MARKING SCHEME
(CONFIDENTIAL)

NAIROBI SCHOOL

Kenya Certificate of Secondary Education

121/2 - MATHEMATICS - Paper 2

(ALT A)

2 ½ hours

THE NAIROBI SCHOOL MOCK EXAMINATIONS

July/August, 2023

Name ______________________ Adm. No__________ Class________

Candidate's Signature ______________ Date ______________

Instructions to candidates

(i) *Write your name, admission number and class in the space provided above.*
(ii) *Sign and write the date of examination in the spaces provided above.*
(iii) *This paper consists of two sections: **Section I** and **Section II**.*
(iv) *Answer **all** the questions in **Section I** and **only five** questions from **Section II**.*
(v) *Show all the steps in your calculations, giving your answer at each stage in the spaces provided below each question.*
(vi) *Marks may be awarded for correct working even if the answer is wrong.*
(vii) *Non – programmable silent electronic calculators and KNEC Mathematical Tables may be used, except where stated otherwise.*
(viii) *This paper consists of 14 printed pages. Candidates should check the question paper to ascertain that all the pages are printed as indicated and that no questions are missing.*

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Section I

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	TOTAL

Section II

17	18	19	20	21	22	23	24	TOTAL

Grand Total

SECTION I (50 Marks)

Answer all the questions in this section in the spaces provided.

1. Solve for x in the equation $0.25^x = \frac{25^x}{25}$

(3 marks)

$$\left(\frac{0.25}{25}\right)^x = \frac{1}{25}$$

$$x \log 0.01 = \log \frac{1}{25} \checkmark M1$$

$$x = \frac{\log \frac{1}{25}}{\log 0.01} \checkmark M1 = 0.6989700043 \approx 0.6990 \checkmark A1$$

2. Use the binomial expansion of $(2 + x)^5$ up to the term in x^3 to estimate the value of $(1.99)^5$, correct to 4 significant figures.

(3 marks)

$$(2+x)^5 = 1 \cdot 2^5 \cdot x^0 + 5 \cdot 2^4 \cdot x^1 + 10 \cdot 2^3 \cdot x^2 + 10 \cdot 2^2 \cdot x^3 + \ldots$$

$$= 32 + 80x + 80x^2 + 40x^3 + \ldots \checkmark B1 \text{ (✓ expansion upto } x^3)$$

$$(2+x)^5 = (2-0.01)^5 \Rightarrow x = -0.01$$

$$1.99^5 \approx 32 + 80(-0.01) + 80(-0.01)^2 + 40(-0.01)^3 \checkmark M1$$

$$\approx 31.21 \text{ (4sf)} \checkmark A1$$

3. The cost, C shillings, of hiring a taxi varies partly as the time, t hours, and partly as the distance, d kilometres, travelled. The cost is sh. 1600 when t = 1 and d = 12. The cost is sh. 3750 when t = 3 and d = 15. John hired the taxi for 1 hour 45 minutes and paid sh. 2500. Determine how far he travelled.

(3 marks)

$$C = at + bd$$

$$\left.\begin{aligned} a + 12b &= 1600 \\ 3a + 15b &= 3750 \end{aligned}\right\} \checkmark B1$$

$$\begin{aligned} 3a + 36b &= 4800 \\ 3a + 15b &= 3750 \\ \hline 21b &= 1050 \\ b &= 50 \\ a &= 1000 \end{aligned} \checkmark M1$$

$$C = 1000t + 50d \qquad 2500 = 1000 \times 1.75 + 50d \checkmark M1$$

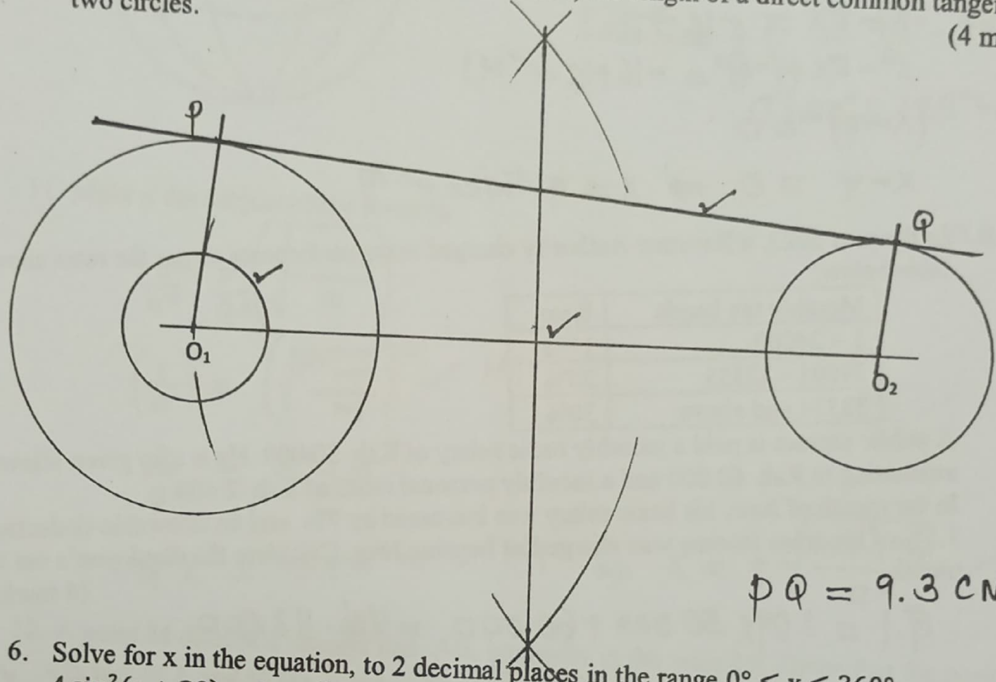
$$d = 15 \text{ km.} \checkmark A1$$

4. A factory packs canned beans in cylindrical cans of radius 5 cm and a height of 15 cm. The tins are then packed in large cylindrical cartons of diameter 50 cm. Determine the minimum height of a carton that should accommodate 100 such tins. (2 marks)

$$\frac{\pi \times 25 \times 25 \times h}{\pi \times 5 \times 5 \times 15} = 100 \checkmark \text{M1}$$

$$h = 60\text{ CM.} \checkmark \text{A1}$$

5. On the line segment O_1O_2 below, draw two circles, centres O_1 and O_2, of radii 2.5 cm and 1.5 cm, respectively. Determine, by construction, the length of a direct common tangent to the two circles. (4 marks)



B1 Circle, r = 1 cm centre O_1

B1 perp. bisector of O_1O_2

B1 ✓ Tangent

$PQ = 9.3\text{ CM} \pm 0.1\text{ CM} \checkmark$ B1

6. Solve for x in the equation, to 2 decimal places in the range $0° \leq x \leq 360°$
$4\sin^2(x+20) = 2\cos^2(x+20)$ (3 marks)

$$\frac{\sin^2(x+20)}{\cos^2(x+20)} = \frac{2}{4}$$

$$\tan^2(x+20) = \frac{1}{2} \checkmark \text{M1}$$

$$\tan(x+20) = \pm\frac{1}{\sqrt{2}}$$

$$(x+20) = \tan^{-1}\left(\frac{1}{\sqrt{2}}\right) = 35.26°, 215.26° \checkmark \text{M1}$$

$$x = 15.26°, 195.26°$$

$$\text{or } (x+20) = \tan^{-1}\left(-\frac{1}{\sqrt{2}}\right) = 144.74°, 324.74°$$

$$= 124.74°, 304.74°$$

$$\Rightarrow x = 15.26°, 124.74°, 195.26°, 324.74° \checkmark \text{A1}$$

7. A right pyramid on a circular base has a radius of 10.5 cm, measured to the nearest 0.1 cm. Given that the height of the pyramid is exact, calculate the percentage error in finding its volume. (3 marks)

$V_{Max.} = \frac{1}{3}\pi \times 10.55^2 \times h = 37.10083\pi h \rightarrow 37.10083\pi h$

$V_{Min.} = \frac{1}{3}\pi \times 10.45^2 \times h = 36.40083\pi h \rightarrow 36.40083\pi h$ ✓M1

$V_{Work.} = \frac{1}{3}\pi \times 10.5^2 \times h = 36.75\pi h.$

$\% E = \dfrac{\frac{1}{2}(37.10083 - 36.40083)\pi h}{36.75\pi h} \times 100$ ✓ M1 $= 0.9524\%$ ✓A1

Alt. 2

$\frac{0.05}{10.5} \times 2 \times 100$

$= 0.9524\%$

8. Solve for x by completing the square method. (2 marks)

$-x^2 + 8x - 16 = 0$

$x^2 - 8x = -16$

$x^2 - 8x + (-4)^2 = -16 + 16$ ✓ M1

$(x-4)^2 = 0$

$x - 4 = 0 \Rightarrow x = 4$ twice ✓A1

9. In the year 2023, a Revenue Authority charged taxes on incomes as per the rates shown in the table below.

Monthly tax bands	Rate
1 – 24000	10%
24001 - 32333	25%
32334 and above	30%

A public servant is paid a monthly basic salary of Ksh. 50 000. He is also given allowances amounting to Ksh. 60 000 and a monthly personal relief of Ksh. 2 400.
In the month of June, his basic salary was increased by 7%, and an allowable deduction of 1.5% of his gross income was charged as housing levy. Calculate the employee's net tax that month. (4 marks)

G.I = $1.07 \times 50\,000 + 60\,000$ = Ksh. 113 500

T.I = $113\,500 \times \left(\frac{100-1.5}{100}\right)$ = Ksh 111,797.50 ✓B1 ✓ Taxable income

1st slab: $0.1 \times 24\,000$ = Ksh. 2 400

2nd slab: $0.25 \times 8\,333$ = Ksh. 2 083.25 ✓ M1

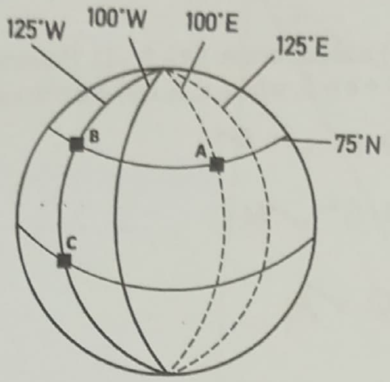
3rd slab: $0.3 \times 79\,464.50$ = Ksh. 23 839.35

Gross tax = Ksh. 28 322.60

Less MPR = Ksh. 2400.00 ✓M1

Net tax = Ksh. 25 922.60 ✓A1

10. The figure below represents a model of the globe. A, B and C are airports in different countries.
An airplane leaves airport A and travels along latitude 75°N using the shortest distance to airport B. From airport B, the airplane then travels due south to another airport C. If the total distance travelled by the airplane is 7796.43 nm, determine the position of airport C on the earth's surface.
(4 marks)



$AB = 60 \times 135 \cos 75°$

$BC = 60\theta$

$\therefore 2096.43 + 60\theta = 7796.43$ ✓M1

$\theta = \frac{5700}{60} = 95°$ ✓A1

Latitude of C $= 95° - 75° = 20°S$ ✓M1

$\therefore C(20°S, 125°W)$ ✓A1

11. Make x the subject of the formula:
(3 marks)

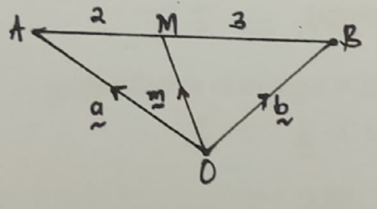
$$\frac{a^2}{a^3} = \frac{a^3}{a^3}\left(\sqrt{\frac{x^2 - m}{m}}\right)$$

$$\left(\frac{1}{a}\right)^2 = \left(\sqrt{\frac{x^2 - m}{m}}\right)^2 \checkmark M1$$

$$\frac{1}{a^2} = \frac{x^2 - M}{M}$$

$$a^2x^2 = m + a^2m \checkmark M1 \Rightarrow x = \pm\sqrt{\frac{m + a^2m}{a^2}} \checkmark A1 \text{ or } \pm\sqrt{m + ma^{-2}}$$

12. A point M divides a straight line, AB, internally in the ratio 2:3. Given that the position vectors of A and M are $3\mathbf{i} - \mathbf{j} - 2\mathbf{k}$ and $\mathbf{i} - \frac{1}{5}\mathbf{j}$, respectively, find the magnitude of $\overrightarrow{AB}$.
(3 marks)



Alt 1.

$\frac{3}{5}\underset{\sim}{a} + \frac{2}{5}\underset{\sim}{b} = \underset{\sim}{m}$

$\frac{3}{5}(3\underset{\sim}{i} - \underset{\sim}{j} - 2\underset{\sim}{k}) + \frac{2}{5}\underset{\sim}{b} = (\underset{\sim}{i} - \frac{1}{5}\underset{\sim}{j})$

$\underset{\sim}{b} = -2\underset{\sim}{i} + \underset{\sim}{j} + 3\underset{\sim}{k}$ ✓B1

$\underset{\sim}{AB} = \underset{\sim}{b} - \underset{\sim}{a} = (-2\underset{\sim}{i} + \underset{\sim}{j} + 3\underset{\sim}{k}) - (3\underset{\sim}{i} - \underset{\sim}{j} - 2\underset{\sim}{k})$

$= -5\underset{\sim}{i} + 2\underset{\sim}{j} + 5\underset{\sim}{k}$

$|\underset{\sim}{AB}| = \sqrt{(-5)^2 + (2)^2 + (5)^2}$ ✓M1 $= 7.348$ Units ✓A1

Alt 3

$|\underset{\sim}{AM}| = \sqrt{(-2)^2 + (\frac{4}{5})^2 + (2)^2}$

$= 2.939387691$

$|\underset{\sim}{AB}| = \frac{5}{2} \times 2.93938769$

$= 7.348$ Units

Alt.2 $\underset{\sim}{AM} = (\underset{\sim}{i} - \frac{1}{5}\underset{\sim}{j}) - (3\underset{\sim}{i} - \underset{\sim}{j} - 2\underset{\sim}{k}) = -2\underset{\sim}{i} + \frac{4}{5}\underset{\sim}{j} + 2\underset{\sim}{k}$

$\underset{\sim}{AB} = \frac{5}{2}\underset{\sim}{AM} = -5\underset{\sim}{i} + 2\underset{\sim}{j} + 5\underset{\sim}{k}$

$|\underset{\sim}{AB}| = \sqrt{(-5)^2 + (2)^2 + (5)^2} = 7.348$ Units.

13. Given that $3x = 2y$, find the ratio $(5x - 2y) : (x + y)$ (2 marks)

$x = \frac{2}{3}y$

$\left[5\left(\frac{2}{3}y\right) - 2y\right] : \left[\frac{2}{3}y + y\right]$ ✓ M1

$\frac{4}{3}y : \frac{5}{3}y \Rightarrow 4 : 5$ ✓ A1

Alt.

$\frac{x}{y} = \frac{2}{3} \Rightarrow x : y = 2 : 3$

$(5\times 2 - 2\times 3) : (2+3)$

$4 : 5$

14. A circle passes through the a point A(4,1) and has centre O(2.5, -1). Determine the equation of the circle in the form $x^2 + y^2 + ax + by + c = 0$, where a, b, and c are integers. (3 marks)

$r = \sqrt{(4-2.5)^2 + (1+1)^2} = 2.5$ ✓ B1

$(x - 2.5)^2 + (y+1)^2 = 2.5^2$ ✓ M1

$\Rightarrow x^2 + y^2 - 5x + 2y + 1 = 0$ ✓ A1

15. Without using calculators or mathematical tables, simplify the surd, and rationalize the denominator (3 marks)

$$\frac{\sqrt{8} + \sqrt{175}}{\sqrt{28} - \sqrt{112}}$$

$\frac{2\sqrt{2} + 5\sqrt{7}}{2\sqrt{7} - 4\sqrt{7}}$

$= \frac{2\sqrt{2} + 5\sqrt{7}}{-2\sqrt{7}}$ ✓ B1

$\frac{2\sqrt{7}(2\sqrt{2} + 5\sqrt{7})}{(-2\sqrt{7})(2\sqrt{7})}$ ✓ M1

$\frac{4\sqrt{14} + 10\times 7}{-4\times 7}$

$= \frac{70 + 4\sqrt{14}}{-28}$ or $-\frac{5}{2} - \frac{1}{7}\sqrt{14}$ ✓ A1

16. A team of 10 contestants took part in a Mathematics contest. The marks (x) scored out of 50 had a mean of 19 and a standard deviation of $\sqrt{13}$. The marks were then converted into percentages before being ranked with the rest of the contestants.
Determine: -

(a) the mean of the converted scores (2 marks)

2×19 ✓ M1

38 ✓ A1

(b) the standard deviation of the converted scores (2 marks)

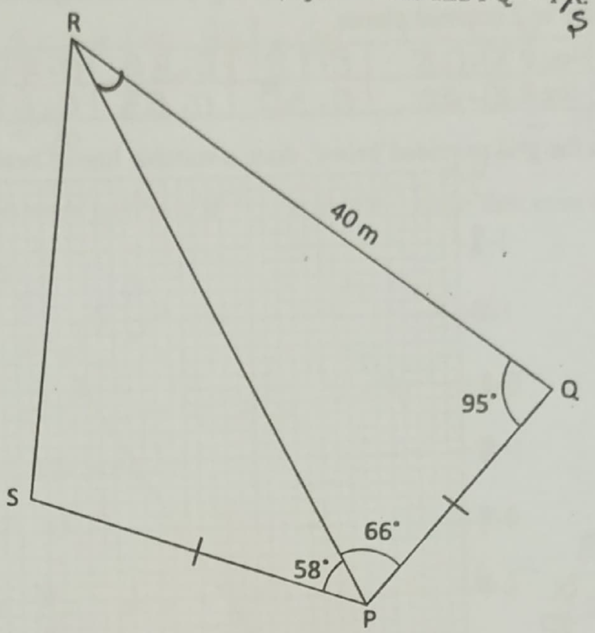
$2 \times \sqrt{13}$ ✓ M1

$2\sqrt{13}$ or 7.211 (4 s.f) ✓ A1

SECTION II (50 MARKS)

*Answer only **five** questions from this section in the spaces provided.*

17. The figure below represents a ranch in the shape of a quadrilateral PQRS. $\angle PQR = 95°$, $\angle QPR = 66°$, $\angle SPR = 58°$, $QR = 40$m and $PQ = P\cancel{R}S$.



Use the diagram to calculate, correct to 2 decimal places:

(a) the length of PR (2 marks)

$\frac{PR}{\sin 95°} = \frac{40}{\sin 66°}$ ✓M1 $\Rightarrow PR = 43.62M$ ✓A1

(b) the length of PQ (2 marks)

$\frac{PQ}{\sin 19°} = \frac{40}{\sin 66°}$ ✓M1 $\Rightarrow PQ = 14.26\,M$ ✓A1

(c) the length of RS (2 marks)

$RS^2 = 14.26^2 + 43.62^2 - 2 \times 14.26 \times 43.62 \cos 58°$ ✓M1

$RS = 38.04M$ ✓A1

(d) the area of triangle PQR in m^2 (2 marks)

$A = \frac{1}{2} \times 40 \times 14.26 \sin 95°$ ✓M1

$= 284.11\,M^2$ ✓A1 (or $284.03M^2$ or $284.12M^2$)

(e) The area of the ranch PQRS in m^2 (2 marks)

$A = 284.11 + \frac{1}{2} \times 14.26 \times 43.62 \sin 58°$ ✓M1

$= 547.86\,M^2$ (or $547.78M^2$ or $547.87M^2$) ✓A1

10

18. The table below represents a relationship between two variables Q and R.

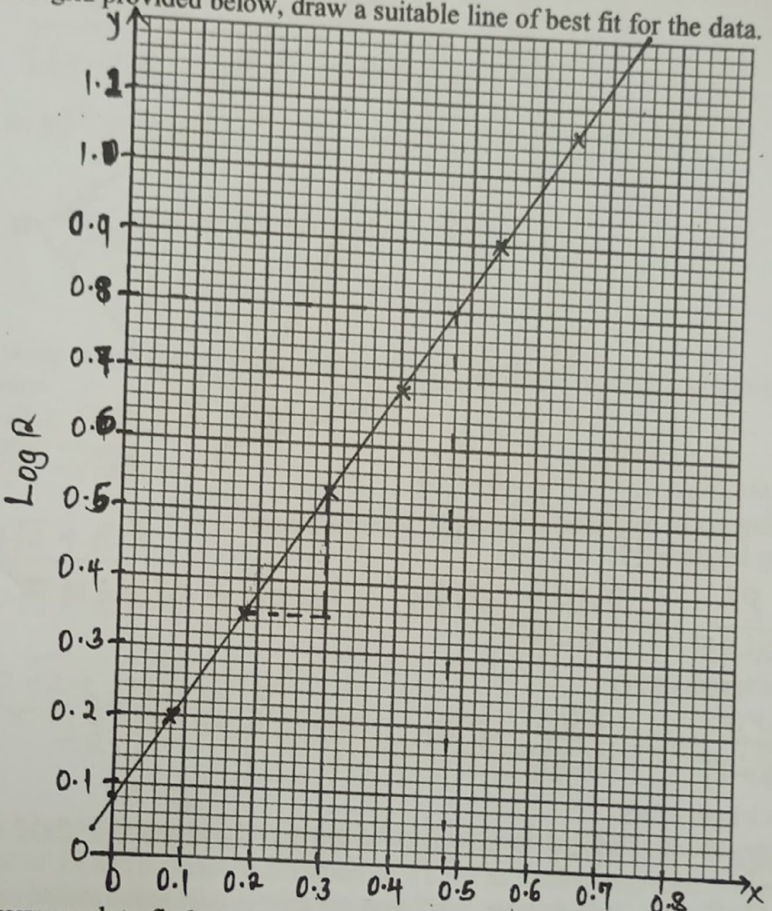
Q	1.2	1.5	2.0	2.5	3.5	4.5
R	1.58	2.25	3.39	4.74	7.86	11.5

The variables are connected by the equation $\log R = k\log Q + \log n$ where k and n are constants.

(a) Fill the table below for values of log Q and corresponding values of log R, correcting each value to 2 decimal places. (2 marks)

x	log Q	0.08	0.18	0.30	0.40	0.54	0.65 ✓B1
y	log R	0.20	0.35	0.53	0.68	0.90	1.06 ✓B1

(b) On the grid provided below, draw a suitable line of best fit for the data. (3 marks)



G1
P1
4

(c) Use your graph to find:

(i) The value of k (1 mark)

$$K = \frac{0.53-0.35}{0.3-0.18} = 1.5$$ ✓B1

(ii) The value of n (2 marks)

$\log n = 0.08$ ✓M1

$n = 10^{0.08} = 1.202264435 \approx 1.202$ (4sf) ✓A1

(iii) The value of R when Q = 3 (2 marks)

$\log 3 = 0.48$

$\Rightarrow \log R = 0.8$ $\quad R = 10^{0.8}$ ✓M1 $= 6.309573445 \approx 6.310$ (4sf) ✓A1

10

19. (a) Find the sum of the first 10 terms of the series
$\log 100 + \log 10000 + \log 1000000 + \cdots$ (3 marks)

Alt. 1

$2 + 4 + 6 + \cdots$ ✓ M1

$S_{10} = \frac{10}{5}\{(2\times 2) + (10-1)2\}$ ✓ M1

$= 110$ ✓ A1

Alt. 2

$S_{10} = \frac{10}{2}\{(2\log 100) + (10-1)\log 100\}$

$= 110$

(b) The first three terms of a GP are 3^{2x+1}, p and 81. If the first term of the GP is 729, determine the: -

(i) value of x (2 marks)

$3^{2x+1} = 3^6$ ✓ M1

$\therefore 2x+1 = 6 \Rightarrow x = 2.5$ ✓ A1

(ii) common ratio of the GP. (3 marks)

$729, p, 81$

$\frac{p}{729} = \frac{81}{p} = r$ ✓ M1

$p^2 = 729 \times 81$

$p = \pm 243$ ✓ A1

$\therefore r = \frac{81}{\pm 243}$

$= \pm \frac{1}{3}$ ✓ B1

(iii) sum of the first 7 terms of the GP. (2 marks)

$S_7 = \frac{729(1-(\frac{1}{3})^7)}{1-\frac{1}{3}} = 1093$ ✓ B1

$S_7 = \frac{729(1-(-\frac{1}{3})^7)}{1-(-\frac{1}{3})} = 547$ ✓ B1

10

20. In this question, use a ruler and a pair of compasses only.
    (a) In the space below, construct a rectangle KLMN in which KL = 80m and LM = 65m, using a suitable scale. (2 marks)

B1
B1 ✓ rectangle constructed

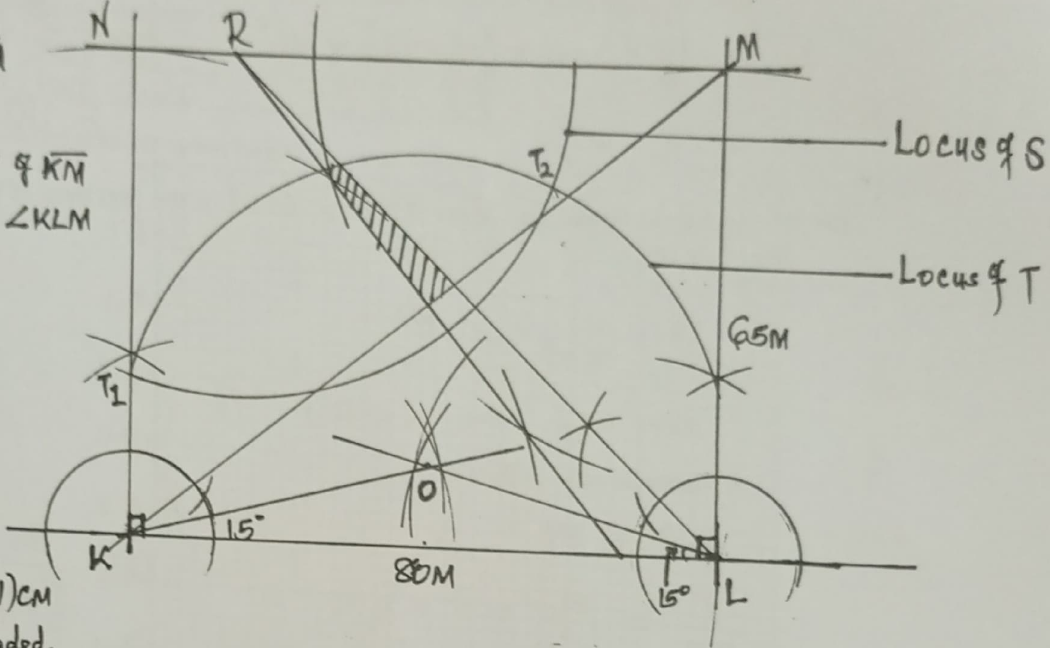
B1 ✓ perp. bisector of $\overline{KM}$
B1 ✓ bisector of ∠KLM
B1 ✓ loc. of R

B1 ✓ arc

B1 ✓ Centre O
B1 ✓ arc

B1 $T_1T_2 = (6.3 \pm 0.1)$cm
B1 ✓ region shaded.

10



(b) Inside the rectangle,
   (i) Construct the locus of a point which is equidistant from K and M. Construct another locus of a point equidistant from LK and LM.
   Let the two loci intersect at a point R. (3 marks)
   (ii) Construct the locus of a point S such that $RS = 46m$. (1 mark)
   (iii) Construct the locus of a point T such that ∠KTL = 75°. (2 marks)

(c) The locus of S and the locus of T intersect at T_1 and T_2. Measure T_1T_2. (1 mark)

$T_1T_2 =$ 6.3 cm ± 0.1 cm

(d) Shade the region bounded by the loci in (b) above, such that RS ≤ 46m and ∠KTL ≥ 75°.

(1 mark)

21. (a) A biased tetrahedron with faces marked 1,2,2 and 3 and a fair die with faces marked 1,2,3,4,5 and 6 are tossed together once and the number on the faces showing up recorded.

(i) Draw a possibility space to show the possible outcomes. (2 marks)

Fair die

Biased tetrahedron	1	2	3	4	5	6
1	1,1	2,1	3,1	4,1	5,1	6,1
2	1,2	2,2	3,2	4,2	5,2	6,2
2	1,2	2,2	3,2	4,2	5,2	6,2
3	1,3	2,3	3,3	4,3	5,3	6,3

✓B1
B1

(ii) What is the probability space if the two faces show the same number? (1 mark)

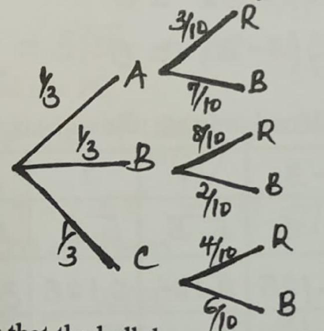
$P(1,1), P(2,2), P(3,3) \Rightarrow \frac{1}{24}, \frac{1}{12}, \frac{1}{24}$ ✓B1

(iii) Determine the probability that the sum of the numbers on the faces showing up is 5. (1 mark)

$P(\text{sum}=5) = \frac{4}{24}$ or $\frac{1}{6}$ ✓B1

(b) Three bags each contain 10 balls of the same shape and size, except for their colour. The first bag contains 3 red and 7 black balls; the second contains 8 red and 2 black balls, and the third contains 4 red and 6 black balls.
A bag is selected at random and a ball drawn from it.

(i) Represent the information above on a probability tree diagram. (2 marks)



✓B1
B1

(ii) Determine the probability that the ball drawn was red. (2 marks)

$P(AR \text{ or } BR \text{ or } CR)$

$= \frac{1}{3}\times\frac{3}{10} + \frac{1}{3}\times\frac{8}{10} + \frac{1}{3}\times\frac{4}{10}$ ✓M1 $= \frac{1}{2}$ ✓A1

Alt.
$P(R) = P(B)$
$P(R) + P(B) = 1$
$\Rightarrow P(R) = \frac{1}{2}$

(iii) What is the probability that the ball was drawn from either the first or the third bag, and was black? (2 marks)

$P(AB \text{ or } CB)$

$\frac{1}{3}\times\frac{7}{10} + \frac{1}{3}\times\frac{6}{10}$ ✓M1 $= \frac{13}{30}$ ✓A1

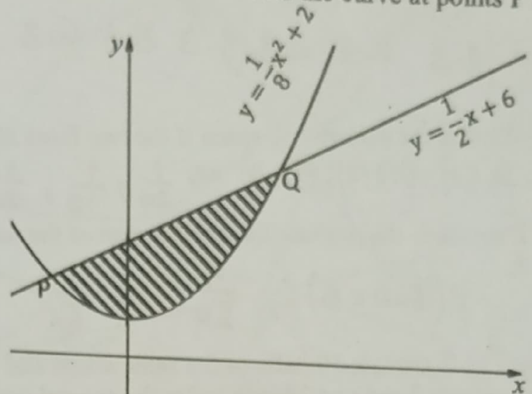
$\overline{10}$

22. (a) Find $\int \left(4 + \frac{1}{2}x - \frac{1}{8}x^2\right) dx$ (2 marks)

$$4x + \frac{1}{2\times2}x^2 - \frac{1}{8\times3}x^3 + c \quad ✓\text{M1}$$

$$4x + \frac{1}{4}x^2 - \frac{1}{24}x^3 + c \quad ✓\text{A1}$$

(b) In the figure below, the shaded region is bounded by the straight line $y = \frac{1}{2}x + 6$ and the curve $y = \frac{1}{8}x^2 + 2$. The straight line intersects the curve at points P and Q.



(i) Determine the coordinates of points P and Q. (2 marks)

$\frac{1}{8}x^2 + 2 = \frac{1}{2}x + 6$

$x^2 - 4x - 32 = 0$

$(x+4)(x-8) = 0$

$x + 4 = 0 \Rightarrow x = -4\ ;\ y = 4$

or $x - 8 = 0 \Rightarrow x = 8\ ;\ y = 10$

$\Rightarrow P(-4, 4)$ ✓B1 ; $Q(8, 10)$ ✓B1

(ii) Using the mid-ordinate rule with six ordinates, estimate the area of the shaded region (4 marks)

$h = \frac{8-(-4)}{6}$

$= 2$

x	−3	−1	1	3	5	7	
$y_1 = \frac{1}{2}x + 6$	4.5	5.5	6.5	7.5	8.5	9.5	
$y_2 = \frac{1}{8}x^2 + 2$	3.125	2.125	2.125	3.125	5.125	8.125	✓B1
$y = y_1 - y_2$	1.375	3.375	4.375	4.375	3.375	1.375	✓B1

$A = 2(1.375 + 3.375 + 4.375 + 4.375 + 3.375 + 1.375)$ ✓M1

$= 36.5$ sq. units. ✓A1

(c) Determine the exact area of the shaded region by integration (2 marks)

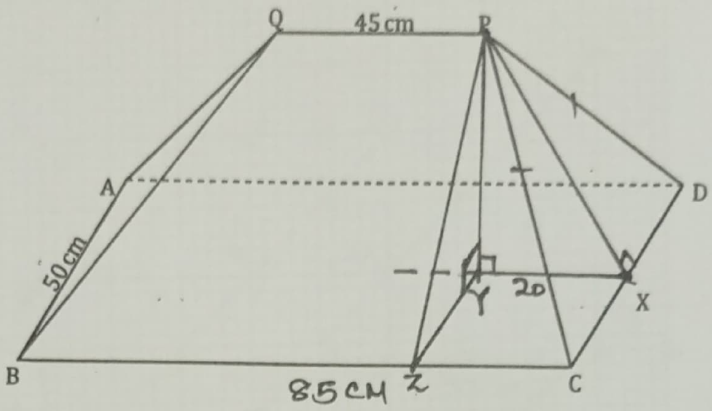
$$\left[4x + \frac{1}{4}x^2 - \frac{1}{24}x^3 + c\right]_{-4}^{8}$$

$$\left(4(8) + \frac{1}{4}(8)^2 - \frac{1}{24}(8)^3\right) - \left(4(-4) + \frac{1}{4}(-4)^2 - \frac{1}{24}(-4)^3\right) \quad ✓\text{M1}$$

$= 36$ sq. units. ✓A1

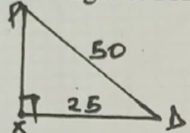
10

23. The figure below shows a model of a roof of a structure with a rectangular base ABCD. BC = 85 cm and AB = 50 cm. The ridge PQ = 45 cm and is centrally placed above the base. The faces ABQ and CDP are equilateral triangles and X is the midpoint of CD.



Calculate:

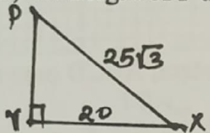
(a) (i) the length of PX ($\cancel{1}$ 2 mark)



$PX = \sqrt{50^2 - 25^2}$ ✓ M1

$= 25\sqrt{3} = 43.30$ CM ✓ A1

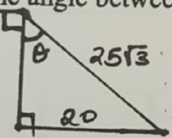
(ii) the height of P above the base ABCD. (2 marks)



$PY = \sqrt{(25\sqrt{3})^2 - 20^2}$ ✓ M1

$= 5\sqrt{59} = 38.41$ CM ✓ A1

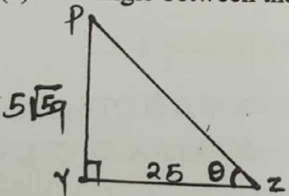
(b) The angle between plane CPD and line PQ (2 marks)



$90^\circ + \sin^{-1}\left(\frac{20}{25\sqrt{3}}\right)$ ✓ M1

$= 90^\circ + 27.51^\circ = 117.51^\circ$ ✓ A1

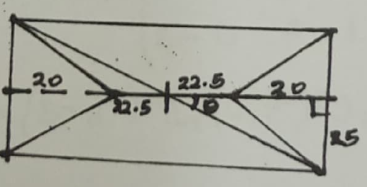
(c) The angle between the planes BCPQ and ABCD ($\cancel{3}$ 2 marks)



$\tan\theta = \frac{5\sqrt{59}}{25}$ ✓ M1

$\theta = 56.94^\circ$ ✓ A1

(d) The acute angle between lines PQ and AC. (2 marks)



$\tan\theta = \frac{25}{42.5}$ ✓ M1

$\theta = \tan^{-1}\left(\frac{25}{42.5}\right)$

$= 30.47^\circ$ ✓ A1

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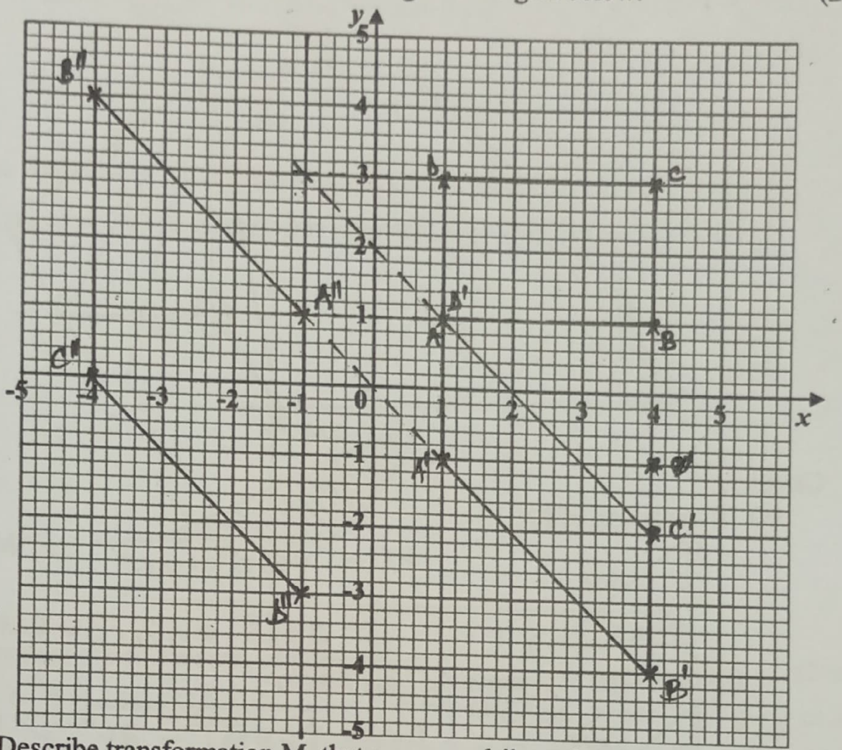
24. The vertices of a quadrilateral ABCD are A(1,1), B(4,1), C(4,3) and D(1,3). The vertices of its image under a transformation M, are A′(1,-1), B′(4, -4), C′(4,-2) and D′(1,1).

(a) Draw quadrilateral ABCD and its image on the grid below. (2 marks)



B1 ✓ object

B1 ✓ Image

B1 ✓ Image

Shear factor
$= -\frac{5}{5} = -1$

(b) (i) Describe transformation M, that maps quadrilateral ABCD onto quadrilateral A′B′C′D′. (2 marks)

B1 ✓ B1

Shear of factor −1 with x = −1 invariant and B(4,1) → B′(4,−4)

* (ii) Determine the matrix of transformation M. (3 marks)

may vary

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 1 & 4 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 4 \\ -1 & -4 \end{pmatrix} \checkmark M1 \quad \left.\begin{matrix} c + d = -1 \\ 4c + d = -4 \end{matrix}\right\} \begin{matrix} c = -1 \\ d = 0 \end{matrix}$$

$$\left.\begin{matrix} a + b = 1 \\ 4a + b = 4 \end{matrix}\right\} a = 1, b = 0 \checkmark M1 \text{ (solving S.E's)} \qquad \therefore \underset{\sim}{M} = \begin{pmatrix} 1 & 0 \\ -1 & 0 \end{pmatrix} \checkmark A1$$

(c) Quadrilateral A′′B′′C′′D′′ is the image of quadrilateral A′B′C′D′ under transformation $N = \begin{pmatrix} -1 & 0 \\ -1 & -2 \end{pmatrix}$. Determine the coordinates of quadrilateral A′′B′′C′′D′′. Hence, plot quadrilateral A′′B′′C′′D′′ on the same grid above. (3 marks)

$$\begin{pmatrix} -1 & 0 \\ -1 & -2 \end{pmatrix}\begin{pmatrix} 1 & 4 & 4 & 1 \\ -1 & -4 & -2 & 1 \end{pmatrix} = \begin{pmatrix} -1 & -4 & -4 & -1 \\ 1 & 4 & 0 & -3 \end{pmatrix} \checkmark B1$$

$\Rightarrow$ A″(−1,1), B″(−4,4), C″(−4,0), D″(−1,−3) ✓ B1

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