NAIROBI SCHOOL

Opener Term 3 Exam

121-Hybrid **Form 4**

MATHEMATICS

Marking Scheme

October. 2022— 150 minutes

FILL IN YOUR PERSONAL DETAILS HERE

Student Name:

Admission Number: | | | | | | **Class:** | 4 | |

Instructions to candidates

(a) Write your name, admission number and class in the spaces provided above.
(b) This paper consists of two sections; **Section I** and **Section II.**
(c) Answer **all** the questions in **Section I** and **any five** questions from **Section II.**
(d) **Show all the steps in your calculations, giving your answers at each stage in the spaces provided below each question.**
(e) **KNEC Mathematical tables** may be used, except where stated otherwise.
(f) **Non-programmable** silent electronic calculators **must not** be used, except where stated otherwise.
(g) This paper consists of **16** printed pages.
(h) Remember to tick the questions you have attempted in **Section II**

For Examiner's Use Only

SECTION I

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	TOTAL

SECTION II *(Please tick the questions you have attempted)*

17	18	19	20	21	22	23	24	TOTAL
								✓

GRAND TOTAL

TURN OVER

SECTION ONE - 50 MARKS

Answer all questions from this section in the spaces provided.

1). The coordinates of two airports **M** and **N** are $(60^\circ N, 35^\circ W)$ and $(60^\circ N, 15^\circ E)$ respectively. Calculate;

(a) the longitude difference. **[1 mark]**

Solution

$\text{longitudinal difference} = 35 + 15$ *sum as the places have different longitude directi*

$= 50^\circ$ ✓ B1

(b) the shortest time an aeroplane whose speed is **250** knots will take to fly from **M** to **N** along a circle of latitude. **[2 marks]**

Solution

$\text{Arc Length} = 60 \times 50 \cos 60$ $\quad 60\theta \cos \alpha$

$= 1500 \text{ nm}$ ✓ M1

$\text{Time} = \dfrac{1500 \text{ nm}}{250 \text{knots}}$ *distance= speed×time*

$= 6 \text{ h}$ ✓ A1

2). Kasyoka and Kyalo working together can do a piece of work in **6** days. Kasyoka, working alone takes **5** days longer than Kyalo. How many days does it take Kyalo to do the work alone? **[4 marks]**

Solution

Let $x =$ the number of days Kyalo takes to finish the job

	Total time in days	Fractional part done in 1 day	Fractional part each person does in 6 days
Kyalo	x	$\frac{1}{x}$	$\frac{6}{x}$
Kasyoka	$x+5$	$\frac{1}{x+5}$	$\frac{6}{x+5}$
Together	6	$\frac{1}{6}$	1

Kasyoka and Kyalo do **1** fractional part in **6** days.

$\frac{6}{x} + \frac{6}{x+5} = 1$ ✓ M1

$6(x+5) + 6x = x(x+5)$ *clear off fractions*

$x^2 - 7x - 30 = 0$

$(x-10)(x+3) = 0$ ✓ M1

$x = 10$ or $x = -3$ (reject) ✓ A1 *as x cannot be negative*

Hence Kyalo takes **10** days to complete the job.✓ B1

3). Find the radius and the centre of the circle whose equation is: **[4 marks]**

$$3x^2 + 3y^2 - 6x + 12y + 3 = 0$$

Solution

$$3x^2 + 3y^2 - 6x + 12y + 3 = 0$$
$$x^2 - 3x + y^2 + 4y = -3$$
$$x^2 - 3x + \left(-\frac{3}{2}\right)^2 + y^2 + 4y + (2)^2 = -3 + \left(-\frac{3}{2}\right)^2 + (2)^2 \checkmark \text{ M1}$$
$$\left(x - \frac{3}{2}\right)^2 + (y+2)^2 = \frac{13}{4} \quad \checkmark \text{ M1}$$

Hence the centre is $\left(\frac{3}{2}, -2\right)$ and radius is $\frac{\sqrt{13}}{2}$. ✓ A2

4). A particle moves along a straight line **AB**. Its velocity **v** metres per second after **t** seconds is given by $v = t^2 - 3t + 5$. Determine distance covered within the third second. **[3 marks]**

Solution

$$s = \int_2^3 t^2 - 3t + 5dt \quad \checkmark \text{ M1}$$
$$= \left[\frac{t^3}{3} - \frac{3t^2}{2} + 5t + c\right]_2^3 \checkmark \text{ M1}$$
$$= \frac{23}{6} = 3\frac{5}{6} \text{ m} \quad \checkmark \text{ A1}$$

5). Ali deposited **KES 100, 000** in a financial institution that paid simple interest at the rate of **12.5%** p.a. Mohamed deposited the same amount of money as Ali in another financial institution that paid compound interest. After **4** years, they had equal amounts of money. Determine the compound interest rate per annum to one decimal place. **[4 marks]**

Solution

Let **r** = compound interest per annum.

$$\text{Simple Interest} = 100000 \times \frac{12.5}{100} \times 4 = \quad \checkmark \text{ M1}$$
$$\text{Amount for Ali} = 100000 + 50000 = 150000$$
$$\text{Amount for Mohamed} = \text{KES } 150000$$
$$\implies 150000 = 100000\left(1 + \frac{r}{100}\right)^4 \quad \checkmark \text{ M1}$$
$$1.5 = \left(1 + \frac{r}{100}\right)^4 \quad \checkmark \text{ M1}$$
$$\implies \frac{r}{100} = \sqrt[4]{1.5} - 1$$
$$r = 100(\sqrt[4]{1.5} - 1) = 10.7\% \quad \checkmark \text{ A1}$$

6). Make $\mathbf{x}$ the subject of the formula. **[3 marks]**

$$\frac{x^4 - 4}{x^2 - 2} = k$$

Solution

$$\begin{aligned}
\frac{x^4 - 4}{x^2 - 2} &= k \\
\frac{(x^2 - 2)(x^2 + 2)}{x^2 - 2} &= k \\
x^2 + 2 &= k \\
x^2 &= k - 2 \\
x &= \pm\sqrt{k - 2}
\end{aligned}$$

7). Solve for $\mathbf{x}$ in the equation. **[3 marks]**

$$2\sin^2 x - 1 = \cos^2 x - \sin^2 x, \quad \text{where } 0^\circ \leq x \leq 360^\circ.$$

Solution

$$\begin{aligned}
2\sin^2 x - 1 &= (1 - \sin^2 x) - \sin^2 x \\
4\sin^2 x &= 2 \\
\sin^2 x &= \frac{1}{2} \quad \checkmark \text{M1} \\
\sin x &= \pm\frac{1}{\sqrt{2}}
\end{aligned}$$

Working from the first quadrant:

$$\implies x = \sin^{-1}\frac{1}{\sqrt{2}} = 45^\circ \quad \checkmark \text{M1}$$

Translating to required quadrants gives:

$$x = 45^\circ,\ 135^\circ,\ 225^\circ,\ 315^\circ \quad \checkmark \text{A1}$$

8). Find $\mathbf{C}$ that divide $\mathbf{AB}$ externally in the ratio $\mathbf{5 : 2}$, given that $\mathbf{A(3, -6, 9)}$ and $\mathbf{B(-15, 3, 12)}$. **[3 marks]**

Solution

$$\overrightarrow{OC} = \frac{-2}{3}\overrightarrow{OB} + \frac{5}{3}\overrightarrow{OA} \quad \checkmark \text{M1} \quad \textit{use ratio theorem}$$

$$= \frac{-2}{3}\begin{pmatrix} -15 \\ 3 \\ 12 \end{pmatrix} + \frac{5}{3}\begin{pmatrix} 3 \\ -6 \\ 9 \end{pmatrix} \quad \checkmark \text{M1}$$

$$= \begin{pmatrix} 10 \\ -2 \\ -8 \end{pmatrix} + \begin{pmatrix} 5 \\ -10 \\ 15 \end{pmatrix} = \begin{pmatrix} 15 \\ -12 \\ 7 \end{pmatrix} \quad \checkmark \text{A1}$$

$$\therefore \quad C(15, -12, 7) \quad \checkmark \text{B1}$$

9). If $\sin \mathbf{x} = \mathbf{2b}$ and $\cos \mathbf{x} = \mathbf{2b\sqrt{3}}$, find the value of $\tan \mathbf{x}$. **[2 marks]**

Solution

$$\tan x = \frac{\sin x}{\cos x}$$
$$= \frac{2b}{2b\sqrt{3}} = \frac{1}{\sqrt{3}} \checkmark \text{M1}$$
$$= \frac{\sqrt{3}}{3} \checkmark \text{A1}$$

must rationalize denominator

10). Solve for $\mathbf{y}$ in the equation: **[3 marks]**

$$(\log_2 y)^2 + \log_2 8 = \log_2 y^4$$

Solution

$$(\log_2 y)^2 + \log_2 8 = \log_2 y^4$$
$$(\log_2 y)^2 + 3 = 4\log_2 y$$

$\log_2 8 = 3$ *and* $\log m^n = n \log m$

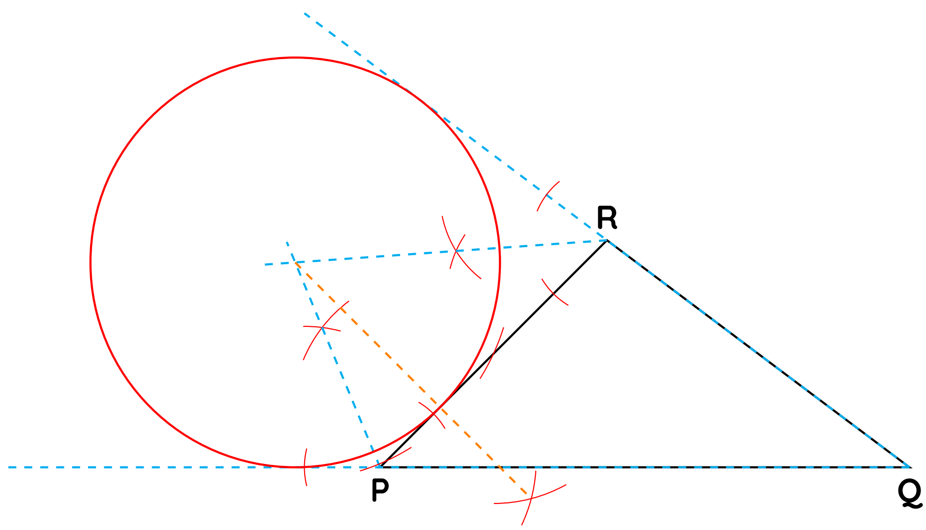
Let $\log_2 y = a$, thus

$$a^2 + 3 = 4a$$
$$a^2 - 4a + 3 = 0$$
$$(a-3)(a-1) = 0$$
$$\implies a = 3 \quad \text{or} \quad a = 1$$

hence $\log_2 y = 3 \implies y = 2^3 = 8$

and $\log_2 y = 1 \implies y = 2^1 = 2$

11). On the triangle **PQR**, draw a circle touching **PR**, **QP** produced and **QR** produced. **[3 marks]**



12). The gradient of a curve at any point given by $\mathbf{2x - 1}$. Given that the curve passes through point $(\mathbf{1, 5})$. Find the equation of the curve. [3 marks]

Solution

$$\frac{dy}{dx} = 2x - 1$$
$$y = \int \frac{dy}{dx} dx$$
$$y = 2x - 1dx = x^2 - x + c \checkmark \text{ M1}$$
$$5 = 1^2 - 1 + c \implies c = 5 \checkmark \text{ M1}$$
$$\therefore y = x^2 - x + 5 \quad \checkmark \text{ A1}$$

13). $\mathbf{w}$ varies directly as the cube of $\mathbf{x}$ and inversely as $\mathbf{y}$. Find $\mathbf{w}$ in terms of $\mathbf{x}$ and $\mathbf{y}$ given that $\mathbf{w = 80}$ when $\mathbf{x = 2}$ and $\mathbf{y = 5}$. [3 marks]

Solution

$$w \propto \frac{x^3}{y} \implies w = \frac{kx^3}{y} \checkmark \text{ M1}$$
$$80 = \frac{k(2)^3}{5} \implies 80 = \frac{8k}{5} \checkmark \text{ M1}$$
$$k = 50$$
$$\therefore \quad w = \frac{50x^3}{y} \quad \checkmark \text{ A1}$$

14). Given that $\mathbf{2 \leq A \leq 4}$ and $\mathbf{0.1 \leq B \leq 0.2}$. Find the minimum value of $\mathbf{\frac{AB}{A - B}}$ as a fraction. [2 marks]

Solution

$$\text{Minimum value} = \frac{A_{min} \times B_{min}}{A_{max} - B_{min}}$$
$$= \frac{2 \times 0.1}{4 - 0.1} \quad \checkmark \text{ M1}$$
$$= \frac{0.2}{3.9} = \frac{2}{39} \quad \checkmark \text{ A1}$$

15). Use matrix method to solve the given simultaneous equation: **[3 marks]**

$$3x + y = 7$$
$$5x + 2y = 12$$

Solution

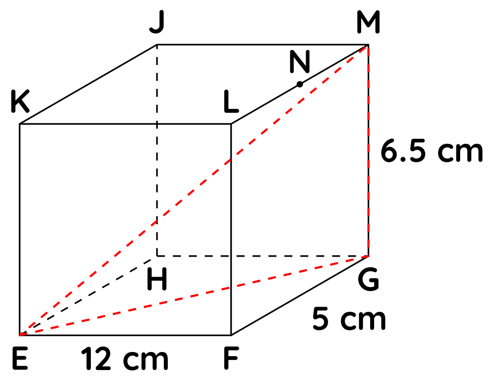
$$\begin{pmatrix} 3 & 1 \\ 5 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 7 \\ 12 \end{pmatrix}$$

$$\begin{pmatrix} 2 & -1 \\ -5 & 3 \end{pmatrix}\begin{pmatrix} 3 & 1 \\ 5 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 & -1 \\ -5 & 3 \end{pmatrix}\begin{pmatrix} 7 \\ 12 \end{pmatrix} \checkmark \text{M1}$$

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \end{pmatrix} \implies \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \end{pmatrix} \quad \checkmark \text{M1}$$

$$\implies x = 2 \quad \checkmark \text{A1}$$
$$y = 1$$

16). The figure below is a cuboid **EFGHJKLM**. **EF** $= 12$ **cm**, **FG** $= 5$ **cm** and **GM** $= 6.5$ **cm**.



(a) State the projection of **EM** on the plane **EFGH**. **[1 mark]**

Solution

projection of **EM** = **EG** ✓ B1

(b) Calculate the angle between **EM** and the plane **EFGH** correct to 2 decimal planes. **[3 marks]**

Solution

$$EG^2 = EF^2 + FG^2$$
$$= 12^2 + 5^2 = 169$$
$$\implies EG = 13$$
$$\therefore \tan\theta = \frac{MG}{EG} = \frac{6.5}{13}$$
$$\implies \theta = \tan^{-1}\frac{1}{2} =$$

SECTION TWO - 50 Marks

*Answer any **five** questions from this section in the spaces provided.*

17). Use Trapezoidal rule to find the area between the curve $y = x^2 + 4x + 4$, the x-axis and the ordinates $x = -2$ and $x = 1$. *(Use 6 strips)*

(a) Complete the table below. **[2 marks]**

x	-2	-1.5	-1	-0.5	0	0.5	1
y	-1	-0.75	0	1.25	3	5.25	8

✓ B2

(b) Find the area enclosed by the curve, the x-axis, lines $x = -2$ and $x = 1$. **[3 marks]**

Solution

$$h = \frac{b-a}{2} = \frac{1--2}{6} = 0.5$$ ✓ B1 $n = 6$ *(number of strips)*

$$\text{Area} = \frac{1}{2}h\{(y_1 + y_7) + 2(y_2 + y_3 + \cdots + y_6)\}$$

$$= \frac{1}{2} \times 0.5\{(1+8) + 2(0.75 + 0 + 1.25 + 3 + 5.25)\}$$ ✓ M1

$$= \frac{1}{4} \times 29.5 = 7.375 \text{ sq units}$$ ✓ A1

(c) Use integration to find the exact area. **[3 marks]**

Solution

$$\text{Area} = \int_{-2}^{1} x^2 + 4x + 4dx$$

$$= \int_{-2}^{-1} x^2 + 4x + 4dx + \int_{-1}^{1} x^2 + 4x + 4dx$$

$$= \left[\frac{x^3}{3} + 2x^2 + 4x + c\right]_{-2}^{-1} + \left[\frac{x^3}{3} + 2x^2 + 4x + c\right]_{-1}^{1}$$ ✓ M1

$$= \left|-\frac{2}{3}\right| + \frac{20}{3}$$ ✓ M1

$$= \frac{22}{3} = 7\frac{1}{3} \text{ sq units}$$ ✓ A1

(d) Hence or otherwise find the percentage error in your approximation correct to 2 significant figures. **[2 marks]**

Solution

$$\text{Percentage Error} = \frac{\text{Absolute error}}{\text{Actual Area}} \times 100\%$$

$$= \frac{7.375 - \frac{22}{3}}{\frac{22}{3}} \times 100\%$$ ✓ M1

$$= 0.5682\% \approx 0.57\%$$ ✓ A1

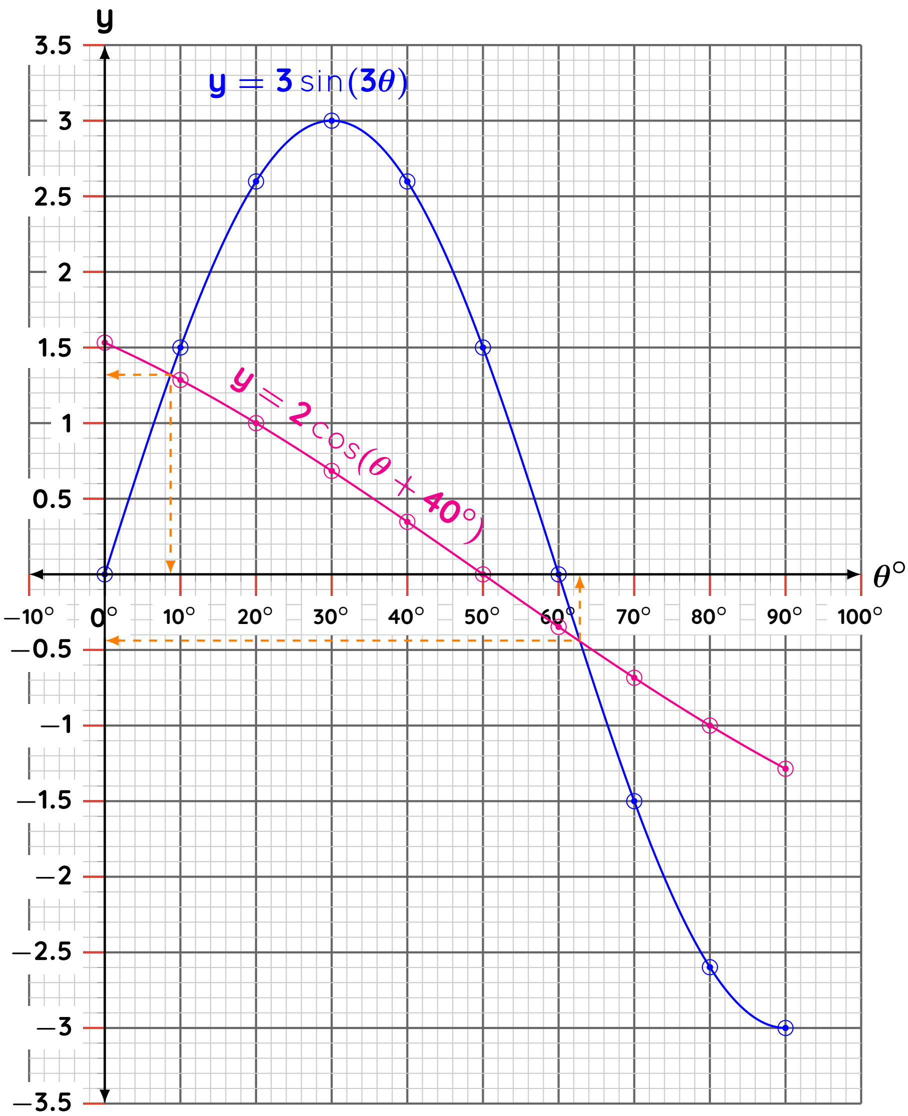
Total: 10 marks

18). (a) Complete the table below for the functions $y = 3\sin 3\theta$ and $y = 2\cos(\theta + 40^\circ)$ **[2 marks]**

θ	0°	10°	20°	30°	40°	50°	60°	70°	80°	90
$3\sin 3\theta$	0.00	1.50	2.60	3.00	2.60	1.50	0.00	−1.50	−2.60	−3.00
$2\cos(\theta + 40^\circ)$	1.53	1.29	1.00	0.68	0.35	0.00	−0.35	−0.68	−1.00	−1.29

(b) On the grid provided, draw the graphs of $y = 3\sin 3\theta$ and $y = 2\cos(\theta + 40^\circ)$ on the same axis. **[5 marks]**

Take **1 cm** to represent 10° on the **x**-axis and **4 cm** to represent **2** unit on the **y** - axis.



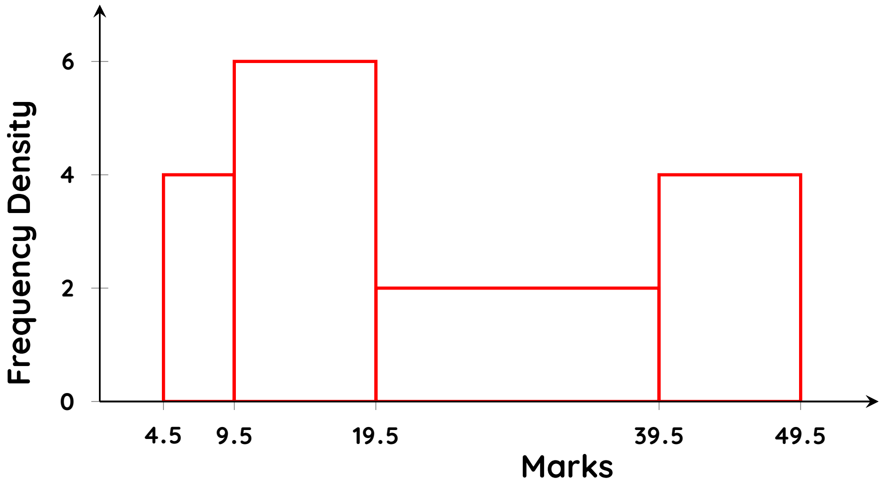
(c) From the graph find the roots of the equation:

(i) $\frac{3}{4}\sin 3\theta = \frac{1}{2}\cos(\theta + 40^\circ)$. **[2 marks]**

(ii) $2\cos(\theta + 40^\circ) = 0$ in the range $0 \leq \theta \leq 90^\circ$. **[1 mark]**

Total: 10 marks

19). The diagram below shows a histogram marks obtained in a certain test.



(a) Develop a frequency distribution table for the data if the first class **5 − 9** has a frequency of **8**. **[3 marks]**

Class	5 − 9	10 − 19	20 − 39	40 − 49
Frequency Density	4	6	2	4
Frequency	8	24	16	16

(b) Fill in the table below, hence or otherwise calculate the mean using an assumed mean of **19.5**. **[3 marks]**

Class	Midpoint(x)	$d = x - 19.5$	$t = \frac{d}{5}$	Frequency(f)	ft	cf
5 − 9	7	−12.5	−2.5	8	−20	8
10 − 19	14.5	−5	−1	24	−24	32
20 − 39	29.5	10	2	16	32	48
40 − 49	44.5	25	5	16	80	64

Solution

$$\text{Mean} = A + \frac{\sum ft}{\sum f} \times 5 \quad ✓ \text{M1}$$

$$= 19.5 + \frac{68}{64} \times 5 = 24.8125 \quad ✓ \text{A1}$$

(c) Calculate interquatile range. **[4 marks]**

Solution

$$\text{IQR} = Q_3 - Q_1$$

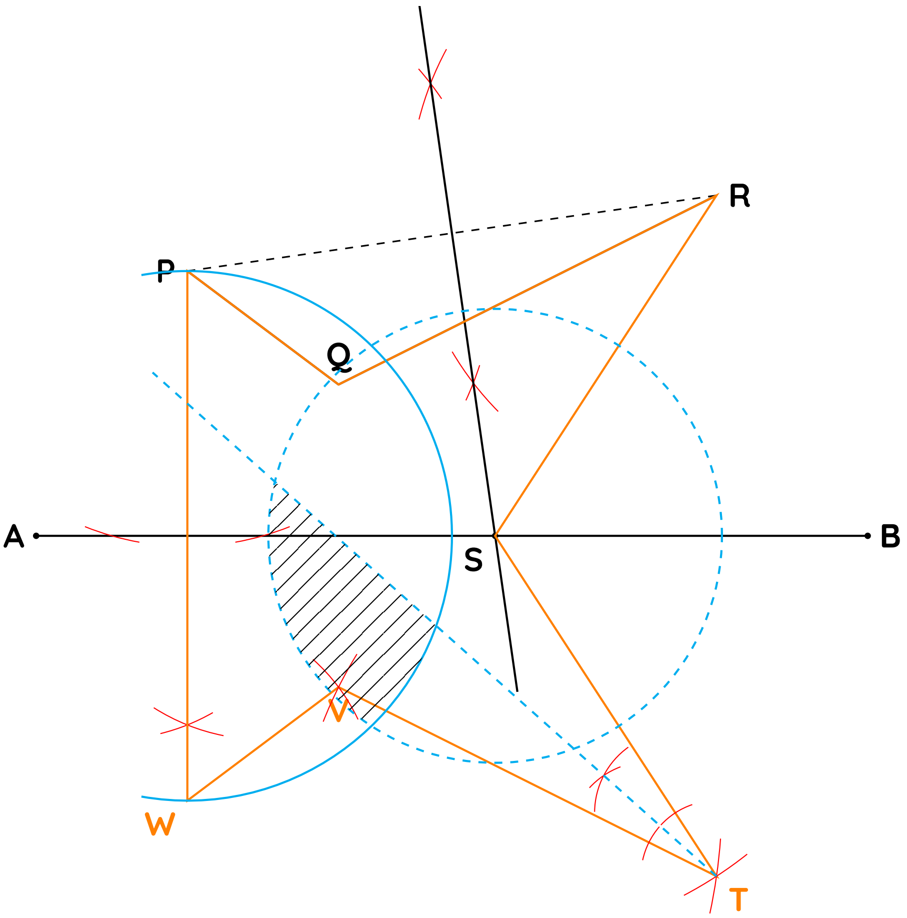
$$= \left\{ LCB + \frac{\frac{3}{4}N - cf}{f} \times cw \right\} - \left\{ LCB + \frac{\frac{1}{4}N - cf}{f} \times cw \right\}$$

$$= \left\{ 19.5 + \frac{\frac{3}{4} \times 64 - 32}{16} \times 20 \right\} - \left\{ 9.5 + \frac{\frac{1}{4} \times 64 - 8}{24} \times 10 \right\}$$

$$= 39.5 - 12.83 = 26.67 \quad ✓ \text{A1}$$

Total: 10 marks

20). In the figure below **AB**, **PQ** and **QR** are straight lines



(a) Use the figure to:

(i) find a point **S** on **AB** such that **S** is equidistant from **P** and **R**. **[1 mark]**

(ii) complete a heptagon **PQRSTVW** with **AB** as its line of symmetry and hence measure **Q** from **S**. **[5 marks]**

(b) shade the region within the heptagon in which a variable point **X** must lie given that **X** satisfies the following conditions:

(i) **X** is nearer to **TV** than to **TS**. **[1 mark]**

(ii) **SX** is less than **3 cm**. **[1 mark]**

(iii) $\angle$**PXW** $\geq$ **90°**. **[2 marks]**

Total: 10 marks

21). The table below shows the income tax rates for a certain year.

Monthly taxable income sh	Tax rates(Percentage)
1 — 9680	10%
9681 — 18800	15%
18801 — 27920	20%
27921 — 37040	25%
37041 — 46160	30%
above 46161	35%

Naliaka earned a basis salary of **KES 30840** and a house allowance of **KES 15000** per month also a commuter allowance amounting to **KES 10480** in a particular month.

(a) Calculate the tax she paid in that month if she is entitled a personal tax relief of **KES 1056** per month. **[7 marks]**

Solution

$$\text{Taxable Income} = 30840 + 15000 + 10480 = \text{KES } 56320 \quad \checkmark \text{M1}$$

$$1^{st} \text{ band} = \frac{10}{100} \times 9680 = \text{KES } 968$$

$$2^{nd} \text{ band} = \frac{15}{100} \times 9120 = \text{KES } 1368 \quad \checkmark \text{M1}$$

$$3^{rd} \text{ band} = \frac{20}{100} \times 9120 = \text{KES } 1824$$

$$4^{th} \text{ band} = \frac{25}{100} \times 9120 = \text{KES } 2280 \quad \checkmark \text{M1}$$

$$5^{th} \text{ band} = \frac{30}{100} \times 9120 = \text{KES } 2736$$

$$6^{th} \text{ band} = \frac{35}{100} \times 10160 = \text{KES } 3556 \quad \checkmark \text{M1}$$

$$\text{Gross tax} = 968 + 1368 + 1824 + 2280 + 2736 + 3556 \quad \checkmark \text{M1}$$

$$= \text{KES } 12732$$

$$\text{Net tax} = 12732 - 1056 \quad \checkmark \text{M1}$$

$$= \text{KES } 11676 \quad \checkmark \text{A1}$$

(b) The following deduction are also made on Naliaka's income:

- NHIF = **KES 1800**
- NSSF = **KES 920**

Calculate the net income in that month. **[3 marks]**

Solution

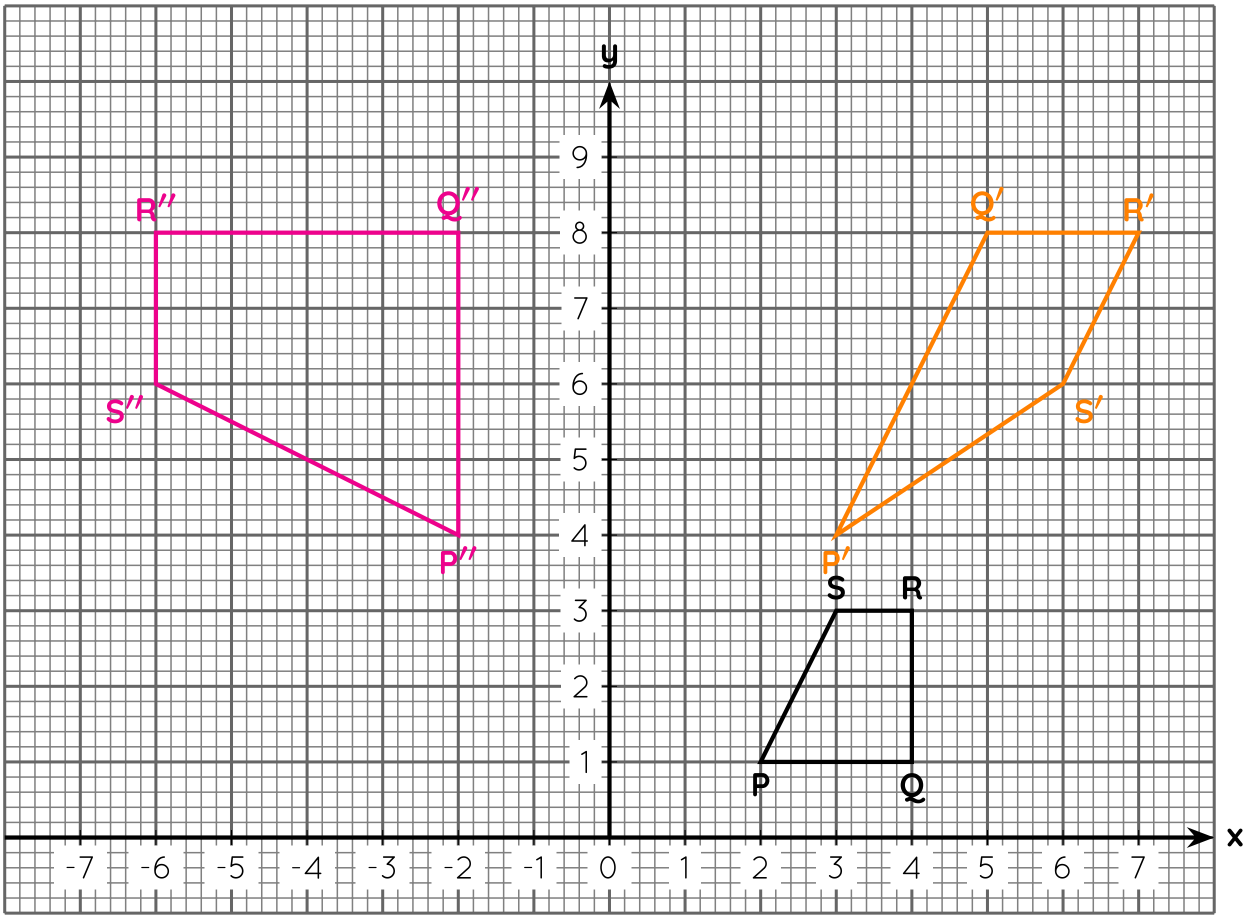
$$\text{Deductions} = 11676 + 1800 + 920 = \text{KES } 14396 \quad \checkmark \text{M1}$$

$$\text{Net income} = 56320 - 14396 \quad \checkmark \text{M1}$$

$$= \text{KES } 41924 \quad \checkmark \text{A1}$$

Total: 10 marks

22). The points $\mathbf{P(2,1)}$, $\mathbf{Q(4,1)}$, $\mathbf{R(4,3)}$ and $\mathbf{S(3,3)}$ are coordinates of a quadrilateral.



(a) Plot the quadrilateral **PQRS** on the grid provided. **[1 mark]**

(b) Find the coordinates of **P′Q′R′S′** the image of **PQRS** under the transformation represented by the matrix $\mathbf{M} = \begin{pmatrix} 1 & 1 \\ 2 & 0 \end{pmatrix}$ **[2 marks]**

Solution

$$\begin{pmatrix} 1 & 1 \\ 2 & 0 \end{pmatrix} \begin{pmatrix} 2 & 4 & 4 & 3 \\ 1 & 1 & 3 & 3 \end{pmatrix} = \begin{pmatrix} 3 & 5 & 7 & 6 \\ 4 & 8 & 8 & 6 \end{pmatrix}$$ ✓ M1

Hence $\mathbf{P'(3,4)}$ $\mathbf{Q'(5,8)}$, $\mathbf{R'(7,8)}$, $\mathbf{S'(6,6)}$ ✓ A1

#22 continues on the next page...

(c) Draw and label **P′Q′R′S′** on the same grid. [1 mark]

(d) Find the coordinates of **P″Q″R″S″** on the image of **P′Q′R′S′** under the transformation represented by the matrix $\mathbf{M} = \begin{pmatrix} -2 & 1 \\ 0 & 1 \end{pmatrix}$ [2 marks]

Solution

$$\begin{pmatrix} -2 & 1 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 3 & 5 & 7 & 6 \\ 4 & 8 & 8 & 6 \end{pmatrix} = \begin{pmatrix} -2 & -2 & -6 & -6 \\ 4 & 8 & 8 & 6 \end{pmatrix} \checkmark \text{M1}$$

Hence $\mathbf{P''}(-2,4)\ \mathbf{Q''}(-2,8),\ \mathbf{R''}(-6,8),\ \mathbf{S''}(-6,6)$ ✓A1

(e) Draw and label **P″Q″R″S″** on the same grid. [1 mark]

(f) Determine the matrix that maps **PQRS** directly onto **P″Q″R″S″**. [3 marks]

Solution

Let $\mathbf{M}$ = the transformation matrix; thus,

$$\text{M}\begin{pmatrix} 2 & 4 & 4 & 3 \\ 1 & 1 & 3 & 3 \end{pmatrix} = \begin{pmatrix} -2 & -2 & -6 & -6 \\ 4 & 8 & 8 & 6 \end{pmatrix} \checkmark \text{M1}$$

$$\text{M}\begin{pmatrix} 2 & 4 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} -2 & -2 \\ 4 & 8 \end{pmatrix}$$

$$\text{M}\begin{pmatrix} 2 & 4 \\ 1 & 1 \end{pmatrix}\frac{1}{-2}\begin{pmatrix} 1 & -4 \\ -1 & 2 \end{pmatrix} = \begin{pmatrix} -2 & -2 \\ 4 & 8 \end{pmatrix}\frac{1}{-2}\begin{pmatrix} 1 & -4 \\ -1 & 2 \end{pmatrix} \checkmark \text{M1}$$ **post multiply** *to solve fo*

$$\text{M}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = -\frac{1}{2}\begin{pmatrix} 0 & 4 \\ -4 & 0 \end{pmatrix}$$

$$\text{M} = \begin{pmatrix} 0 & -2 \\ 2 & 0 \end{pmatrix} \checkmark \text{A1}$$

Total: 10 marks

23). A supermarket is stocked with plates which come from two suppliers **A** and **B**. They are bought in the ratio **3 : 5** respectively, **10%** of plates from **A** are defective and **6%** of the plates from B are defective.

(a) A plate is chosen by a buyer at randon. Find the probability that:

(i) it is from **A**. [1 mark]

Solution

$$P(A) = \frac{3}{8} \checkmark \text{ B1}$$

(ii) it is from **B** and it is defective. [2 marks]

Solution

$$P(B \text{ and } D) = \frac{5}{8} \times 0.06 \checkmark \text{ M1}$$
$$= \frac{3}{80} \checkmark \text{ A1}$$

(iii) it is defective. [2 marks]

Solution

$$P(D) = P(A \text{ and } D) + P(B \text{ and } D)$$
$$= \frac{3}{8} \times 0.1 + \frac{3}{80} \checkmark \text{ M1}$$
$$= \frac{6}{80} = \frac{3}{40} \checkmark \text{ A1}$$

(b) Two plates are chosen at random. Find the probability that:

(i) both are defective. [2 marks]

Solution

$$P(AD \text{ and } BD) = P(A \text{ and } D) \times P(B \text{ and } D)$$
$$= \frac{3}{80} \times \frac{3}{80} \checkmark \text{ M1}$$
$$= \frac{9}{6400} \checkmark \text{ A1}$$

(ii) at least one is defective. [3 marks]

Solution

$$P(\text{at least one } D) = 1 - \left\{ P(A \text{ and } D') \times P(B \text{ and } D') \right\} \checkmark \text{ M1}$$
$$= 1 - \left\{ \frac{27}{80} \times \frac{27}{80} \right\} \checkmark \text{ M1}$$
$$= \frac{5671}{6400} \checkmark \text{ A1}$$

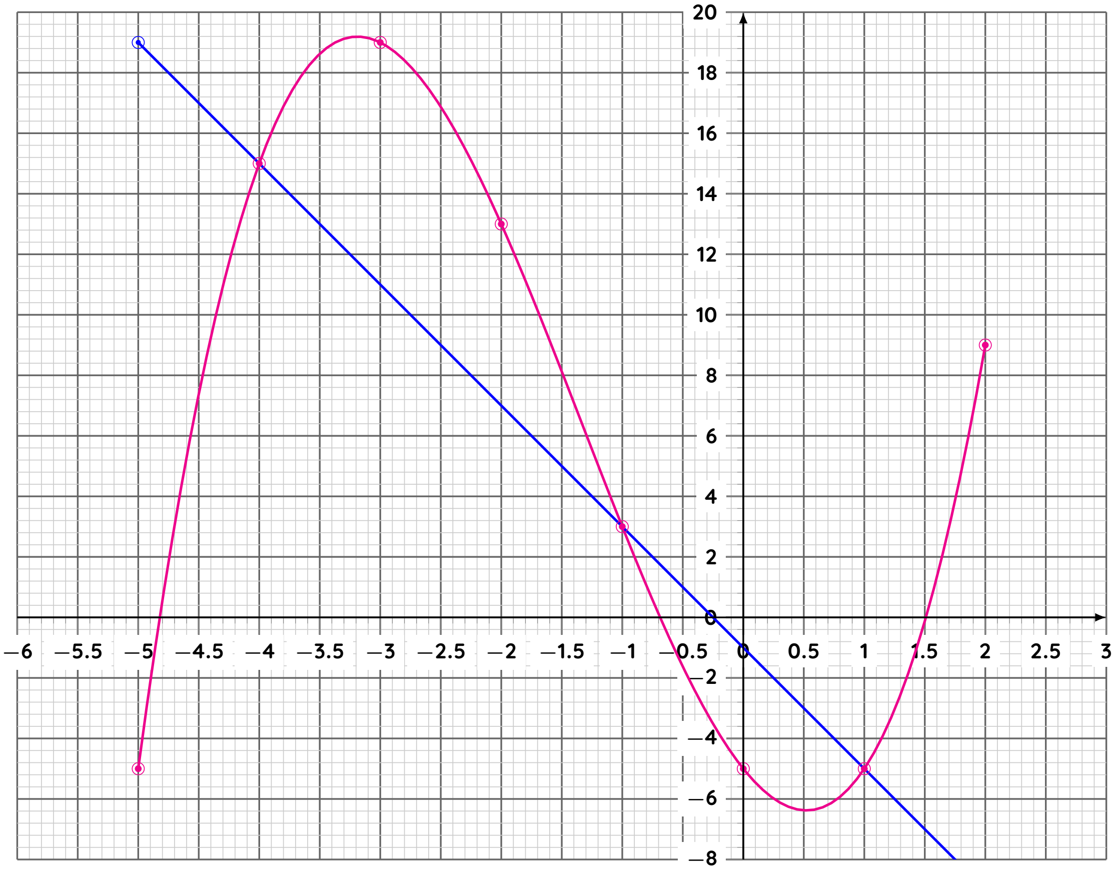
Total: 10 marks

24). (a) Complete the table below for $y = x^3 + 4x^2 - 5x - 5$. [2 marks]

x	-5	-4	-3	-2	-1	0	1	2
y	-5	15	19	13	3	-5	-5	9

(b) On the grid provided, draw the graph of: [3 marks]

$$y = x^3 + 4x^2 - 5x - 5 \quad \text{for} \quad -5 \le x \le 2.$$



(c) **(i)** Use the graph to solve the equation: [2 marks]

$$x^3 + 4x^2 - 5x - 5 = 0$$

Solution

Read the $x-$ intercepts on the graph of $y = x^3 + 4x^2 - 5x - 5$

$x_1 = 1.5$, $x_2 = -0.7$ and $x_3 = -4.8$ ✓ B2

(ii) By drawing a suitable straight line on the graph, solve the equation [3 marks]

$$x^3 + 4x^2 - 5x - 5 = -4x - 1$$

Solution

$y = x^3 + 4x^2 - 5x - 5$

$-4x - 1 = x^3 + 4x^2 - 5x - 5$

$\Longrightarrow y = -4x - 1$ ✓ B1

Plot the straight line graph $y = -4x - 1$ ✓ L1

The line intersects with $y = x^3 + 4x^2 - 5x - 5$ at

$x = -4$, $x = -1$ and $x = 1$ ✓ B1

Total: 10 marks