NAME:.....M. SCHEME......................ADM NO.....001.........STREAM..................

SCHOOL..INDEX NO.......................DATE:.............

PINNACLE CLUSTER EXAMINATIONS

Kenya Certificate of Secondary Education

121/1

MATHEMATICS

PAPER 1

TIME: 2 ½ HOURS

JULY 2023

INSTRUCTIONS TO CANDIDATES

1. Write your name, index number, class and school in the spaces provided above.
2. This paper consists of TWO sections I & II
3. Answer **ALL** the questions in section I and only **FIVE** questions from section II
4. All answers and working must be written on the question paper in the spaces provided below each question.
5. Show all the steps in your calculations giving your answers at each stage in the spaces below each question.
6. Marks may be given for correct working even if the answer is wrong.
7. Non-programmable silent electronic calculators and KNEC mathematical tables may be used except where stated otherwise.

FOR EXAMINERS USE ONLY

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	TOTAL

17	18	19	20	21	22	23	24	TOTAL

GRAND TOTAL	

SECTION I (50 Marks)

Answer all the Questions in this section in the spaces provided.

1. Evaluate: $\dfrac{2\frac{1}{2}\ of\ 1\frac{3}{4}-5\frac{1}{4}}{1\frac{2}{5}+2(1\frac{1}{4}-2\frac{3}{4})}$ (3mks)

Numerator

$= \frac{5}{2}$ of $\frac{7}{4} - \frac{21}{4}$

$= \frac{35}{8} - \frac{21}{4}$

$= -\frac{7}{8}$ M1

Denominator

$= \frac{7}{5} + 2\left(\frac{5}{4} - \frac{11}{4}\right)$

$= \frac{7}{5} + 2 \times -\frac{6}{4}$

$= -\frac{8}{5}$ M1

Hence; $-\frac{7}{8} \times -\frac{5}{8}$

$= \frac{35}{64}$ A1

03

2. Madam Veronica has 36 chemistry books, 32 biology books and 28 physics books. She wishes to arrange the books in groups such that each group has the same number of each book without any book being left out. Calculate the least number of books that can be found in each group. (3mks)

2	36	32	28
2	18	16	14
	9	8	7

GCD = 2 × 2 = 4. M1

No. of books $= \frac{36}{4} + \frac{32}{4} + \frac{28}{4}$ M1

$= 9 + 8 + 7$

$= 24.$ A1

03

3. Simplify the expression: $\dfrac{3x^2 - 4xy + y^2}{9x^2 - y^2}$ (3mks)

Numerator

$= 3x^2 - 4xy + y^2$

$= (3x^2 - 3xy) - (xy + y^2)$

$= 3x(x - y) - y(x - y)$

$= (3x - y)(x - y)$ M1

Denominator

$= 9x^2 - y^2$

$= (9x^2 + 3xy) - (3xy - y^2)$

$= 3x(3x + y) - y(3x + y)$

$= (3x - y)(3x + y)$ M1

$\dfrac{(3x - y)(x - y)}{(3x - y)(3x + y)}$

03

$= \dfrac{x - y}{3x + y}.$ A1

4.Given that **OA** = 3i + 4**j** and **OB** = 9**i** – 5**j**, a point T is on **AB** such that **AT** = 2 **TB**. Calculate the magnitude of **OT** to 4 significant figures. (3mks)

$\underset{\sim}{AB} = \underset{\sim}{AO} + \underset{\sim}{OB}$

$= \begin{pmatrix}3\\4\end{pmatrix} + \begin{pmatrix}9\\-5\end{pmatrix}$

$= \begin{pmatrix}6\\-9\end{pmatrix}$

$AT = \frac{2}{3}\underset{\sim}{AB}$

$\underset{\sim}{AT} = \frac{2}{3}\begin{pmatrix}6\\-9\end{pmatrix} = \begin{pmatrix}4\\-6\end{pmatrix}$

$\underset{\sim}{OT} = \underset{\sim}{OA} + \underset{\sim}{AT}$

$= \begin{pmatrix}3\\4\end{pmatrix} + \begin{pmatrix}4\\-6\end{pmatrix}$

$= \begin{pmatrix}7\\-2\end{pmatrix}$ M1

$|\underset{\sim}{OT}| = \sqrt{(7)^2 + (-2)^2}$ M1

$= \sqrt{49+4}$

$= \underline{7.280 \text{ units}}$ A1

$\overline{\underline{03}}$

5. A football match last 90 minutes with a break of 15 minutes at half – time. If a referee allows five minutes extra for injuries and stoppages, what time does a match which kicks off at 4:30 p.m. end? (3mks)

Total time taken = 90 + 15 + 5

= 110 minutes = 1 hour 50 minutes. M1

Time to end = 4.30 + 1.50 M1

= $\underline{6.20 \text{ pm.}}$ A1

$\overline{\underline{03}}$

6. A laptop has a mass of 0.8kg and a density of 0.8g/cm^3. If its length is 30cm and breadth 20cm, calculate its thickness. (3mks)

$\rho = \frac{\text{mass}}{\text{Volume}}$

Mass = 0.8 × 1000

= 800g.

$0.8 = \frac{800}{x}$

Volume = $x = \frac{800}{0.8} = 1000 \text{ cm}^3$ M1

1000 = 30 × 20 × y M1

y = $\underline{1.667 \text{ cm}}$ or $\underline{1\frac{2}{3} \text{ cm}}$ A1

$\overline{\underline{03}}$

7. The masses of two similar bars of soaps are 343 g and 1331 g. If the surface area of the smaller bar is 196 cm^2. Calculate the surface area of the longer bar. (3mks)

$V.S.F = \frac{1331}{343}$

$L.S.F = \left(\frac{1331}{343}\right)^{1/3} = 11/7$ M1

$A.S.F = \left(\frac{11}{7}\right)^2 = \frac{121}{49}.$

$\frac{121}{49} = \frac{x}{196}.$ M1

$x = \frac{121 \times 196}{49}$

$= 484\ cm^2.$ A1

03

8. It would take 15men 8days to dig a trench of 240m long. Find how many days it would take 18men to dig a trench 360meters long working at the same rate. (3mks)

$15/18 \times 8 \times \frac{360}{240}$

$= 10$ days A1

03

9. Find the values of x and y in the equation $2^{\frac{3x}{2}} \times 3^{2y} = 5184.$ (3mks)

$2^{3/2 x} \times 3^{2y} = 2^6 \times 3^4.$ M1

$\frac{3x}{2} = 6$

$x = 4$ A1

$3^{2y} = 3^4$

$2y = 4$

$y = 2.$ A1

03

10. A regular polygon is such that its exterior angle is one eighth the size of interior angle. Find the number of sides of the polygon. (3mks)

Let the exterior be $\frac{x}{8}$

Interior $= x$.

$8\left(\frac{x}{1}\right)+\left(\frac{x}{8}\right)^{8}=\left(\frac{180}{1}\right)^{8}$ ✓

$9x = 1440$

$x = 160$. M1

Exterior $= \frac{160}{8} = 20°$

No. of sides $= \frac{360}{20}$ ✓ M1

$= \underline{\underline{18 \text{ sides}}}$ ✓ A1

$\underline{03}$

11. Find all the integral values of x which satisfy the inequality. (3mks)

$3(1 + x) < 5x - 11 < x + 45$

$3 + 3x < 5x - 11$ ... (1)

$3x - 5x < -11 - 3$

$-2x < -14$

$x > 7$ M

$5x - 11 < x + 45$

$4x < 56$

$x < 14$ M

$7 \leq x < 14$

Integral values are; 8, 9, 10, 11, 12 and 13

A1

{For all the integral values} A1

$\underline{03}$

12. A translation vector $\begin{pmatrix} x-1 \\ 2-y \end{pmatrix}$ maps a point A (4,6) onto A' (9,12). Find the value of x and y. (3mks)

$A + T = I$

$\begin{pmatrix} x-1 \\ 2-y \end{pmatrix} + \begin{pmatrix} 4 \\ 6 \end{pmatrix} = \begin{pmatrix} 9 \\ 12 \end{pmatrix}$ M1

$x + 3 = 9$

$x = 9 - 3$

$x = 6$ B1

$2 - y + 6 = 12$

$-y + 8 = 12$

$-y = 4$

$y = -4$. B1

$\underline{03}$

(*) 13. Amoit bought 2 pens and 5 exercise books at a cost of sh. 275. Allan bought 4 such pens and 5 exercise books from the same shop at a cost of sh. 415. By letting sh. x and y to be the costs of a pen and a book respectively, find the cost of each item (4mks)

$2x + 5y = 275 \; \text{----- (i)}$

$4x + 5y = 415 \; \text{--- (ii)}$ MB M1

$2x + 5y = 275$

$4x + 5y = 415$ −

$-2x = 140$

$x = 70.$ A1

$2(140) + 5y = 275$

$5y = 275 - 140.$

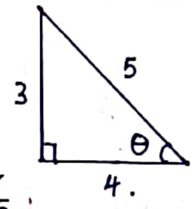
$5y = 135$

$y = 27$ A1 { Each values of x and y }.

04

14.Given that $\sin\alpha = \frac{3}{5}$, Evaluate $\frac{\sin\alpha + \tan\alpha}{\cos\alpha - \tan\alpha}$ without using tables or Calculator. (3mks)

SOHCAHTOA.



$\sin\theta = 3/5.$

$\frac{\sin\alpha + \tan\alpha}{\cos\alpha - \tan\alpha} = \frac{\frac{3}{5} + \frac{3}{4}}{\frac{4}{5} - \frac{3}{4}}$ MB

Num $= \frac{3}{5} + \frac{3}{4} = \frac{27}{20}$

Denom $= \frac{4}{5} - \frac{3}{4} = \frac{1}{20}$ B1

$= \frac{27}{20} \times \frac{20}{1}$

$= 27$ A

03

15. A circle of radius 28cm is divided into ten equal sectors. In each sector, find:

a) The area of the triangle. (2mks)

$360/10 = 36$

Area $= \frac{1}{2} \times 28 \times 28 \times \sin 36°$ M1.

$= 230.4118 \text{ cm}^2$ A1

02

(b)The area of the segment. (Take $\pi = {}^{22}/_7$) (2mks)

Area $= \left(\frac{36}{360} \times \frac{22}{7} \times 28 \times 28\right) - \left(\frac{1}{2} \times 28 \times 28 \times \sin 36°\right)$

$= 246.4 - 230.4118$ M1

$= 15.9882 \text{ cm}^2.$ A1

02

16. Use the exchange rates below to answer this question.

	Buying	Selling
1 US dollar	63.00	63.20
1 UK £	125.30	125.95

A tourist arriving in Kenya from Britain had 9600 UK Sterling pounds (£). He converted the pounds to Kenya shillings at a commission of 5%. While in Kenya, he spent ¾ of this money. He changed the balance to US dollars after his stay. If he was not charged any commission for this last transaction, calculate to the nearest US dollars, the amount he received. (3 marks)

KEFOSE :
FOKEBU:

If 1£ = 125.30.
9600 £ = ?

$\frac{9600 \times 125.30}{1}$

$= 1180\,80 \times \frac{95}{100}$

$= 112\,176$ /= M1

Remaining $= \frac{1}{4} \times 112176 = 28\,044$/=

If 1 US dollar = 63.20/=
?? = 28044/=

$\frac{28044 \times 1}{6320}$ M1

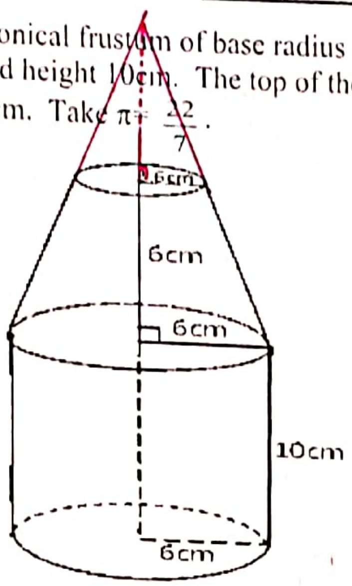
= 444 dollars A1

03

SECTION II (50 Marks)

Attempt only 5 questions in this section

17. A right conical frustum of base radius 6cm is mounted on top of a cylinder of the same base radius and height 10cm. The top of the solid frustum is of radius 3.6cm. The height of frustum is also 6cm. Take $\pi = \frac{22}{7}$.



$\frac{3.6}{6} = \frac{h}{6+h}$

$21.6 + 3.6h = 6h.$

$2.4h = 21.6$

$h = 9$ M_1

Calculate:

(a) The total surface area of the solid. (6mks)

S·Area of the frustum $= (\pi RL - \pi rl).$

$= \left(\frac{22}{7} \times 6 \times 16.16\right) - \left(\frac{22}{7} \times 3.6 \times 9.693\right) + \left(\frac{22}{7} \times 3.6^2\right)$

$= (304.7314 - 109.66937) + (40.731429) = 235.793459.$

S·Area of the cylinder $= 2\pi rh + \pi r^2$

$= \left(2 \times \frac{22}{7} \times 6 \times 10\right) + \left(\frac{22}{7} \times 6 \times 6\right)$

$= 490.28571\ cm^2$

Total S·Area $= 235.793459 + 490.28571.$

$= 726.079$

(b) The volume of the solid. (4mks)

Volume of the frustum $= \frac{1}{3}\pi R^2 H - \frac{1}{3}\pi r^2 h$ $= \left(\frac{1}{3} \times \frac{22}{7} \times 6 \times 6 \times 15\right) - \left(\frac{1}{3} \times \frac{22}{7} \times 3.6 \times 3.6 \times 9\right)$

$= 565.7142 - 122.1942$ $= 443.52$

Volume of the cylinder $= \frac{22}{7} \times 6 \times 6 \times 10 = 1131.428571$

Total volume $= 443.52 + 1131.428571$

$= 1574.95\ cm^3$

18. (a) Find the inverse of the matrix $A=\begin{pmatrix}4 & 3\\ 3 & 2\end{pmatrix}$. (1mk)

$\det A = 2.4 - 3.3$
$= 8 - 9$
$= -1$

$A^{-1} = \frac{1}{-1}\begin{pmatrix}2 & -3\\ -3 & 4\end{pmatrix}$
$= \begin{pmatrix}-2 & 3\\ 3 & -4\end{pmatrix}$ B1

01

b) Peter bought 20 bags of oranges and 15 bags of mangoes for a total of Ksh. 9,500. Wilson bought 30 bags of oranges and 20 bags of mangoes for a total of Ksh. 13,500. If the price of a bag of oranges is x and that of mangoes is y;

i) Form two equations to represent the information above. (2mks)

$20x + 15y = 9500$ ------ (i) B1
$30x + 20y = 13500$ ------ (ii) B1

02

or

$4x + 3y = 1900$ ---- (i)
$3x + 2y = 1350$ ---- (ii)

02

ii) Hence use the matrix A^{-1} above to find the price of one bag of each item. (4mks)

$\begin{pmatrix}4 & 3\\ 3 & 2\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix} = \begin{pmatrix}1900\\ 1350\end{pmatrix}$ M1

$\begin{pmatrix}-2 & 3\\ 3 & -4\end{pmatrix}\begin{pmatrix}4 & 3\\ 3 & 2\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix} = \begin{pmatrix}-2 & 3\\ 3 & -4\end{pmatrix}\begin{pmatrix}1900\\ 1350\end{pmatrix}$ M1

04

$\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix} = \begin{pmatrix}250\\ 300\end{pmatrix}$ M1

$x = 250$
$y = 300$. A1 B

c) The price of each bag of oranges increased by 10% and that of mangoes reduced by 10%. The businessman (Peter and Wilson) bought as many oranges and as many mangoes as they bought earlier. Find by matrix method, the total cost of oranges and mangoes that the businessmen bought after the percentage change. (3mks)

New prices:

$\left.\begin{array}{l}\text{Orange} \Rightarrow \frac{110}{100} \times 250 = 275/=\\ \text{Mango} \Rightarrow \frac{90}{100} \times 300 = 270/=\end{array}\right\}$ M1

03

$\begin{pmatrix}20 & 15\\ 30 & 20\end{pmatrix}\begin{pmatrix}275\\ 270\end{pmatrix} =$

Peter $= \begin{pmatrix}20 & 15\end{pmatrix}\begin{pmatrix}275\\ 270\end{pmatrix} = 9550/=$ A1

Wilson $\begin{pmatrix}30 & 20\end{pmatrix}\begin{pmatrix}275\\ 270\end{pmatrix} = 13650/=$ A1

Total cost = 23 200

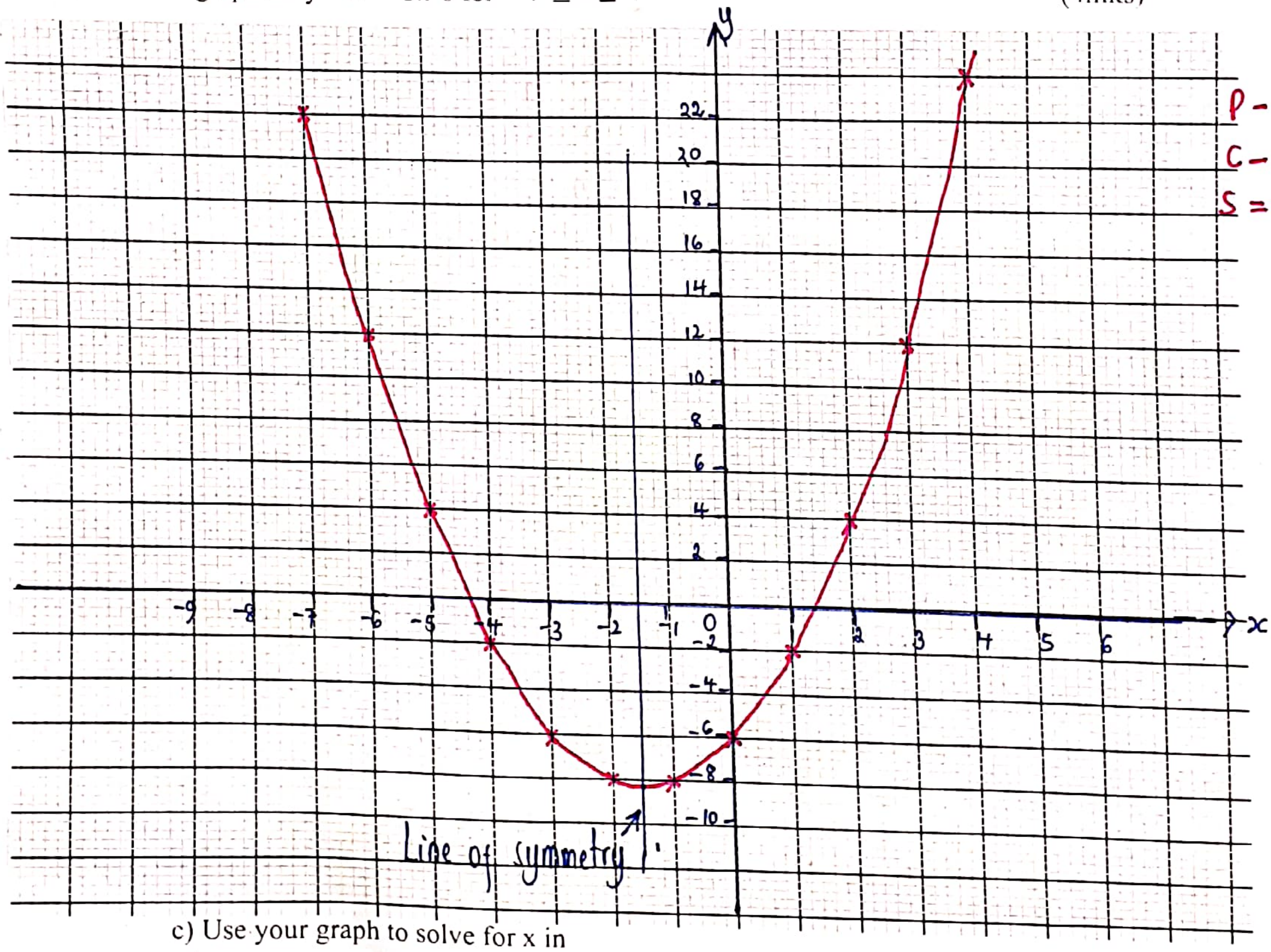
19. a) Complete the table below for the equation $Y = x^2 + 3x - 6$ where $-7 \leq x \leq 4$

x	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4
y	22	12	4	-2.	-6	-8	-8.	-6	-2	4	12	22

(2mks)

- Full table filled B_2
- More than half values correct B_1

b)Using the scale 1 cm to represent 1 unit on the X- axis and 1cm to represent 2 units on the Y - axis, draw the graph of $y = x^2 + 3x - 6$ for $-7 \leq x \leq 4$ (4mks)



P - B_1
C - B_1
S = B_1

c) Use your graph to solve for x in
$x^2 + 3x - 6 = 0$

$x = -4.4 \pm 0.1$ B_1 or $x = 1.4 \pm 0.1$. B_1 $\frac{}{02}$ (2mks)

d)State the Equation of the line symmetry (2mk)

$x = -1.5$ B_2 $\frac{}{02}$

10

20. The table below shows the marks scored in a chemistry test.

Marks	Frequency	x	fx	Cf. B1	f.d
5 -14	3	9.5	28.5	3	0.3
15 -34	19	24.5.	465.5	22	0.95
35 -54	50	44.5.	2225 B1	72	2.5
55 -84	26	69.5.	1807	98.	0.8667
85 -94	2	89.5.	179	100	0.2.
	$\Sigma f = 100.$		$\Sigma fx = 4705.$		

(a) State the modal class. (1 mk)

35-54.

01

(b) Calculate the mean mark. (3mks)

$$\text{Mean } (\bar{x}) = \frac{\Sigma fx}{\Sigma f} = \frac{4705}{100} \text{ M1}$$

$$= \underline{47.05} \text{ A1}$$

03

(c)Find the median mark. (3 mks)

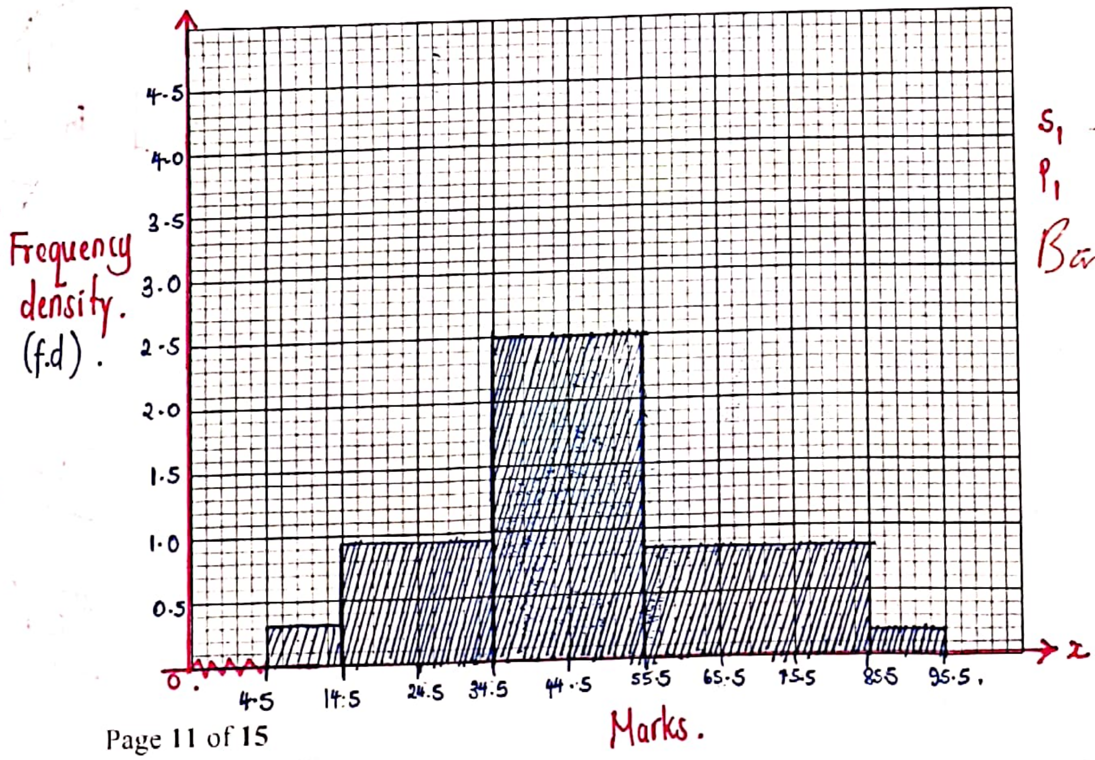
$$\text{Median} = l_0 + \left(\frac{N/2 - cf}{f}\right) i$$

$$= 34.5 + \left(\frac{50-22}{50}\right) 20 \text{ M1}$$

$$= \underline{45.7.} \text{ A1}$$

03

(d)Draw a histogram to represent the above information. (3 mks)



S1 ✓

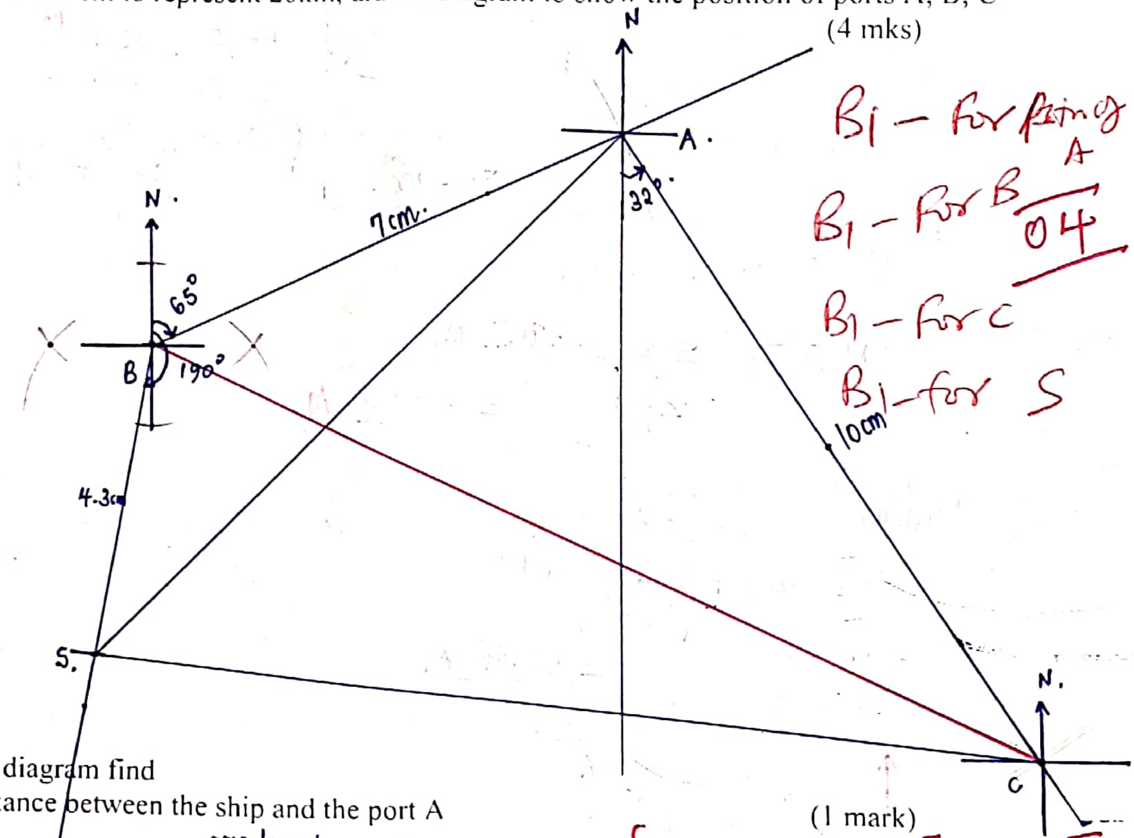
P1 ✓ 03

Bars - 1 ✓

10

21. Three ports A, B and C are situated in such a way that port A is 140km on a compass bearing of N65^{0}E from port B. Port C is 200km on a compass bearing of S32^{0}E from A. A ship S is docked in the sea, 86km on a bearing of 190^0 from port B.

(a) Using a scale of 1cm to represent 20km, draw a diagram to show the position of ports A, B, C and ship S. (4 mks)



B1 – for fixing A

B1 – for B

B1 – for C

B1 – for S

04

(b) Using your diagram find

(i) The distance between the ship and the port A (1 mark)

~~200km~~ ± $(10 \pm 0.1) \times 20$ or [202 km or 198 km.] B1 01

200 km

(ii) The bearing of the ship from port C (1 mark)

$275° \pm 0.1$ B1 01

(iii) The distance from B to C (1 mark)

$(13 \pm 0.1) \times 20$

= 260km or 258 km or 262 km. B1 01

(iv) Find how far C is south of A (2 marks)

$(5.5 \pm 0.1) \times 20$

110km or 108km or 112 km. 02

(v) Compass bearing of S from A (1 mark)

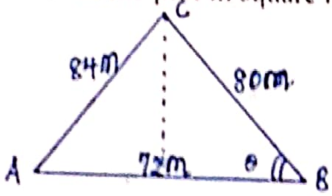
S 44° W B1 01

10

22. A triangular plot ABC is such that AB=72m, BC=80m and AC=84m.

a) Calculate the:

i) Area of the plot in square meters. (3mks)



$Area = \sqrt{s[(s-a)(s-b)(s-c)]}$

$= \sqrt{118[46 \times 34 \times 38)}$ M

$= 2648.20241\ m^2$ A

03

$S = \frac{1}{2} \times (72+84+80)$

$= 118.$ M

ii) Acute angle between the edges AB and BC. (3mks)

$2648.202409 = \frac{1}{2} \times 72 \times 80 \sin \theta.$ M1

$2640.202409 = 2880 \sin \theta$

$\theta = \sin^{-1} \frac{2640.202409}{2880}$ M1

$\theta = 66.45^0$ A

03

iii) Perpendicular height from A to the line BC. (2mks)

$\sin 66.45 = \frac{x}{80}$

$x = 80 \sin 66.45$ M1

$x = 73.33895m$ or $73.34m.$ A1

02

b) A water tap is to be installed inside the plot such that the tap is equidistant from each of the vertices A,B and C. Calculate the distance of the tap from the vertex A. (2mks)

$\frac{84}{\sin 66.45^0} = 2R$ M1

02

$2R = 91.6293$

$R = 45.81$ A1 (Atleast 4 s.fs).

10

23. A bus left Kisii at 6.45am towards Nairobi at an average speed of 60Km/h. a matatu left Nairobi for Kisii at 8.15am at an average speed of 75km/h. the distance between Kisii and Nairobi is 530km. determine;

i) The time taken before the two vehicles meet. (4mks)

234.44km

530km.

90km | 440km.

Bus Kisii Nairobi.

S.T. 6.45am → ← Matatu

Speed = 60km/h. S.T = 8.15am

Speed = 75km/h.

Distance travelled by the bus before the matatu left = $60 \times 3/2$. M1

= 90km.

Time taken to the meeting point

$= \frac{440}{R.S = 135}$ M1

Relative speed (R.S) = 75+60 = 135 km M

$= \frac{440}{135}$ M1

= 3 hours 15 minutes 33.3 seconds. A1

(Accept 3 hours 16 minutes).

04

ii) The distance between two vehicles 40 minutes after meeting. (2mks)

Distance to the meeting point = 3.259×75

= 244.4km from Nairobi.

Distance by the bus after the meeting point = $40/60 \times 60$ = 40km

Distance covered by the matatu after the meeting point = $40/60 \times 75$

= 50

Distance between the bodies = 50+40 M1

= 90 km A1

02

iii) At 9.00am, a car left Kisii for Nairobi at an average speed of 120km/h. determine the time the car caught up with the bus. (4mks)

Distance travelled by the bus before the car took off = $9/4 \times 60$ = 135km. M1

Time to catch up = $\frac{135}{60}$ M1

= 2 hours 15 min

= 9.00
+2.15
11.15 M1

= 11.15 am A1

04

10.

24. (a) Simplify $y = (x - 1)(x^2 - 3x + 2)$. (2 mks)

$y = x^3 - 3x^2 + 2x - x^2 + 3x - 2$. M1

02

$y = x^3 - 4x^2 + 5x - 2$. A1

(b) Hence find the turning points of the curve; $y = (x - 1)(x^2 - 3x + 2)$. (6 mks)

06

At stationary point: $\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = 3x^2 - 8x + 5 = 0$ M1

$= (3x^2 - 3x) - (5x + 5) = 0$

$3x(x - 1) - 5(x - 1) = 0$

$(3x - 5)(x - 1) = 0$ M1

$x = 1 \mid x = 5/3$. A1

When $x = 5/3$

$y = (5/3)^3 - 4(5/3)^2 + 5(5/3) - 2$ M1

$y = -4/27$

Hence $(5/3, -4/27)$. A1

When $x = 1$

$y = (1)^3 - 4(1)^2 + 5(1) + 2$

$y = 0$

Hence ; $(1, 0)$ A1

{A1 for all the coordinates of the turning points }.

(c) Determine the nature turning points. (2 marks)

To test the nature;

$\frac{d^2y}{dx^2} = 6x - 8$.

When $x = 1$; $\frac{d^2y}{dx^2} = 6(1) - 8$

$= -2$

Hence $(1, 0)$ is a maximum point. B1

When $x = 5/3$; $\frac{d^2y}{dx^2} = 6(5/3) - 8$

$= 10 - 8$

$= 2$

02

Hence $(5/3, -4/27)$ is a minimum point. B1

10