NAIROBI SCHOOL

End Term 1 Exam

121/2 Form 4

MATHEMATICS

Question Paper

April. 2023— 2 hours 30 minutes

FILL IN YOUR PERSONAL DETAILS HERE

Student Name:

Admission Number: | | | | | |

Class: 4

Instructions to candidates

(a) Write your name, admission number and class in the spaces provided above.
(b) This paper consists of two sections; **Section I** and **Section II**.
(c) Answer **all** the questions in **Section I** and **any five** questions from **Section II.**
(d) **Show all the steps in your calculations, giving your answers at each stage in the spaces provided below each question.**
(e) **KNEC Mathematical tables** may be used, except where stated otherwise.
(f) **Non-programmable** silent electronic calculators **must not** be used, except where stated otherwise.
(g) This paper consists of **16** printed pages.

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SECTION I(50 Marks)

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	TOTAL

SECTION II(50 Marks)

17	18	19	20	21	22	23	24	TOTAL

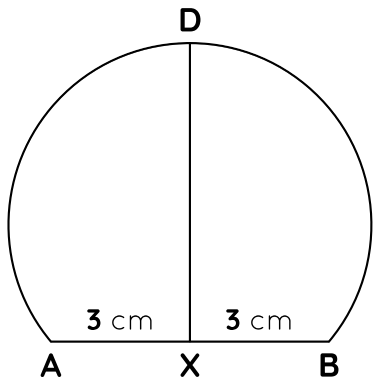
GRAND TOTAL

TURN OVER

SECTION ONE - 50 MARKS

Answer all questions from this section in the spaces provided.

1). In the figure below, not drawn to scale $\mathbf{AX} = \mathbf{XB} = \mathbf{3}$. Given that the circle has a radius of $\mathbf{4.5}$ cm.



Calculate to 2 decimal places, the length **XD**. **(3 marks)**

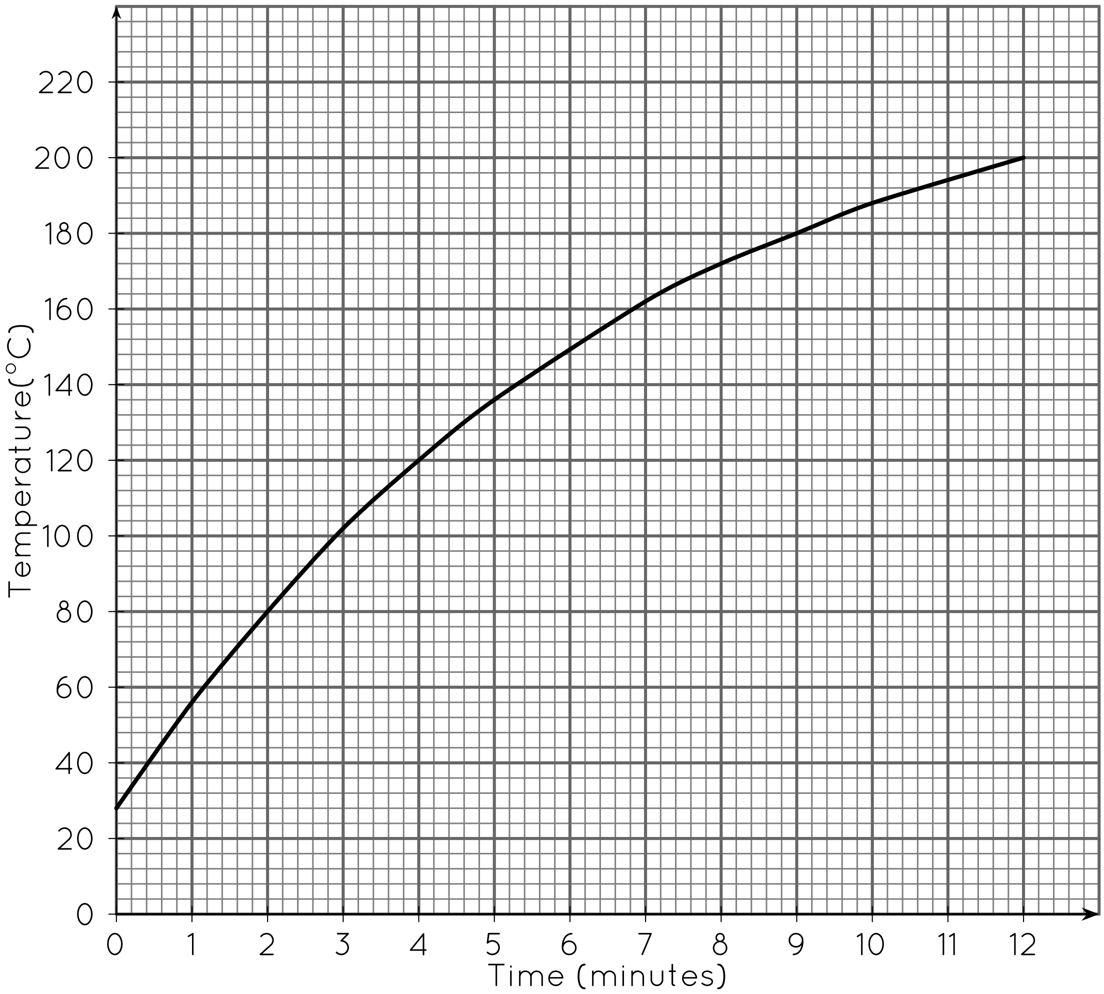
2). Solve $\mathbf{2}\sin^{\mathbf{2}}\theta - \cos^{\mathbf{2}}\theta = \mathbf{1} + \sin\theta$ for $\mathbf{0}^{\circ} \leq \theta \leq \mathbf{360}^{\circ}$ correct to 2 decimal places. **(4 marks)**

3). Tap **A** takes **3** hours to fill a tank when empty, Tap **B** takes **4** hours to fill the same tank when empty. Tap **C** takes **6** hours to empty the same tank when full. Tap **A** is opened, one hour later Tap **B** and Tap **C** are opened simultaneously. Calculate the total time it takes to fill the tank. **(4 marks)**

4). An object has an area of **16** cm^2. It is transformed using the matrix $\begin{pmatrix} 1 & 2 \\ -3 & 2 \end{pmatrix}$, find the area of the image formed. **(3 marks)**

5). The data below represents the ages in months at which **6** babies started walking; **9**, **11**, **12**, **13**, **11** and **10**. Without using a calculator, find the **exact value** of the variance of the data. **(3 marks)**

6). The graph below shows the rate of change of heating of a metal with respect to time.



Determine the average rate of heating of the metal between the **4**th and the **10**th minute correct to 2 decimal places. (2 marks)

7). Simplify: $$\frac{2}{\sqrt{14}-2\sqrt{3}}$$ (2 marks)

8). The cost (**C**) of hiring a venue for a delegates conference is partly fixed and partly varies inversely to the number **N** of delegates. When **200** delegates attend the cost is KES **4500** per delegate while for **150** delegates the cost is KES **5500** per delegate. Calculate the fixed cost. **(3 marks)**

9). The length of a rectangle is $\mathbf{y}$ cm. The width of the rectangle is $(\mathbf{x} - \mathbf{1})$ cm. Given that the perimeter and the area of the rectangle of the rectangle are **32** cm and **48** cm^{2} respectively, determine the values of $\mathbf{x}$ and $\mathbf{y}$. **(4 marks)**

10). Given that $\mathbf{y} = \mathbf{2}\cos(\mathbf{2x} - \mathbf{15})$, find

(a) Amplitude. **(1 mark)**

(b) Period. **(1 mark)**

11). Three boats **P**, **Q** and **R** are situated such that boat **Q** is **450** m on a bearing of **120°** from boat **P**. Boat **R** is **600** m on a bearing of **030°** from boat **Q**.

(a) Draw a sketch showing the positions of **P**, **Q** and **R**. **(1 mark)**

(b) Calculate the distance of boat **R** from boat **P**. **(2 marks)**

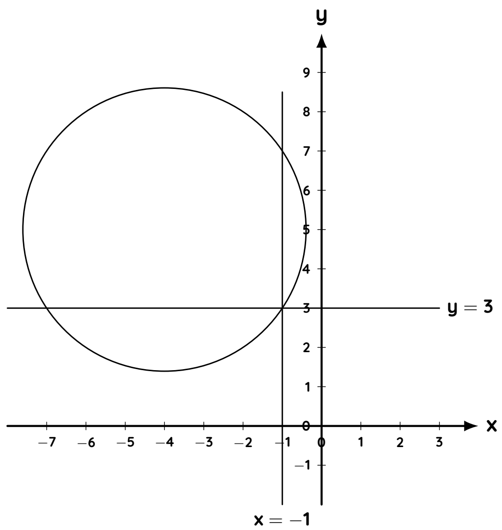
12). The table below shows income tax rates in the year 2018.

Monthly Income (KES)	Tax rates (%)
0 – 9680	**10**
9681 – 18800	**15**
18801 – 27920	**20**

In April 2018, the tax on Mutuku's monthly income after tax relief of KES **1162** was KES **2714**. Calculate Mutuku's monthly income. **(4 marks)**

13). A motorist travelling at a steady speed of **120** km/h covers a section of a highway in **10** minutes. To minimize accidents a speed limit is imposed. Travelling at the maximum speed allowed, the motorist takes **5** minutes longer to cover the same section. Calculate the speed limit imposed. **(3 marks)**

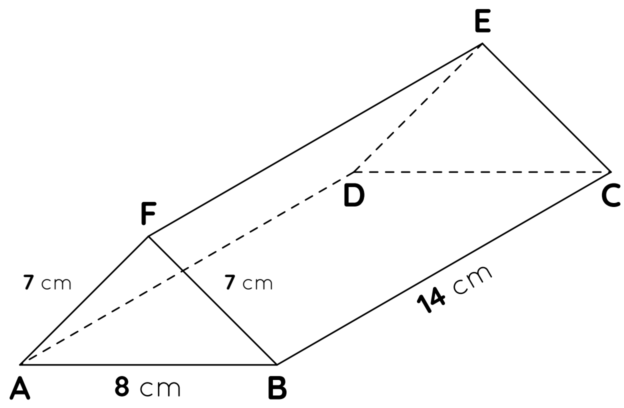
14). The circle shown below cuts the line $\mathbf{y} = -\mathbf{1}$ at $(-\mathbf{1}, \mathbf{3})$ and $(-\mathbf{1}, \mathbf{7})$. It also cuts the line $\mathbf{y} = \mathbf{3}$ at $(-\mathbf{7}, \mathbf{3})$ and $(-\mathbf{1}, \mathbf{3})$.



(a) Find the radius of the circle, leaving your answer in surd form. **(2 marks)**

(b) Determine the equation of the circle in the form $\mathbf{x^2} + \mathbf{y^2} + \mathbf{ax} + \mathbf{by} + \mathbf{c} = \mathbf{0}$. **(2 marks)**

15). The figure below represents a triangular prism. The faces **ABCD**, **ADEF** and **CBFE** are rectangles. **AB** $=$ **8** cm, **BC** $=$ **14** cm, **BF** $=$ **7** cm and **AF** $=$ **7** cm.



Calculate the angles between faces **BCEF** and **ABCD** correct to 1 decimal place. **(3 marks)**

16). A plane leaves airport **P**(**60**°N, **38**°W) at **9** am local time and flies due east at a speed of **400** knots to airport **Q**. The distance from **P** to **Q** is **3000** nm. Determine the local time in 12 hour clock system at airport **Q** when the aircraft lands there. **(3 marks)**

SECTION TWO - 50 Marks

*Answer any **five** questions from this section in the spaces provided.*

17). (a) Complete the table given below correct to **2** d.p **(2 marks)**

x	$0°$	$20°$	$40°$	$60°$	$80°$	$100°$	$120°$	$140°$	$160°$	$180°$
$y = 2\cos(2x - 40)$	1.53			0.35	-1.00		-1.88			1.53
$y = 3\sin 3x$		2.60		0		-2.60		2.60	2.60	

(b) Using the grid provided draw on the same axes the graph of $y = 2\cos(2x - 40)$ and $y = 3\sin 3x$ for $0° \leq x \leq 180°$.
Take **2** cm to represent **20°** on the **x**-axis and **2** cm for **1** unit on the y-axis. **(5 marks)**

(c) Use your graph to solve the equation $2\cos(2x - 40)3\sin 3x = 0$ **(2 marks)**

(d) State the period of the function $y = 2\cos(2x - 40)$ **(1 mark)**

Total: 10 marks

18). Bag **A** contains **2** green balls and **3** red balls while bag **B** contains **3** green balls and **4** red balls. Bag **A** has a probability of **25**% of being selected while **B**'s probability is **75**%. A bag is selected and two balls are drawn from the bag at random, one at a time without replacement.

(a) Draw a tree diagram to represent this information. **(2 marks)**

(b) Find the probability that the two balls are green and from bag **B**. **(2 marks)**

(c) Find the probability that the two balls are of different colours. **(3 marks)**

(d) What is the probability that the second ball is red. **(3 marks)**

Total: 10 marks

19). The table below shows the masses to the nearest kilogram of Form four students in a certain school.

Mass(kg)	35 – 39	40 – 44	45 – 49	50 – 54	55 – 59	60 – 64	65 – 69	70 – 74	75 – 79	80 – 84
Number of students	4	10	10	19	20	20	7	6	3	1

(a) State the median class. **(1 mark)**

(b) Taking an assumed mean of **62**kg calculate:

(i) The actual mean. **(4 marks)**

(ii) The variance of the distribution. **(3 marks)**

(iii) Hence or otherwise determine the standard deviation. **(2 marks)**

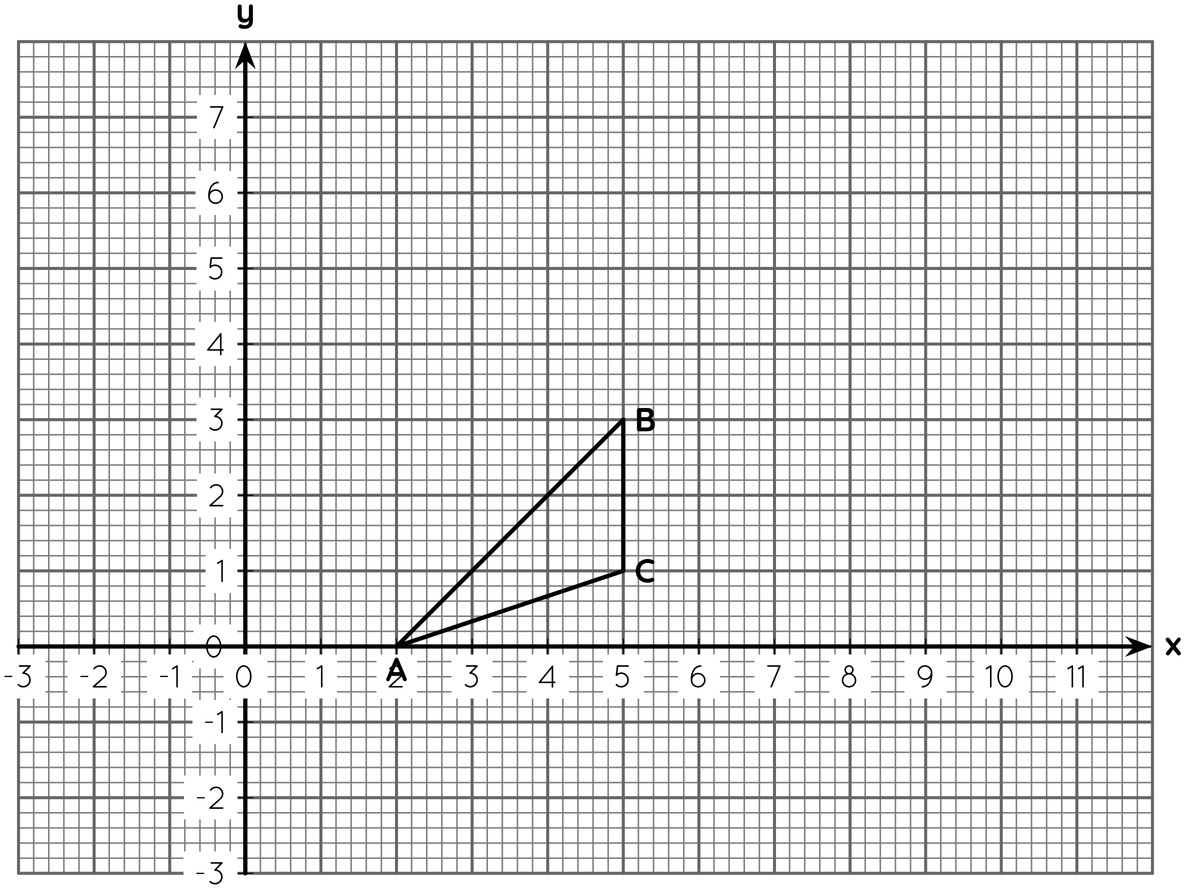
Total: 10 marks

20). The vertices of the triangle shown below are $\mathbf{A}(\mathbf{2},\mathbf{0})$, $\mathbf{B}(\mathbf{5},\mathbf{3})$ and $\mathbf{C}(\mathbf{5},\mathbf{1})$.

(a) Find the coordinates of triangle $\mathbf{A'B'C'}$, the image of triangle **ABC** after a transformation by the matrix $\mathbf{T} = \begin{pmatrix} -1/2 & 3/2 \\ 3/2 & -1/2 \end{pmatrix}$. **(2 marks)**

(b) Find the coordinates of triangle **ABC**, the image of triangle $\mathbf{A'B'C'}$ after a transformation by the matrix $\begin{pmatrix} 2 & 1 \\ 1 & 0 \end{pmatrix}$ **(2 marks)**

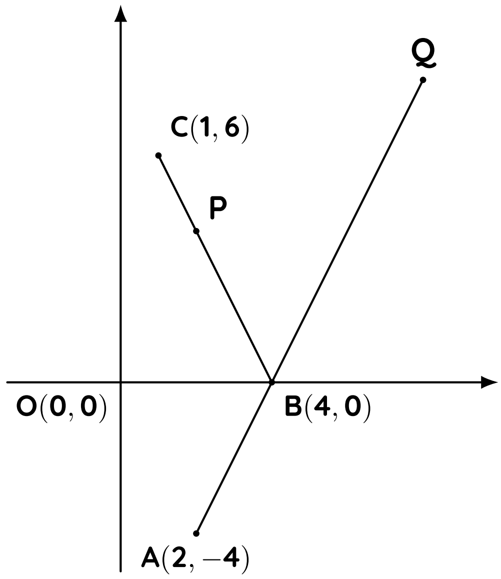
(c) Draw both triangle $\mathbf{A'B'C'}$ and triangle $\mathbf{A''B''C''}$ on the same grid as triangle **ABC**. **(2 marks)**



(d) Determine the single matrix can map triangle $\mathbf{A''B''C''}$ onto triangle **ABC**. **(4 marks)**

Total: 10 marks

21). In the diagram below, the coordinates of points A, B and C are as shown.



(a) Use a vector method to find the coordinates of point **D** given that **ABCD** is a parallelogram. **(2 marks)**

(b) Given that point **P** is on **BC** such that $\mathbf{BP} : \mathbf{PC} = \mathbf{2} : \mathbf{1}$, find the coordinates of **P**. **(2 marks)**

(c) If point **Q** lies on line **AB** is produced to such that $\mathbf{BQ} = \mathbf{2AB}$

(i) The position vector of **Q**. **(2 marks)**

(ii) Show that the points **D**, **P** and **Q** are collinear. **(4 marks)**

Total: 10 marks

22). An aircraft leaves town **P**(30°S, 17°E) and flies due north to **Q**(60°N, 17°E) . It then flies at an average speed of **300** knots for **8** hours due west to town **R**. Determine:

(a) The distance **PQ** in nautical miles. **(2 marks)**

(b) The position of town **R**. **(4 marks)**

(c) The local time at **R** if the local time at **Q** is **3.12** pm. **(2 marks)**

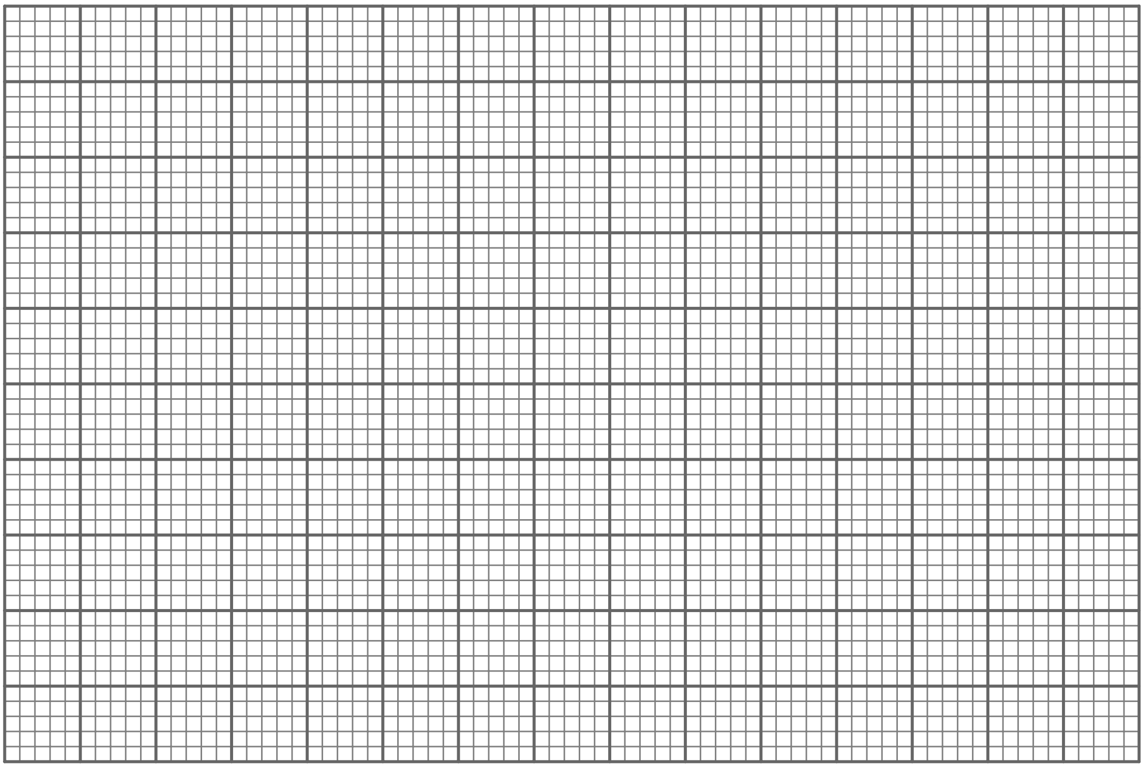
(d) The distance travelled by the aircraft from **Q** to **R** to the nearest kilometre. ($\pi = \frac{22}{7}$, $R = 6370$ km) **(2 marks)**

Total: 10 marks

23). A bus company runs a fleet of two types of buses operating between Nairobi and Nyeri. Type **A** bus has a capacity to take **70** passengers and **2000**kg luggage. Type **B** can carry **50** passengers and **3000**kg of luggage. On a certain day, at most **500** passengers with at least **35000**kg of luggage to be transported. The company could only use a maximum of **15** buses altogether.

(a) If the company uses **x** buses of type A and **y** buses of type B write down all the inequalities satisfying the given conditions. **(4 marks)**

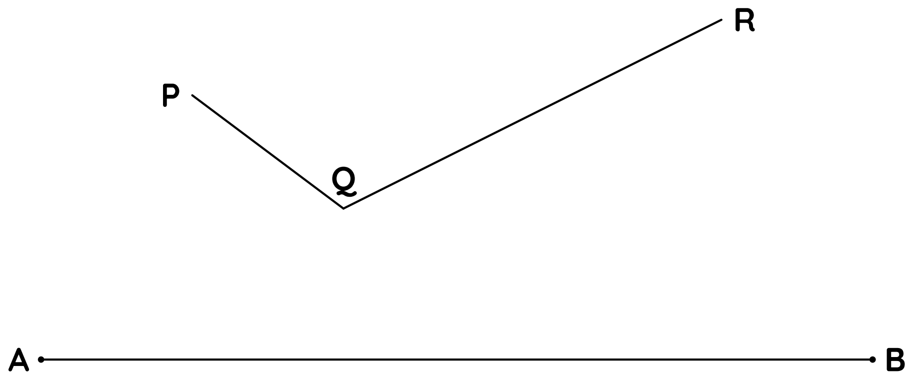
(b) Represent the inequalities graphically and use your graph to determine the least number of buses that could be used. **(4 marks)**



(c) If the cost of running one bus of type A is KES **7200** and that of running one bus of type B KES **6000**. Find the minimum cost of running the buses. **(2 marks)**

Total: 10 marks

24). In the figure below **AB**, **PQ** and **QR** are straight lines



(a) Use the figure to:

(i) find a point **S** on **AB** such that **S** is equidistant from **P** and **R**. **(1 mark)**

(ii) complete a heptagon **PQRSTVW** with **AB** as its line of symmetry and hence measure **Q** from **S**. **(5 marks)**

(b) shade the region within the heptagon in which a variable point **X** must lie given that **X** satisfies the following conditions:

(i) **X** is nearer to **TV** than to **TS**. **(1 mark)**

(ii) **SX** is less than **3** cm. **(1 mark)**

(iii) $\angle$**PXW** $\geq$ **90°**. **(2 marks)**

Total: 10 marks