SCHEME

Name........Mr. Calvince..........................Adm No ....Stream..................

School.......................................Date...........................Signature.....................

MATHEMATICS

PAPER 2

TIME: $2^1/_2$ HOURS

JULY 2023

PINNACLE CLUSTER EXAMINATION

Kenya Certificate of Secondary Education (K.C.S.E)

INSTRUCTIONS TO THE CANDIDATES

- *Write **your name** and **Admission number** in the spaces provided above*
- *This paper contains two sections; **Section I** and **Section II**.*
- *Answer all the questions in **section I** and only **five** questions from **Section II***
- *All workings and answers must be written on the question paper in the spaces provided below each question.*
- *Marks may be given for correct working **even if** the answer is wrong.*
- *Non programmable silent electronic calculators and KNEC Mathematical tables may be used **EXCEPT** where stated otherwise*
- *Show all the steps in your calculations, giving your answers at each stage in the spaces below each question.*

FOR EXAMINERS'S USE ONLY

Section I

Question	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	Total
Marks																	

Section II

GRAND TOTAL

Question	17	18	19	20	21	22	23	24	**Total**
Marks									

SECTION I(50MARKS)

Answer all questions in this section

1. Find the value of x in the equation $\log_{10}(2x-1) + \log_{10}3 = \log_{10}(8x-1)$ (2mks)

$\log_{10} 3(2x-1) = \log_{10} 8x-1$ ✓ M1

$6x-3 = 8x-1$

$6x-8x = -1+3$

$-2x = 2$

$x = -1$ ✓ A1

02

2. By correcting each number to the nearest one significant figure, approximates the value of 699 × 0.003, hence calculate the percentage error arising for the approximation. (3marks)

Actual $= 699 \times 0.003 = 2.097$

Approximation $= 700 \times 0.003 = 2.1$ M1

% error $= \frac{2.1 - 2.097}{2.097} \times 100$ ✓

$= 0.143061516$ % ✓

03

3. . Simplify by rationalizing the denominator

$$\frac{\sqrt{2}+\sqrt{3}}{\sqrt{6}-\sqrt{3}}$$

(3 mks)

$\frac{(\sqrt{2}+\sqrt{3})(\sqrt{6}+\sqrt{3})}{(\sqrt{6}-\sqrt{3})(\sqrt{6}+\sqrt{3})}$ ✓

$\frac{\sqrt{2}(\sqrt{6}+\sqrt{3})+\sqrt{3}(\sqrt{6}+\sqrt{3})}{6-3}$

$\frac{\sqrt{12}+\sqrt{6}+\sqrt{18}+3}{6-3}$ ✓

$\frac{2\sqrt{3}+\sqrt{6}+3\sqrt{2}+3}{3}$ ✓ or $\frac{2\sqrt{3}}{3}+\frac{\sqrt{6}}{3}+\sqrt{2}+1$

03

4. The points with the coordinates (5,5) and (-3,1) are the ends of a diameter of a circle Centre A. determine:

(a) The coordinate of A. (1mk)

$\left(\frac{5+-3}{2}, \frac{5+1}{2}\right) = (1, 3)$ ✓ B1

01

(b) The equation of the circle in the form $x^2 + y^2 + ax + by + c = 0$ when a, b and c are constants. (2 marks)

$r = \sqrt{(5-1)^2+(5-3)^2}$

$= \sqrt{20}$

$(x-1)^2+(y-3)^2 = 20$ ✓

$x^2-2x+1+y^2-6y+9=20$

$x^2+y^2-2x-6y+10=20$

$x^2+y^2-2x-6y-10=0$ ✓

02

5. a) Expand $(x+y)^4$ (2mks)

1
1 2 1
1 3 3 1
1 4 6 4 1

$x^4(y)^0+4(x)^3(y)^1+6(x)^2(y)^2$
$+4(x)^1(y)^3+y^4$ ✓

$=x^4+4x^3y+6x^2y^2+$
$4xy^3+y^4$ ✓

b) Use your expansion to evaluate $(1.99)^4$ correct to five significant figures. (2mks)

$(x+y)^4=(2-0.01)^4$

$\Rightarrow x=2,\ y=-0.01$

$2^4+4(2)^3(-0.01)+6(2)^2(-0.01)^2$
$+4(2)(-0.01)^3+(-0.01)^4$ ✓

$=15.682$ ✓ A

6. A quantity A is partly constant and partly varies inversely as a quantity B. Given that A = -10 when B= 2.5 and A = 10 when B = 1.25, find the value of A when B = 1.5.

(4mks)

$A \propto C+\frac{1}{B}$

$A=C+\frac{K}{B}$

$-10=C+\frac{K}{2.5}$

$10=C+\frac{K}{1.25}$

$20=\frac{K}{1.25}-\frac{K}{2.5}$ ✓ m

$\frac{2}{5}K=20$
$K=50$

$C+\frac{50}{2.5}=-10$
$C=-10-20$
$\Rightarrow C=-30$ ✓ values of K & C

Eqtn
$A=-30+\frac{50}{B}$

$A=-30+\frac{50}{1.5}$ ✓

$=3\frac{1}{3}$ ✓

or 3.333

7. Given that $S=\frac{a(1-r^n)}{1-r}$ make **n** the subject of the formula. (3 marks)

$\frac{S(1-r)}{a}=\frac{\cancel{a}(1-r^n)}{\cancel{a}}$

$1-r^n=\frac{S-Sr}{a}$ ✓

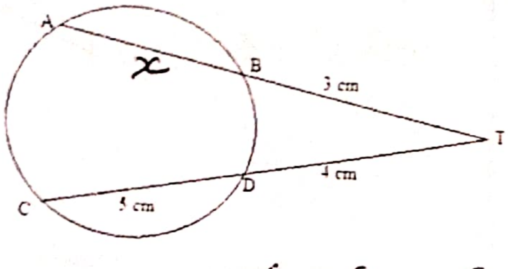
$r^n=1-\frac{S-Sr}{a}$ ✓

$r^n=\frac{a-S+Sr}{a}$

$n=\frac{\log\left(\frac{a-S+Sr}{a}\right)}{\log r}$ ✓ or

$n=\frac{\log(a+Sr-S)-\log a}{\log r}$

8. In the figure below, the chords CD and AB intersect externally at T. DT = 4 cm, BT = 3 cm and CD = 5 cm, calculate the length AB. (3marks)



$4(5+4) = (x+3)3$

$36 = 3x + 9$

$3x = 27$

$x = 9$

9. If the area of an object is 10 square units, calculate the area of the image after a transformation whose matrix is $\begin{pmatrix} -2 & 3 \\ 5 & 1 \end{pmatrix}$ (3mks)

Det $(-2\times1) - (5\times3)$

$-2-15 = -17$

$= \frac{\text{Area of image}}{\text{Area of object}}$

Area of image

$= 17\times10 = 170$ square units

10. Calculate the quartile deviation for set of data below. (3 marks)

16,13,24,40,6,20,18,17

6, 13, 16, 17, 18, 20, 24, 40

14.5 Q_1 17.5 Q_2 $Q_3 = 22$

M1 (Arange in ascending/descending order)

$\frac{22-14.5}{2} = 3.75$

11. It would take 18 men 12 days to dig a piece of land, if they work for 8 hours a day. How long will it take 24 men if they work 12 hours a day to cultivate three quarters of the same land. (3 marks)

Men	days	hrs	fraction
18	12	8	1
24		12	0.75

$= 6$ days $\times \frac{3}{4}$

$\frac{18}{24} \times 12 \times \frac{8}{12} \times \frac{3}{4}$

$= 4\frac{1}{2}$ days

12. Solve the equation $2x^2 + 4x + 1 = 0$ using completing square method (3mks)

$x^2 + 2x = -\frac{1}{2} + c$

$x^2 + 2x + 1 = -\frac{1}{2} + 1$ M1

$(x+1)^2 = \frac{1}{2}$

$x + 1 = \pm\sqrt{0.5}$ M1

$x = -1 \pm \sqrt{0.5}$

$= -1 \pm 0.7071$ A1 Both

$= -0.2929$ or -1.7071

13. Use logarithm tables to evaluate. (4 marks)

$$\sqrt[4]{\frac{0.8465 \times 12.14}{214.5 \div 9.067}}$$

No	log
0.8465	$\bar{1}.9276$ +
12.14	1.0842
	1.0118
214.5	2.3314 +
9.067	0.9575
	1.3739 −
	$\bar{1}.6379$

$\frac{\bar{1}.6379}{4}$

$\frac{\bar{4} + 3.6379}{4}$

$\bar{1}.9095$

$10^{-1} \times$ Antilog 0.9095

$10^{-1} \times 8.119$

$= 0.8119$

Also accept 0.8118

M1 - Correct logs

M1 - correct substraction & addition

M1 - Correct division

A1

14. The gradient function of a curve is given by the expression $2x + 1$. If the curve passes through the point $(-4, 6)$; find the equation of the curve (3mks)

$\frac{dy}{dx} = 2x + 1$

$y = \frac{2x^2}{2} + x + c$

$y = x^2 + x + c$

$6 = (-4)^2 + (-4) + c$

$6 = 16 - 4 + c$

$6 - 12 = c \Rightarrow c = -6$

Equation

$y = x^2 + x - 6$

15. Solve $8\cos^2 x - 2\cos x - 1 = 0$ for $0^0 \leq x \leq 360^0$ (3marks)

Let $\cos x = t$.

$8t^2 - 2t - 1 = 0$ ✓

$8t^2 - 4t + 2t - 1 = 0$

$4t(2t-1) + 1(2t-1) = 0$

$(4t+1)(2t-1) = 0$ ✓

$t = \frac{-1}{4}$ or $t = \frac{1}{2}$

$\cos x = -\frac{1}{4}$, $x = 104.5^0$, 255.5

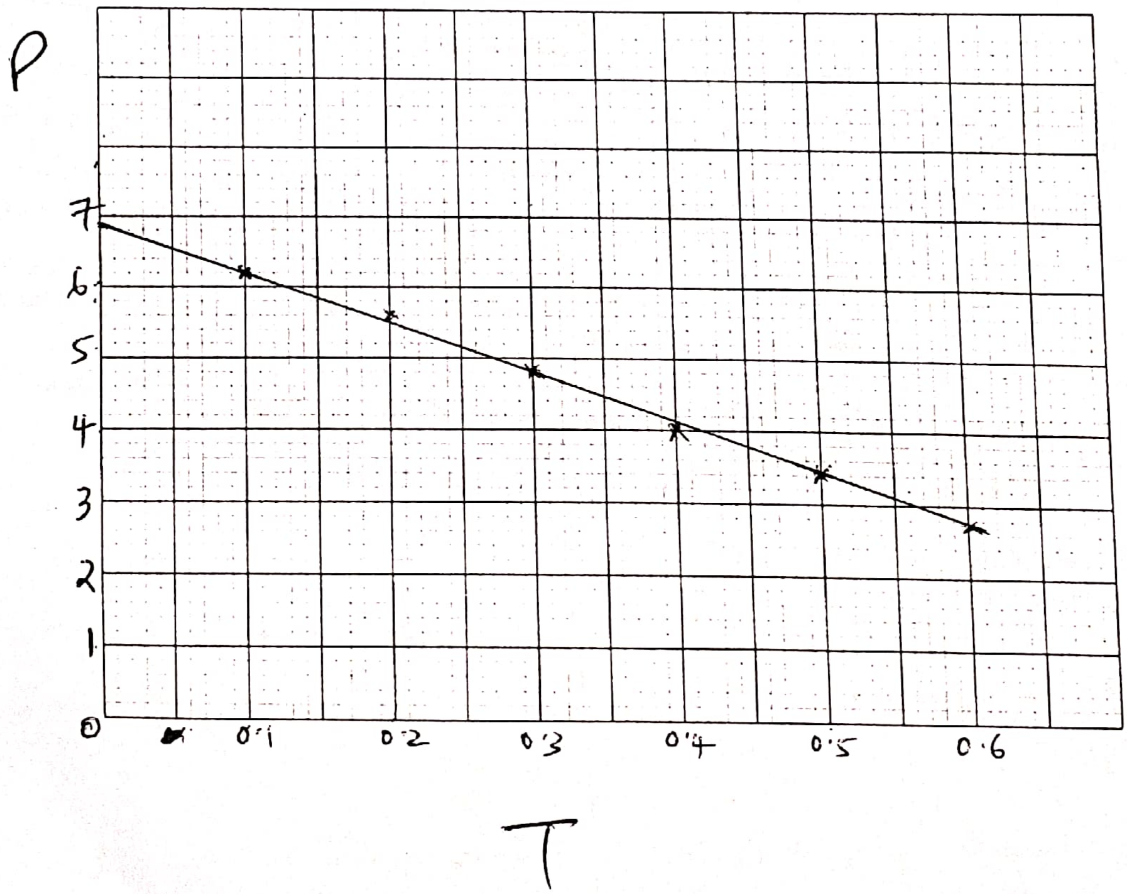
$\cos x = 0.5$, $x = 60^0$, 300^0

$\Rightarrow x = 60^0, 104.5^0, 255.5^0, 300^0$ ✓

16. The table below represents a relationship between two variables P and T connected by the equation $P = aT + b$ where a and b are constants.

T	0.1	0.2	0.3	0.4	0.5	0.6
P	6.2	5.6	4.8	4.0	3.4	2.7

On the grid provided, draw the line of best fit for the data (3mks)



P_1

S_1

L_1

SECTION II (50MKS)

Attempt ANY five (5) questions ONLY

17. An arithmetic progression is such that the first term is −5, the last term is 135 and the sum of the progression is 975.

(a) Calculate

(i) The number of terms in the series (4 marks)

$S_n = \frac{n}{2}(a+l)$

$975 = \frac{n}{2}[-5+135]$

$\frac{n}{2}(130) = 975$

$65n = 975$

$n = 15$

(ii) The common difference of the progression (2 marks)

$135 = -5 + (15-1)d$

$135 = -5 + 14d$

$140 = 14d$

$\Rightarrow d = 10$

(c) The sum of the first three terms of a geometric progression is 27 and first term is 36. Determine the common ratio and the value of the fourth term (4 marks)

$a + ar + ar^2 = 27$

$36 + 36r + 36r^2 = 27$

$36r^2 + 36r + 9 = 0$

$4r^2 + 4r + 1 = 0$ M1

$4r^2 + 2r + 2r + 1 = 0$

$2r(2r+1) + 1(2r+1) = 0$

A1

$r = -\frac{1}{2}$

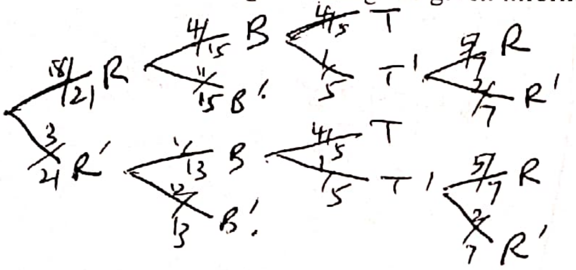
4^{th} term $= 36\left(-\frac{1}{2}\right)^3$ M1

$= 36\left(-\frac{1}{8}\right)$

$= -4.5$ A1

18. Mr. Maiyo, who works in a sugarcane plantation, owns a bicycle which he sometimes rides to work. Out of the 21 working days in a month, he rides to work for 18 days. If he rides to work, the probability that he is bitten by a rabid dog is $^{4}/_{15}$ otherwise it is only $^{1}/_{13}$. When he is bitten by the dog, the probability that he will get treated is $^{4}/_{5}$ and if he does not get treated, the probability that he will get rabies is $^{5}/_{7}$.

a. Draw a tree diagram using the given information. (3mks)



B 3

b. Using the tree diagram in (a) above, determine the probability that;

i. Maiyo will not be bitten by a rabid dog. (2mks)

$$\left(\frac{18}{21}\times\frac{11}{15}\right)+\left(\frac{3}{21}\times\frac{12}{13}\right)$$

$$\frac{22}{35}+\frac{12}{91}=\frac{346}{455}$$

ii. He will get rabies. (3mks)

$$\left(\frac{18}{21}\times\frac{4}{15}\times\frac{1}{5}\times\frac{5}{7}\right)+\left(\frac{3}{21}\times\frac{1}{13}\times\frac{1}{5}\times\frac{5}{7}\right)$$

$$\frac{8}{245}+\frac{1}{637}=\frac{109}{3185}$$

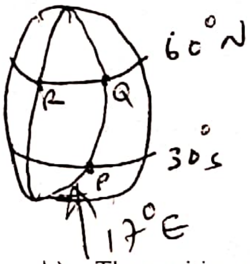
iii. He will not get rabies. (2mks)

$$\left(\frac{18}{21}\times\frac{4}{15}\times\frac{1}{5}\times\frac{2}{7}\right)+\left(\frac{3}{21}\times\frac{1}{13}\times\frac{1}{5}\times\frac{2}{7}\right)$$

$$\frac{16}{1225}+\frac{2}{3185}=\frac{218}{15925}$$

19. An aircraft leaves town P (30°S, 17°E) and moves directly towards Q (60°N, 17°E). It then moved at an average speed of 300 knots for 8 hours Westwards to town R. Determine

a) The distance PQ in nautical miles. 2400 nm (2mks)



$\theta = 60 + 30 = 90^\circ$

$D(nm) = 60\,\theta$

$= 60 \times 90^\circ = 5400\ nm$

b) The position of town R. (4mks)

$\frac{60\alpha \cos 60}{60} = \frac{2400}{60}$

$\alpha (0.5) = 40$

$\alpha = \frac{400}{5} = 80^\circ$

$80 - 17 = 63^\circ$

$R(60^\circ N, 63^\circ W)$

c) The local time at R if local time at Q is 3.12p.m (2mks)

$1^\circ = 4mins$

$80^\circ = ?$

$= 320 mins$

$\frac{320}{60} = 5hrs\ 20mins$

1512
520
0952hrs

or 9.52 a.m

d) The total distance moved from P to R in kilometers. (Take 1nm = 1.853km) (2 marks)

$2400 + 5400 = 7800\ nm$

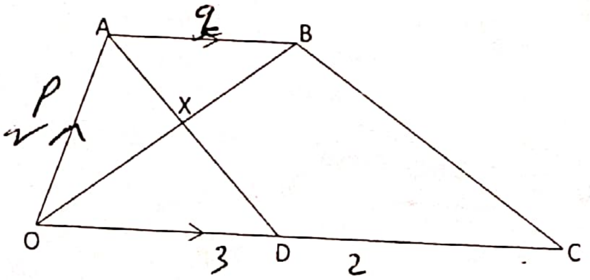
$1nm = 1.853 KM$

$7800nm = ?$

$7800 \times 1.853 = 14453.4\ KM$

20. In the figure below, OABC is a trapezium. AB is parallel to OC and OC = 5AB. D is a point on OC such that OD: DC = 3:2

$\frac{OC}{AB} = \frac{5}{1}$



a) Given that $OA = p$ and $AB = q$, express in terms of p and q

i) **OB** (1mk)

$p + q$ ✓ B1

ii) **AD** (2mks)

$-p + \frac{3}{5}(5q) = -p + 3q$ ✓M1 ✓A1 or $3q - p$

iii) **CB** (2mks)

$-5q + p + q = -4q + p$ ✓M1 ✓A1 or $p - 4q$

b) Lines OB and AD intersect at point X such that **AX** = k**AD** and **OX** = r**OB** where k and r are scalars. Determine the values of k and r. (5mks)

$\vec{OX} = r(p + q)$ ✓B1

$= rp + rq$ --- ①

$\vec{OX} = \vec{OA} + \vec{AX}$

$= p + k\vec{AD}$

$= p + k(-p + 3q)$

$= (1-k)p + 3kq$ ✓B1 --- ②

$r = 1 - k$

$r = 3k$

$1 - k = 3k$ ✓M1

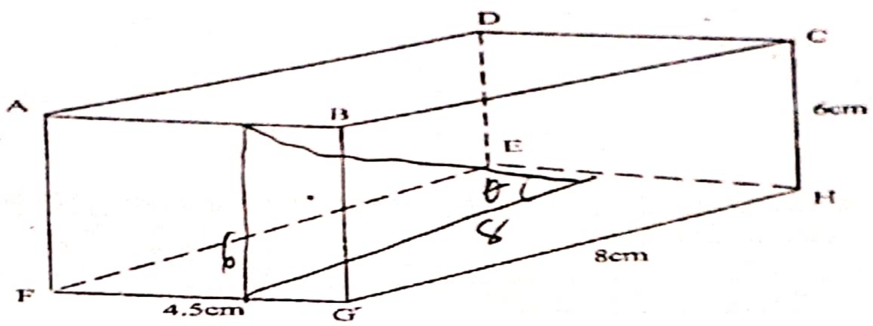
$4k = 1$

$k = \frac{1}{4}$ ✓A1

$r = 3\left(\frac{1}{4}\right)$

$= \frac{3}{4}$ B1

21. The diagram below represents a cuboid ABCDEFGH in which FG= 4.5 cm. GH=8cm and HC=6 cm



Calculate:

a) The length of FC (2mks)

$FH = \sqrt{4.5^2 + 8^2} = 9.179$

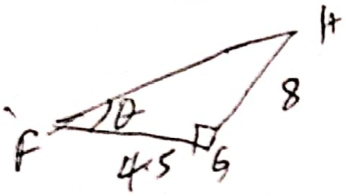
$FC = \sqrt{9.179^2 + 6^2}$

$= \underline{\underline{10.97}}$

b) (i) The size of the angle between the lines FC and FH (2mks)

$\tan\theta = \frac{6}{9.179} \Rightarrow \theta = \underline{\underline{33.17^\circ}}$

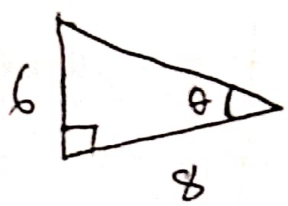
(ii) The size of the angle between the lines AB and FH (3mks)



$\tan\theta = 8/4.5$

$\Rightarrow \theta = 60.64^\circ$

c) The size of the angle between the planes ABHE and the plane FGHE (3mks)



$\tan\theta = 6/8$

$\theta = \underline{\underline{36.87^\circ}}$

22. The table below shows the rates of taxation in the year 2004

Income in K£ pa	Rate in Ksh per K£
1 – 3900	2
3901 – 7800	3
7801 – 11700	4
11701 – 15600	5
15601 – 19500	7
Above 19500	9

In that period, Juma was earning a basic salary of sh. 21,000 per month. In addition, he was entitled to a house allowance of sh. 9000 per month. and a personal relief of ksh.1056 per month. He also has an insurance scheme for which he pays a monthly premium of sh. 2000. He was also entitled to a tax relief of 15% of the premium paid.

a) Calculate how much income tax Juma paid per month. (7 marks)

Taxable income p.a $= \frac{(21000+9000)12}{20}$

$= K£\ 18000$

1st slab $= 3900 \times 2 = Ksh.\ 7800$

2nd slab $= 3900 \times 3 = Ksh.\ 11700$

3rd slab $= 3900 \times 4 = Ksh.\ 15600$

4th slab $= 3900 \times 5 = Ksh.\ 19500$

5th slab $= (18000 - 15600)7 = Ksh.\ 16800$

Gross tax p.a $= Ksh\ 71400$

Gross tax p.m $= Ksh.\ 5950$

Net tax = Gross tax − Reliefs

$= 5950 - \left[1056 + \frac{15}{100} \times 2000\right]$

$= 5950 - 1356$

$= 4594$

b) Juma's other deductions per month were cooperative society contributions of sh. 2000 and a loan repayment of sh. 2500. Calculate his net salary per month. (3 mks)

Taxable income $= 21000 + 9000$

$= 30,000$

Net salary = Taxable income − All deductions

$= 30,000 - [4594 + 200 + 2500 + 2000]$

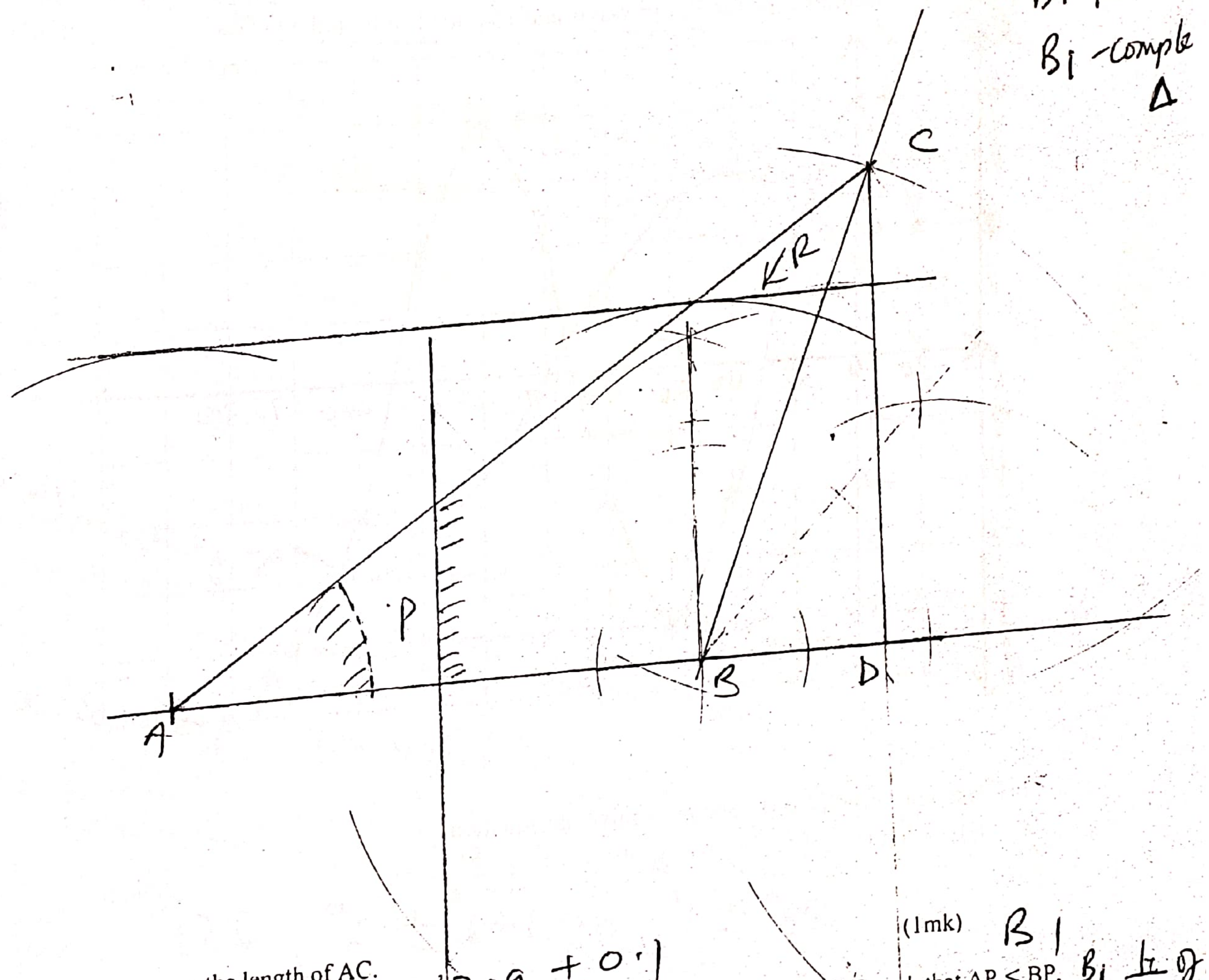
$= 30,000 - 11094$

$= 18906$

23. Use a ruler and compasses only for all construction in this question.

a) Construct a triangle ABC in which AB = 8cm, BC = 7.5cm and $\angle ABC = 112\frac{1}{2}^{\circ}$. (3 marks)

B1 for ∠
B1 for ∠112
B1 - complete Δ



(1mk) B1

b) Measure the length of AC. 12·9 ± 0·1

c) By shading the unwanted region show the locus of P within the triangle ABC such that AP ≤ BP, AP > 3cm. Mark the required region as P. (3mks)

B1 ⊥ bi
B1 for dot arc
B1 for P

d) Construct a normal from C to meet AB produced at D. ~~(1mk)~~

e) Locate the locus of R in the same diagram such that the arc of triangle ARB is $\frac{3}{4}$ the arc of the triangle ABC. (2mks)

$\frac{3}{4} \times 7 = 5.25$ ✓ B1
≈ 5.3
Location of R ✓ B1

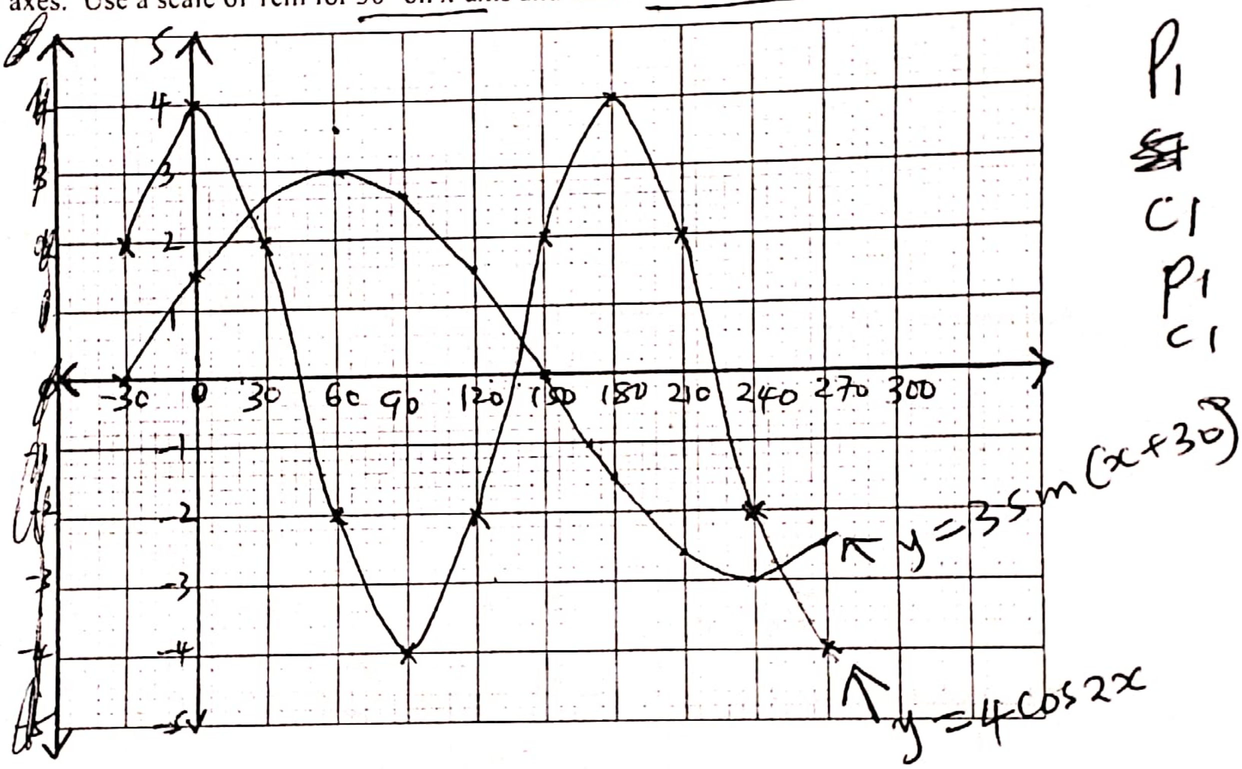
24. (a) Complete the table below for the functions y = 4 Cos 2x and y = 3 Sin $(2x + 30^0)$ giving the values to 1 decimal place. (2mks)

B2 - All correct values

B1 - for atleast 5 correct values

x	-30^0	0^0	30^0	60^0	90^0	120^0	150^0	180^0	210^0	240^0	270^0
y= 4 Cos 2x	2.0	4.0	2.0	-2.0	-4.0	-2.0	2.0	4.0	2.0	-2.0	-4.0
y=3 Sin $(x+30^0)$	0.0	1.5	2.6	3.0	2.6	1.5	0	-1.5	-2.6	-3.0	-2.6

(b) Draw the graphs of y = 4 Cos $2x^0$ and y = 3 Sin $(x + 30^0)$ for $-30^0 \leq x \leq 270^0$ on the same axes. Use a scale of 1cm for 30^0 on x-axis and 1cm for 1 unit on the y-axis. (4mks)



P1

S1

C1

P1

C1

(c) Use your graphs in (b) above to solve the equation:

(i) 3 Sin $(x + 30^0)$ – 4 Cos 2x = 0. (2mks)

B1 B1

$23^0 \pm 1^0$ or $142^0 \pm 2^0$, 252^0

(ii) 3 Sin $(2x + 30^0)$ + 1 = 0 (1mk) B1

3sin(2x+30) = -1, $x = 168^0 \pm 1^0$

(d) Determine the period of the function y = 4 Cos 2x. (1mk)

180^0

B1