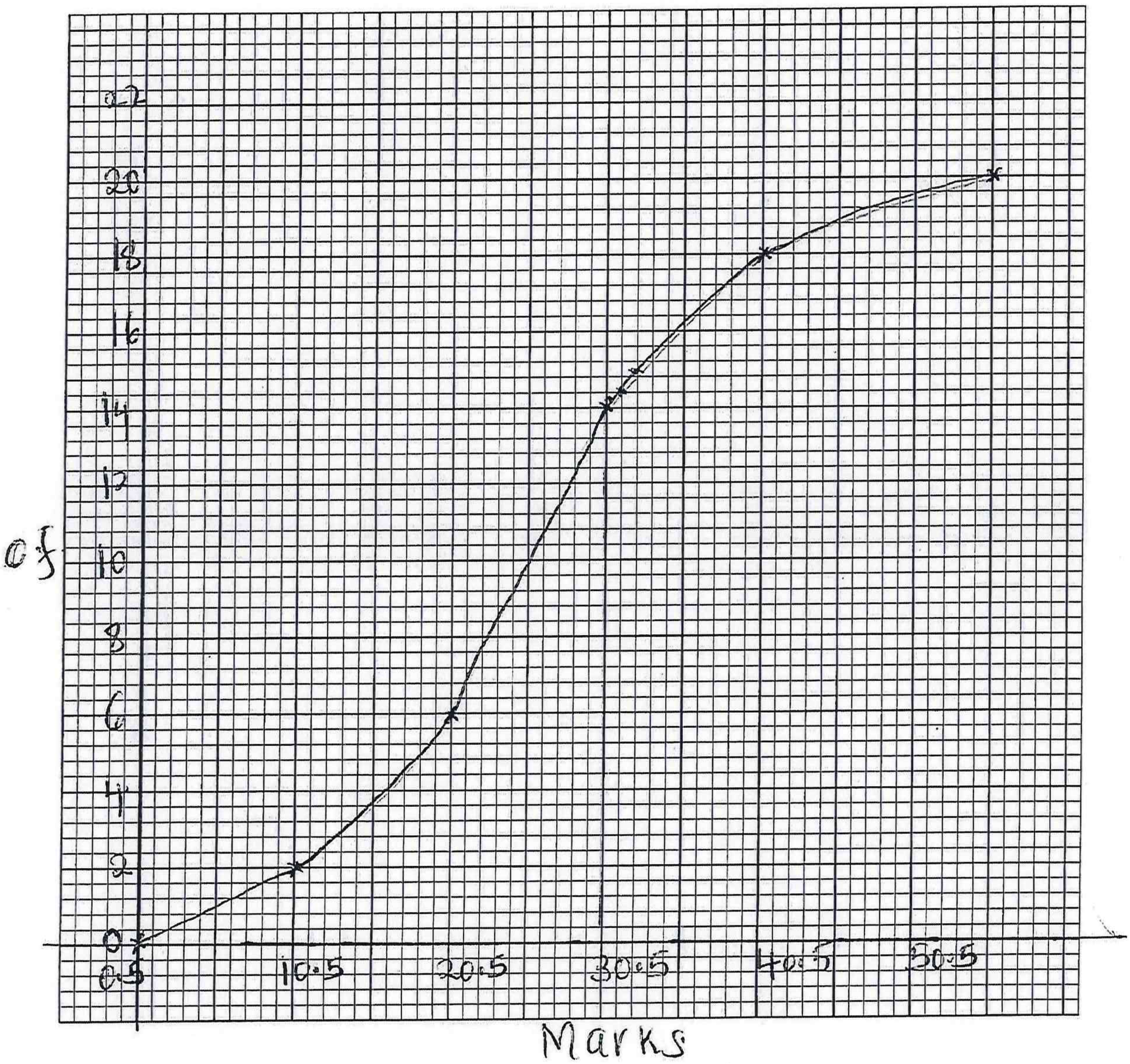
$= \frac{10}{2}(2 \times 3 + (10-1)2)$ M1

$= 120$ A1

20. The table below shows the frequency distribution of marks scored by students in a test.

cf	2	6	14	18	20
Marks	1-10	11-20	21-30	31-40	41-50
Frequency	2	4	8	4	2

a). On the grid provided, draw a cumulative frequency curve for the data. (4 mks)



b). Use your graph to determine;

(i). The pass mark if only 6 students passed the exam. (2 mks)

$20-6$
$=14$ M1

$=30.5$ A1

(ii). The upper quartile mark (1 mk)

$\frac{3}{4} \times 50$

$= 31.5$ marks

$\frac{3}{4} \times 20$

$= 15^{th}$ value

c). Find the percentage change if the upper quartile in b(ii) above was found by calculation. (3 mks)

$= 30.5 + \left(\frac{15-14}{4}\right) 10$ M1

$= 30.5 + 2.5$ M1

$= 33$ M1

$\left(\frac{33-31.5}{31.5}\right) \times 100 = 4.76\%$ A1

21. A gold urn contains 3 red balls and 4 white balls and a silver urn contains 5 red balls and 2 white balls. A die is rolled and if a 6 shows, balls will be selected at random from the gold urn. Otherwise balls are selected from the silver urn.

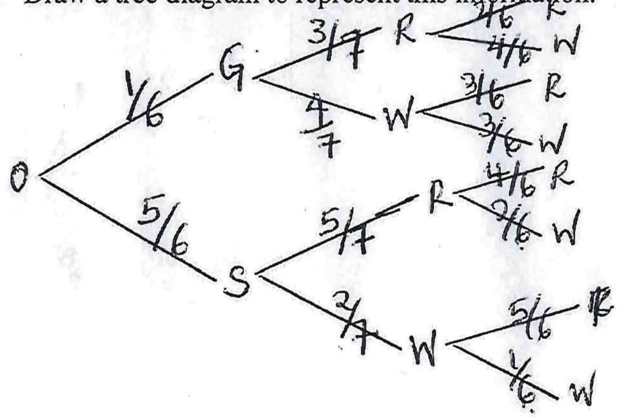
a. Find the probability of selecting a red ball. (3marks)

$\frac{1}{6}\times\frac{3}{7}$ or $\frac{5}{6}\times\frac{5}{7}$ M1

$\left(\frac{1}{14}+\frac{25}{42}\right)$ M1 $=\frac{2}{3}$ A1

b. If two balls are selected at random without replacement,

i. Draw a tree diagram to represent this information. (2marks)



B1

B1

ii. Find the probability that two balls are white. (2marks)

P(SWW) or P(GWW)

$=\left(\frac{5}{6}\times\frac{2}{7}\times\frac{1}{6}\right)+\left(\frac{1}{6}\times\frac{4}{7}\times\frac{3}{6}\right)$ M1

$\frac{5}{126}+\frac{1}{21}$ $=\frac{11}{126}$ A1

iii. Find the probability that there is at most one white ball from the silver urn. (3marks) P(0) or P(1)

P(SRR) or P(SWR) or P(SRW)

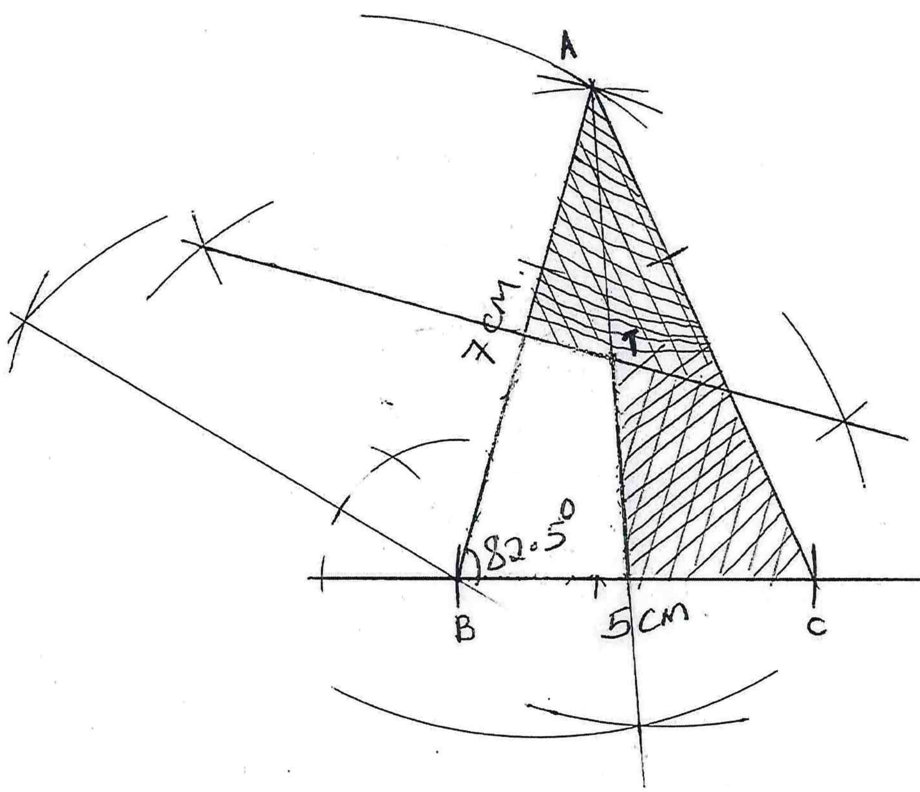
$=\frac{5}{6}\times\frac{5}{7}\times\frac{4}{6}+\frac{5}{6}\times\frac{2}{7}\times\frac{5}{6}+\frac{5}{6}\times\frac{5}{7}\times\frac{2}{6}$ M1

$=\frac{25}{63}+\frac{25}{126}+\frac{25}{126}$ M1

$=\frac{50}{63}$ A1

22. a) Using a ruler and a compass only construct triangle **ABC** where $AB = 7cm$, Angle $CBA = 82.5^0$ and $BC = 5cm$ (4mks)

(a)
- $B_1 \Rightarrow \overline{BC}$
- $B_1 \Rightarrow \overline{AB}$
- $B_1 \Rightarrow \angle CBA$
- $B_1 \Rightarrow$ complete $\triangle ABC$.



(b)
- $B_1 \Rightarrow$ Bisecting AB
- $B_1 \Rightarrow$ Bisecting $\angle BAC$.
- $B_1 \Rightarrow$ locating T

(c) A_1

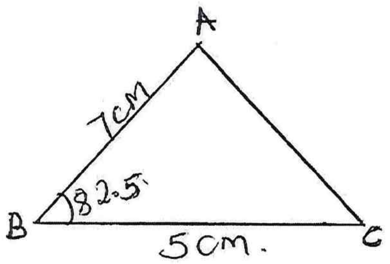
(d) $B_1 B_1 \Rightarrow$ locating region
$\Rightarrow$ shading the unwanted region.

b) i) Locate a point T inside the triangle which is equidistant from points A and B and also equidistant from lines AB and AC (3mks)

ii) Measure TB (1mk)

(3.7 ± 0.1) cm.

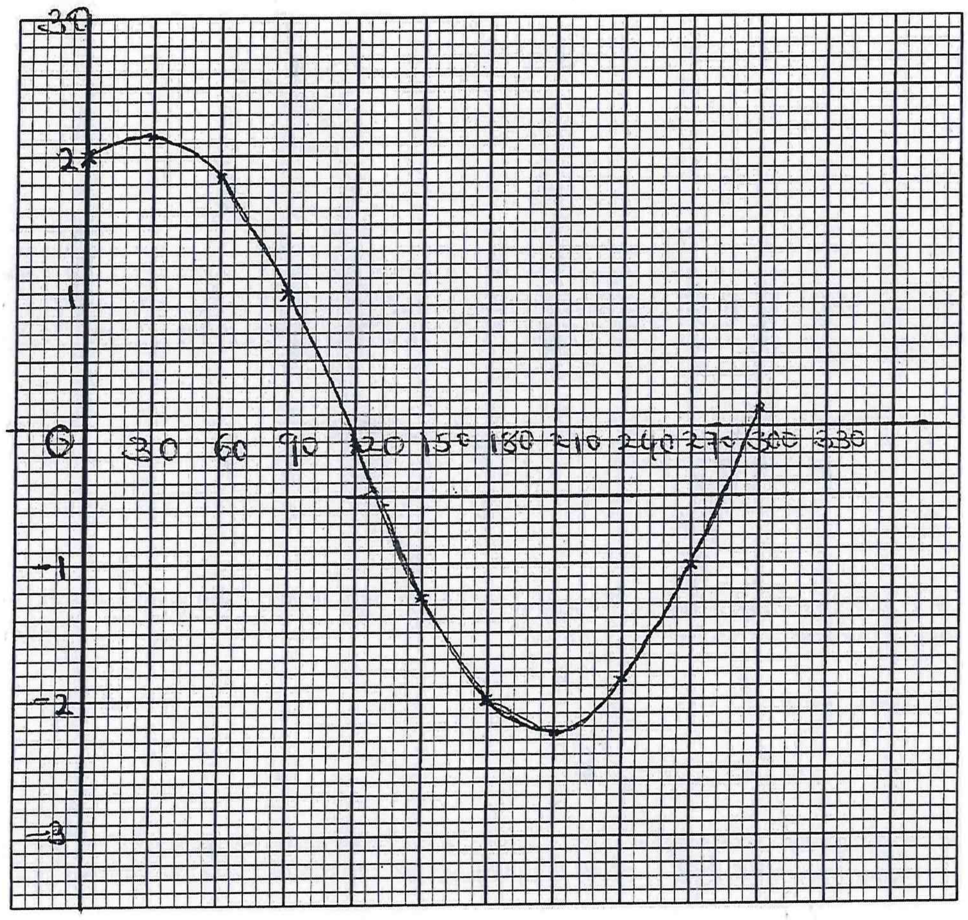
c) By shading the unwanted region show the area inside the triangle where P lies if it is nearer to point B than to point A and also nearer to the line AB than line AC. (2mks)



23. (a) Complete the table for $y = Sin\,x + 2\,Cos\,x$. (2marks)

x^0	0	30	60	90	120	150	180	210	240	270	300
$Sin\,x$	0.00	0.50	0.87	1.00	0.87	0.50	0.00	-0.50	-0.87	-1.00	-0.87
$2\,cos\,x$	2.00	1.73	1.00	0.00	-1.00	-1.73	-2.00	-1.73	-1.00	0.00	1.00
y	2.00	2.23	1.87	1.00	-0.13	-1.23	-2.00	-2.23	-1.87	-1.00	0.13

(b) Draw the graph of $y = Sin\,x + 2\,cos\,x$. (3marks)



P1
C1
S1.

c). Solve $sin x + 2\,cos\,x = 0$ using the graph. (2marks)

114 ± 2 B1 ; 294 ± 2 B1

d). Find the range of values of x for which $y < -0.5$ (3marks).

$132^0 - 280^0$

24. A triangle **ABC** with vertices at **A (1,-1)** ,**B (3,-1)** and **C (1, 3)** is mapped onto triangle **A¹B¹C¹** by a transformation whose matrix is $\begin{pmatrix}-1 & 0\\ 0 & 1\end{pmatrix}$.Triangle **A¹B¹C¹** is then mapped onto **A¹¹B¹¹C¹¹** with vertices at **A¹¹ (2, 2)**, **B¹¹ (6, 2)** and **C¹¹ (2,-6)** by a second transformation.

(i) Find the coordinates of **A¹B¹C¹** M1 (3 marks)

$$\begin{pmatrix}-1 & 0\\ 0 & 1\end{pmatrix}\begin{pmatrix}1 & 3 & 1\\ -1 & -1 & 3\end{pmatrix} = \begin{array}{ccc} A^1 & B^1 & C^1 \\ \end{array} \begin{pmatrix}-1 & -3 & -1\\ -1 & -1 & 3\end{pmatrix} M_1$$

A1

$A^1(-1,-1)$, $B^1(-3,-1)$, $C^1(-1,3)$

(ii) Find the matrix which maps **A¹B¹C¹** onto **A¹¹B¹¹C¹¹**. (3 marks)

$$\begin{pmatrix}a & b\\ c & d\end{pmatrix}\begin{pmatrix}-1 & -3 & -1\\ -1 & -1 & 3\end{pmatrix} = \begin{pmatrix}2 & 6 & 2\\ 2 & 2 & -6\end{pmatrix} M_1$$

A1

$$M_1 \left\{ \begin{array}{ll} -a-b=2 & -c-d=2 \\ -3a-b=6. & -3c-d=2 \\ a=-2\ ,\ b=0 & c=0 \\ & d=-2. \end{array} \right. \quad \begin{pmatrix}-2 & 0\\ 0 & -2\end{pmatrix}$$

(iii) Determine the ratio of the area of triangle **A¹B¹C¹** to triangle **A¹¹B¹¹C¹¹**. (1 mark)

$$\begin{pmatrix}-2 & 0\\ 0 & -2\end{pmatrix}$$

determinant = 4 A1

(iv) Find the transformation matrix which maps **A¹¹B¹¹C¹¹** onto **ABC** (3 marks)

$M = \begin{pmatrix}-1 & 0\\ 0 & 1\end{pmatrix}$ $N = \begin{pmatrix}-2 & 0\\ 0 & -2\end{pmatrix} \rightarrow$ N·M

$$\begin{pmatrix}-2 & 0\\ 0 & -2\end{pmatrix}\begin{pmatrix}-1 & 0\\ 0 & +1\end{pmatrix} = \begin{pmatrix}2 & 0\\ 0 & -2\end{pmatrix} M_1$$

Inverse of $\begin{pmatrix}2 & 0\\ 0 & -2\end{pmatrix} = -\frac{1}{4}\begin{pmatrix}-2 & 0\\ 0 & 2\end{pmatrix} M_1$

det = −4

$$= \begin{pmatrix}\frac{1}{2} & 0\\ 0 & -\frac{1}{2}\end{pmatrix} A_1$$

→ other method allowed