NAIROBI SCHOOL

Opener Exam, Term 2

121 **Form 4**

MATHEMATICS

Marking Scheme

July. 2022— 150 minutes

FILL IN YOUR PERSONAL DETAILS HERE

Student Name:

Admission Number: | | | | | | **Class:** 4 |

Instructions to candidates

(a) Write your name, admission number and class in the spaces provided above.
(b) This paper consists of two sections; **Section I** and **Section II**.
(c) Answer **all** the questions in **Section I** and **any five** questions from **Section II.**
(d) **Show all the steps in your calculations, giving your answers at each stage in the spaces provided below each question.**
(e) **KNEC Mathematical tables** may be used, except where stated otherwise.
(f) **Non-programmable** silent electronic calculators **must not** be used, except where stated otherwise.
(g) This paper consists of **16** printed pages.
(h) Remember to tick the questions you have attempted in **Section II**

For Examiner's Use Only

SECTION I

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	TOTAL

SECTION II(Please tick the questions you have attempted)

17	18	19	20	21	22	23	24	TOTAL
								✓

GRAND TOTAL

By Mr Matoke

TURN OVER

SECTION ONE - 50 MARKS

Answer all questions from this section in the spaces provided.

1). Solve $2\sin^2\theta + 3\cos\theta = -1$ for $0^\circ \leq \theta \leq 360^\circ$ **(3 marks)**

Solution

$$\begin{aligned}
2\sin^2\theta + 3\cos\theta &= -1\\
2(1-\cos^2\theta) + 3\cos\theta &= -1\\
2 - 2\cos^2\theta + 3\cos\theta &= -1\\
2\cos^2\theta - 3\cos\theta - 3 &= 0\\
\cos\theta &= \frac{3 \pm \sqrt{(-3)^2 - 4\cdot 2\cdot(-3)}}{2\cdot 2}\\
\cos\theta &= 2.18614\ldots,\ x = -0.68614\ldots\\
\implies \cos\theta &= -0.68614\\
\theta &= 46.67^\circ \qquad \text{working on first quadrant}\\
\theta &= 180^\circ - 46.67^\circ,\ 180^\circ + 46.67^\circ\\
&= 133.33^\circ,\ 226.67^\circ
\end{aligned}$$

2). Given that $A = \sqrt[4]{\dfrac{d - c^2g}{b + c^2f}}$ make c the subject of the formula. **(3 marks)**

Solution

$$\begin{aligned}
A = \sqrt[4]{\frac{d-c^2g}{b+c^2f}} \implies A^4 &= \frac{d-c^2g}{b+c^2f}\\
A^4(b + c^2f) &= d - c^2g\\
A^4b + A^4c^2f &= d - c^2g\\
A^4c^2f + c^2g &= d - A^4b\\
c^2(A^4f + g) &= d - A^4b\\
c^2 &= \frac{d - A^4b}{A^4f + g}\\
c &= \pm\sqrt{\frac{d - A^4b}{A^4f + g}}
\end{aligned}$$

3). A sum of Ksh. 8000 was partly lent at 10% p.a simple interest and 12.5% p.a simple interest. The total interest after 2 years was Ksh. 1775. How much was lent at 10% simple interest? **(3 marks)**

Solution

Let the amount lent at 10% p.a simple interest be x:

$$\begin{aligned}
\left(x \times \frac{10}{100} \times 2\right) + \left([8000 - x] \times \frac{12.5}{100} \times 2\right) &= 1775\\
0.2x + 2000 - 0.25x &= 1775\\
-0.05x &= -225\\
x &= \frac{-225}{-0.05} = 4500
\end{aligned}$$

Hence amount lent at 10% interest is Ksh. 4500.

4). Solve the following simultaneous equations **(4 marks)**

$$\log_3(3x+4y) = 2$$
$$\log_2(2x+y) = 1$$

Solution

$$\log_3(3x+4y) = 2 \implies 3x+4y = 3^2$$
$$\log_2(2x+y) = 1 \implies 2x+y = 2^1$$
$$3x+4y = 9$$
$$2x+y = 2 \implies 8x+4y = 8$$
$$-5x = 1 \implies x = -\frac{1}{5}$$
$$y = 2 - 2\left(-\frac{1}{5}\right) = \frac{12}{5}$$

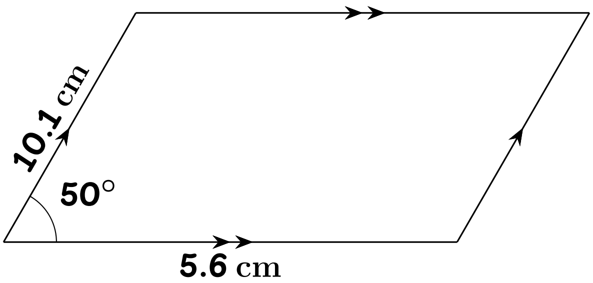
Hence $x = -\frac{1}{5}$ and $y = 2\frac{2}{5}$

5). The position vectors for points **P** and **Q** are $6\mathbf{i} - 3\mathbf{j} + 9\mathbf{k}$ and $3\mathbf{i} - 6\mathbf{j} - 3\mathbf{k}$ respectively. **R** divides line **PQ** in the ratio $1 : 2$. Find the position vector of **R** and express it in terms of unit vector **i**, **j** and **k**. **(3 marks)**

Solution

$$\overrightarrow{OR} = \frac{1}{3}\overrightarrow{OQ} + \frac{2}{3}\overrightarrow{OP}$$
$$= \frac{1}{3}(6\mathbf{i} - 3\mathbf{j} + 9\mathbf{k}) + \frac{2}{3}(3\mathbf{i} - 6\mathbf{j} - 3\mathbf{k})$$
$$= 2\mathbf{i} - \mathbf{j} + 3\mathbf{k} + 2\mathbf{i} - 4\mathbf{j} - 2\mathbf{k}$$
$$= 4\mathbf{i} - 5\mathbf{j} + \mathbf{k}$$

6). The parallelogram below has adjacent sides of lengths 5.6 cm and 10.1 cm respectively while the angle between them is $50°$.



Calculate the percentage error of finding its area. **(3 marks)**

Solution

$$\text{Minimum area} = \frac{1}{2} \times 5.55 \times 10.05 \sin 50° = 21.36402\ldots$$
$$\text{Actual area} = \frac{1}{2} \times 5.6 \times 10.1 \sin 50° = 21.6637 \text{ cm}^2$$
$$\text{Maximum area} = \frac{1}{2} \times 5.65 \times 10.15 \sin 50° = 21.96536\ldots$$
$$\text{Absolute error} = \frac{21.9653 - 21.6637}{2} = 0.1508$$
$$\text{Percentage error} = \frac{0.1508}{21.6637} \times 100\% = 0.696\%$$

7). Simplify completely $\dfrac{9x^2-16x+7}{162x^2-98}$ **(3 marks)**

Solution

$$\begin{aligned}\frac{9x^2-16x+7}{162x^2-98} &= \frac{9x^2-9x-7x+7}{2(81x^2-49)}\\ &= \frac{(9x-7)(x-1)}{2(9x-7)(9x+7)}\\ &= \frac{x-1}{2(9x+7)}\end{aligned}$$

8). Without using mathematical tables or a calculator, express $\sin 45^\circ$ in surd form. Hence simplify $\dfrac{\sqrt{8}}{1+\sin 45^\circ}$ leaving your answer in surd form. **(3 marks)**

Solution

$$\begin{aligned}\sin 45^\circ &= \frac{1}{\sqrt{2}}\\ \frac{\sqrt{8}}{1+\sin 45^\circ} &= \frac{2\sqrt{2}}{1+\frac{1}{\sqrt{2}}} = \frac{2\sqrt{2}}{1+\frac{\sqrt{2}}{2}}\\ &= \frac{2\sqrt{2}}{1+\frac{\sqrt{2}}{2}} \times \frac{1-\frac{\sqrt{2}}{2}}{1-\frac{\sqrt{2}}{2}}\\ &= \frac{2\sqrt{2}-2}{1-\frac{1}{2}} = \frac{2\sqrt{2}-2}{\frac{1}{2}}\\ &= 4\sqrt{2}-4\end{aligned}$$

9). (a) Expand $\left(1-\frac{1}{4}x\right)^5$ up to the 4^{th} term. **(2 marks)**

Solution

Apply binomial theorem: $(a+b)^n = \sum_{i=0}^{n}\binom{n}{i}a^{(n-i)}b^i$ where $a=1,\ b=-\frac{1}{4}x$

$$\begin{aligned}\left(1-\frac{1}{4}x\right)^5 &= \sum_{i=0}^{5}\binom{5}{i}\cdot 1^{(5-i)}\left(-\frac{1}{4}x\right)^i\\ &= 1\cdot 1^5\left(-\frac{1}{4}x\right)^0 + 5\cdot 1^4\left(-\frac{1}{4}x\right)^1 + 10\cdot 1^3\left(-\frac{1}{4}x\right)^2 + 10\cdot 1^2\left(-\frac{1}{4}x\right)^3 + 5\cdot 1^1\left(-\frac{1}{4}x\right)^4\\ &= 1-\frac{5x}{4}+\frac{5x^2}{8}-\frac{5x^3}{32}+\frac{5x^4}{256}-\frac{x^5}{1024}\\ &= 1-\frac{5x}{4}+\frac{5x^2}{8}-\frac{5x^3}{32}+\cdots\end{aligned}$$

(b) Use the expansion in part (a) above to find the approximate value of $(1.25)^5$. **(2 marks)**

Solution

To find 1.25^5, equate $1-\frac{1}{4}x = 1.25 \implies x=-1$. substitute $x=-1$ into the expression $1-\frac{5x}{4}+\frac{5x^2}{8}-\frac{5x^3}{32}+\cdots$

$$\begin{aligned}1.25^5 &= 1-\frac{5(-1)}{4}+\frac{5(-1)^2}{8}-\frac{5(-1)^3}{32}\\ &= \frac{97}{32} = 3.03125\end{aligned}$$

10). A bus travelling at an average speed of x km/h left a station at **8.15** a.m . A car, travelling at an average speed of **80** km/h left the same station at **9.00** a.m. and caught up with the bus at **10.45** a.m. Find the value of x. **(3 marks)**

Solution

	Time (hours)	Rate (speed)	Distance
Bus	$\frac{5}{2}$	x	$\frac{5}{2} \times x$
Car	$\frac{7}{4}$	80	$\frac{7}{4} \times 80$

The bus travelled the same distance as the car hence:

$$\frac{5}{2}x = \frac{7}{4} \times 80$$
$$\frac{5}{2}x = 140$$
$$\implies x = 140 \times \frac{2}{5}$$
$$= 56 \text{ km/h}$$

11). The data below represents the ages in months at which **6** babies started walking; **9, 11, 12, 13, 11** and **10**. Without using a calculator, find the exact value of the variance of the data. **(3 marks)**

Solution

$$\bar{x} = \frac{9+11+12+13+11+10}{6} = 11$$
$$d: -2 \quad 0 \quad 1 \quad 2 \quad 0 \quad -1$$
$$d^2: \quad 4 \quad 0 \quad 1 \quad 4 \quad 0 \quad 1$$
$$s^2 = \frac{\sum d^2}{N} = \frac{4+0+1+4+0+1}{6}$$
$$s^2 = \frac{10}{6} = \frac{5}{3} = 1\frac{2}{3}$$

12). A triangle **PQR** has an area of **3.2** cm^2 .It's image under a transformation matrix $\begin{pmatrix} 4x^2 & 5 \\ -x & 1 \end{pmatrix}$ has an area of **19.2** cm^2 .Find the value of x **(3 marks)**

Solution

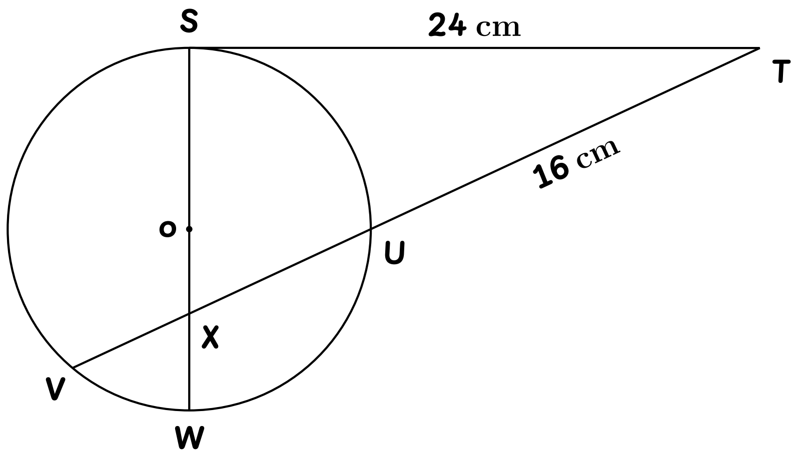
$$\text{ASF} = \frac{19.2}{3.2} = 6$$
$$\text{ASF} = \det \mathbf{M}$$
$$6 = 4x^2 - 5(-x)$$
$$0 = 4x^2 + 5x - 6$$
$$0 = 4x^2 + 8x - 3x - 6$$
$$0 = (x+2)(4x-3)$$
$$\implies x = -2 \quad \text{or} \quad x = \frac{3}{4}$$

13). The scale of a map is given as $1 : 50,000$. Find the actual area in hectares of the region represented by a rectangle of sides 6 cm by 7 cm. **(3 marks)**

Solution

$$\begin{aligned}1 \text{ cm} &\text{ rep } 50000 \text{ cm}\\ 1 \text{ cm} &\text{ rep } 500 \text{ m}\\ 1 \text{ cm}^2 &\text{ rep } 250000 \text{ m}^2\\ 1 \text{ cm}^2 &\text{ rep } 25 \text{ ha}\\ \text{Scale Area } &= 6 \times 7 = 42 \text{ cm}^2\\ \text{Actual area } &= 25 \text{ ha} \times 42\\ &= 1050 \text{ ha}\end{aligned}$$

14). In the figure below, the tangent **ST** meets chord **VU** produced at **T**. chord SW passes through the centre **O** of the circle and intersects chord **VU** at **X**. line $ST = 24$ cm and $UT = 16$ cm.



(a) Calculate the length of chord **VU**. **(1 mark)**

Solution

Let $x = VU$

$$\begin{aligned}ST^2 &= VT \cdot TU\\ 24^2 &= (16 + x)16\\ 576 &= 256 + 16x\\ \implies 16x &= 320\\ VU = x &= 20 \text{ cm}\end{aligned}$$

(b) If $WX = 6$ cm and $VX : XU = 2 : 3$ find **SX** **(2 marks)**

Solution

$$\frac{VX}{XU} = \frac{WX}{SX} = \frac{2}{3}$$

$$\frac{6}{SX} = \frac{2}{3} \implies SX = 6 \times \frac{3}{2}$$

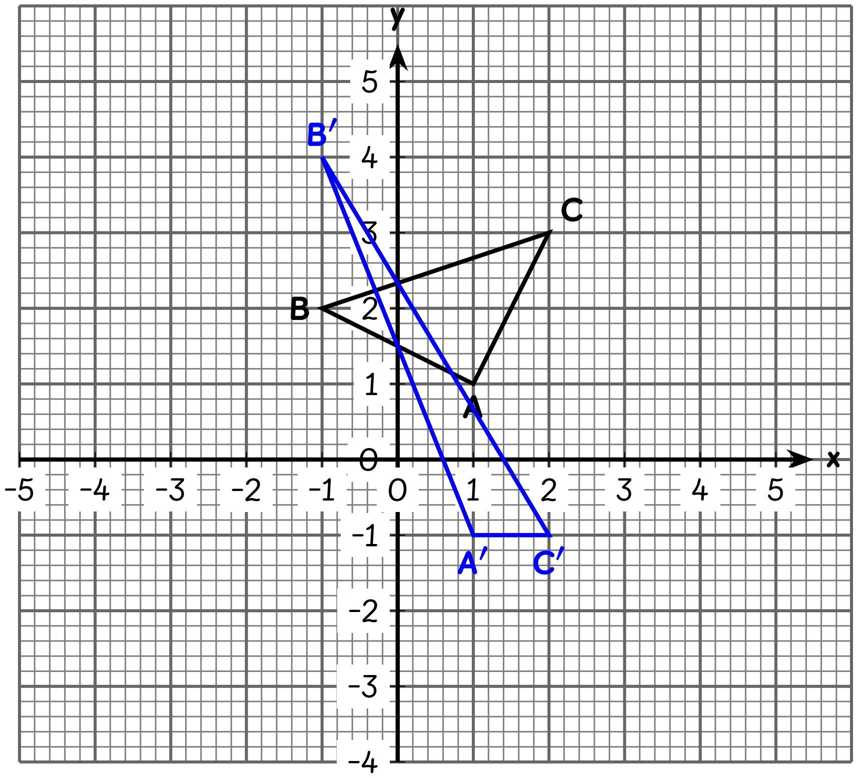
$$SX = 9 \text{ cm}$$

15). Find the value of **x** in the following equation, $2^{2x-1} + 4^{x+1} = 36$ **(3 marks)**

Solution

$$2^{2x-1} + 4^{x+1} = 36$$
$$2^{2x-1} + 2^{2(x+1)} = 36$$
$$\frac{1}{2} \cdot 2^{2x} + 2^2 \cdot 2^{2x} = 36$$
$$\frac{1}{2} \cdot 2^{2x} + 4 \cdot 2^{2x} = 36$$
$$\frac{9}{2} \cdot 2^{2x} = 36$$
$$2^{2x} = 8 = 2^3$$
$$\implies 2x = 3$$
$$x = \frac{3}{2} = 1\frac{1}{2}$$

16). Triangle **ABC** is shown on the coordinate plane below. Given that $\mathbf{A}(1,1)$ is mapped onto $\mathbf{A'}(1,-1)$ by a shear with the **y − axis** invariant , draw triangle **A′B′C′** the image of triangle **ABC** under the shear. **(3 marks)**



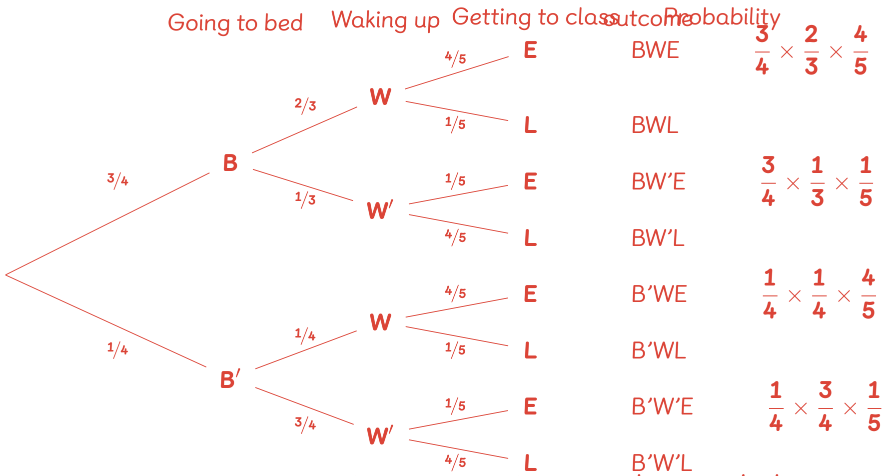
SECTION TWO - 50 Marks

Answer any **five** questions from this section in the spaces provided.

17). (a) The probability that Nina goes to bed on time is $3/4$. If she goes to bed on time, the probability that she wakes up on time is $2/3$, otherwise the probability that she wakes up on time is $1/4$. If she wakes up late, her probability of getting to class on time is $1/5$ otherwise her probability of getting to class on time is $4/5$. Find the probability that:

(i) She wakes up late. **(3 marks)**

Solution



$$P(\text{wake up late}) = P(BW') + P(B'W') = \frac{3}{4}\times\frac{1}{3}+\frac{1}{4}\times\frac{3}{4} = \frac{1}{4}+\frac{3}{16}=\frac{7}{16}$$

(ii) She gets to class on time **(3 marks)**

Solution

$$P(\text{arrives on time}) = P(BWE)+P(BW'E)+P(B'WE)+P(B'W'E)$$
$$= \left(\frac{3}{4}\times\frac{2}{3}\times\frac{4}{5}\right)+\left(\frac{3}{4}\times\frac{1}{3}\times\frac{1}{5}\right)+\left(\frac{1}{4}\times\frac{1}{4}\times\frac{4}{5}\right)+\left(\frac{1}{4}\times\frac{3}{4}\times\frac{1}{5}\right)$$
$$=\frac{2}{5}+\frac{1}{20}+\frac{1}{20}+\frac{3}{80}=\frac{43}{80}$$

(b) A die and a coin are cast simultaneously.

(i) Draw a table to show all the possible outcomes **(2 marks)**

Solution

	1	2	3	4	5	6
H	H1	H2	H3	H4	H5	H6
T	T1	T2	T3	T4	T5	T6

(ii) What is the probability of a tail and a number less than **4** showing up. **(2 marks)**

Solution

$$P(T \text{ and } X<4) = P(T1)+P(T2)+P(T3) = \frac{1}{12}+\frac{1}{12}+\frac{1}{12}=\frac{3}{12}=\frac{1}{4}$$

18). (a) The first term of an arithmetic progression (AP) is **6**. The sum of the first **7** terms of the AP is **126**.

(i) Find the common difference of the AP **(2 marks)**

Solution

First term is $T_1 = 6$ and sum of first 7 terms is $S_7 = 126$.

$$S_7 = 126 = \frac{7}{2}(2(6) + d(7-1))$$
$$126 = 7(6 + 3d)$$
$$18 = 6 + 3d$$
$$12 = 3d \implies d = 4$$

(ii) Find the 19^{th} term of the AP. **(1 mark)**

Solution

First term is $T_1 = 6$ and common difference $d = 4$.

$$T_n = a + d(n-1)$$
$$T_{19} = 6 + 4(19-1) = 78$$

(b) The 2^{nd}, 3^{rd} and 11^{th} terms of an increasing arithmetic progression(AP) form the first **3** terms of a geometric progression (GP). The first term of the AP is -2.

(i) Find the common difference of the AP and the common ratio(r) of the GP. **(4 marks)**

Solution

Let the first term of the GP be a and common ratio be r and d be the common difference of the AP.

$$T_1 = ar^{1-1} = b + (2-1)d \implies a = -2 + d$$
$$T_2 = ar^{2-1} = b + (3-1)d \implies ar = -2 + 2d$$
$$T_3 = ar^{3-1} = b + (11-1)d \implies ar^2 = -2 + 10d$$

Solve equations for d by dividing consecutive terms in the GP.

$$\frac{T_2}{T_1} = \frac{T_3}{T_2} \implies \frac{-2+2d}{-2+d} = \frac{-2+10d}{-2+2d}$$
$$(-2+2d)^2 = (-2+d)(-2+10d)$$
$$4 - 8d + 4d^2 = 4 - 22d + 10d^2$$
$$-6d^2 + 14d = -2d(3d-7) = 0$$
$$\implies d = 0 \quad \text{or} \quad d = \frac{7}{3}$$

We are know that $d = 6$ hence the common ratio:

$$r = \frac{T_2}{T_1} = \frac{-2+2d}{-2+d} = \frac{-2+2(7/3)}{-2+7/3}$$
$$\implies r = 8$$

(ii) Find the sum of the first **5** terms of the geometric progression(GP). **(3 marks)**

Solution

We are at a GP with $n = 5$ terms, first term $a = -2 + \frac{7}{3}$ and the common ratio $r = 8$

$$T_n = \frac{a(r^n - 1)}{r-1}$$
$$T_5 = \frac{\frac{1}{3}(8^5 - 1)}{8-1} = \frac{4681}{3}$$

19). An aircraft leaves town **P**$(30^\circ\text{S}, 17^\circ\text{E})$ and flies due north to **Q**$(60^\circ\text{N}, 17^\circ\text{E})$. It then flies at an average speed of **300** knots for **8** hours due west to town **R**. Determine:

(a) The distance **PQ** in nautical miles. **(2 marks)**

Solution

$$\begin{aligned}\text{latitude difference } &= 30 + 60 = 90^\circ\\ \text{Arc length } \mathbf{PQ} &= 60 \times 90\\ &= 5400 \text{ nm}\end{aligned}$$

(b) The position of town **R**. **(4 marks)**

Solution

$$\begin{aligned}\text{arc length } \mathbf{QR} &= 300 \times 8 = 2400 \text{ nm}\\ 2400 &= 60\theta\cos 60\\ \implies \theta &= 80^\circ\\ \text{longitude difference } &= 17 + x = 80\\ &\implies x = 63\end{aligned}$$

Hence position of **R** is $(60^\circ\text{N},\ 63^\circ\text{W})$.

(c) The local time at **R** if the local time at **Q** is **3.12** pm. **(2 marks)**

Solution

$$\begin{aligned}\text{time difference } &= 60 \times 4 = 240 \text{ min}\\ \text{local time at } \mathbf{R} &= 3.12 \text{ pm} + 4 \text{ h}\\ &= 7.12 \text{ pm}\end{aligned}$$

(d) The distance travelled by the aircraft from **Q** to **R** to the nearest kilometre. $(1 \text{ km} = 0.539957 \text{ nm})$ **(2 marks)**

Solution

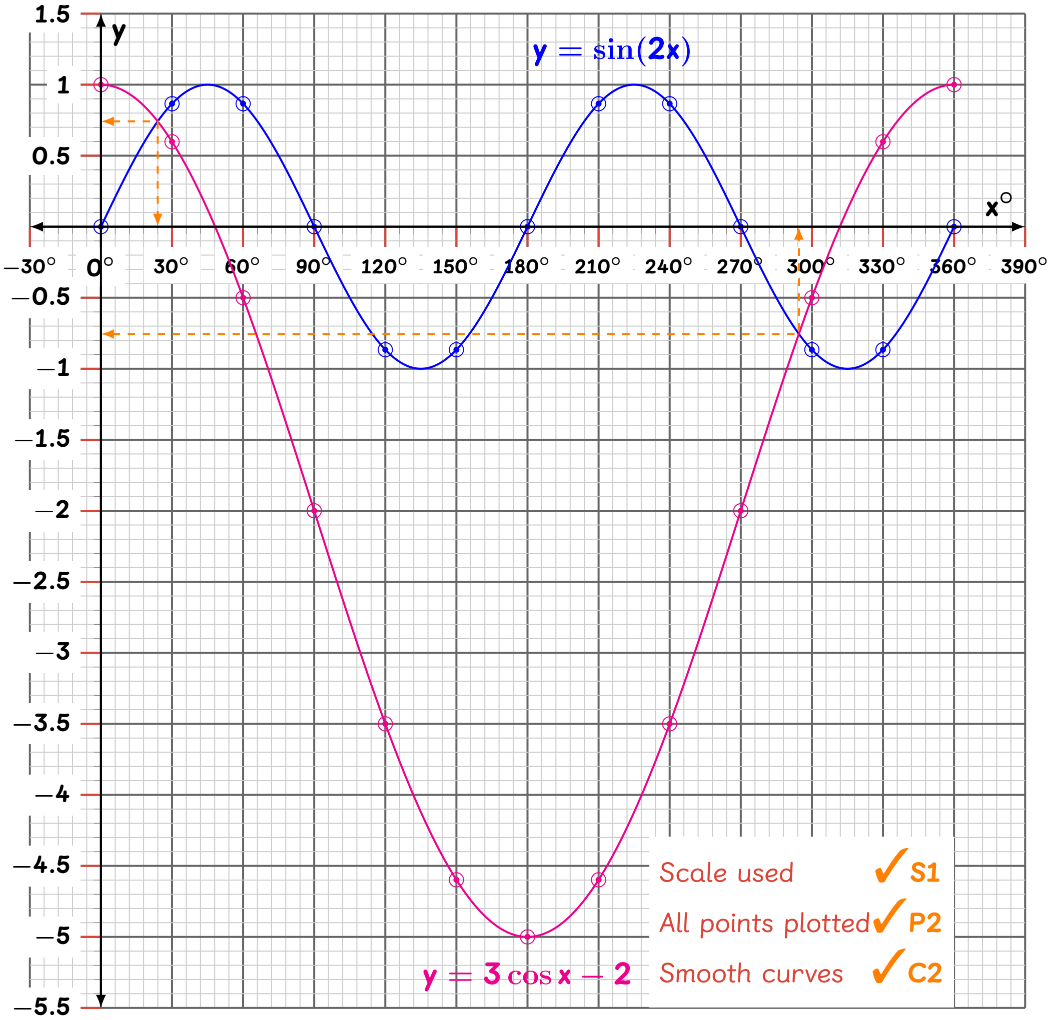
$$\begin{aligned}\text{arc length } \mathbf{QR} &= 2400 \times 1.852\\ &= 4444.8 \text{ km}\end{aligned}$$

20). (a) Complete the table below, giving the values correct to **2** decimal places. **(2 marks)**

x	0°	30°	60°	90°	120°	150°	180°	210°	240°	270°	300°	330°	360°
$\sin 2x$	0	0.50	0.87	1	−0.87	0.5	0	0.87	0.87	-1	-0.87	-0.5	0
$3\cos x - 2$	1	0.60	-0.5	−2	−3.5	-4.60	-5	−4.60	-3.5	-2	−0.5	0.60	1

✓ B2

(b) On the grid provided, draw the graphs of $y = \sin 2x$ and $y = 3\cos x - 2$ for $0^\circ \leq x \leq 360^\circ$ on the same axes. Use a scale of **1 cm** to represent 30° on the x-axis and **2 cm** to represent **1** unit on the y-axis. **(5 marks)**



(c) Use the graph in **(b)** above to solve the equation $3\cos x - \sin 2x = 2$. **(2 marks)**

Solution

$$3\cos x - \sin 2x = 2 \implies \sin 2x = 3\cos x - 2$$ ✓ B1 rearrange the equation

the curve $y = \sin 2x$ intersects with $y = 3\cos x - 2$ at:

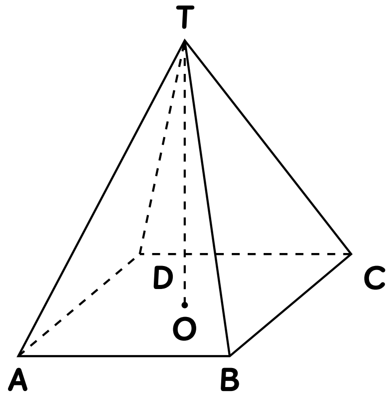
$$x = 24, \; x = 294^\circ$$ ✓ B1

(d) State the amplitude of $y = 3\cos x - 2$. **(1 mark)**

Solution

Amplitude $= 3$ ✓ B1

21). The figure below is of a right pyramid on a rectangle base. **TC** = **TB** = **TA** = **TD** = **17** cm, and **TO** = **15** cm.**AB** is twice **BC**.



Calculate:

(i) The length **AB** **(4 marks)**

Solution

$$OA^2 = AT^2 - OT^2 = 17^2 - 15^2 = 64$$
$$\implies OA = \sqrt{64} = 8$$

Note that **AB** = **2BC**, hence

$$AC^2 = AB^2 + BC^2$$
$$(2OA)^2 = (2BC)^2 + BC^2$$
$$(2 \cdot 8)^2 = 4BC^2 + BC^2 \implies 256 = 5BC^2$$
$$BC = \frac{16}{\sqrt{5}} = 3.2\sqrt{5}$$
$$\implies AB = 2(3.2\sqrt{5}) \approx 14.32 \text{ cm}$$

(ii) The angle between **TC** and plane **ABCD** **(2 marks)**

Solution

$$\sin \angle C = \frac{OT}{TC} = \frac{15}{17}$$
$$\implies \angle C = \sin^{-1} \frac{15}{17} = 61.93^\circ$$

(iii) The angle between **TD** and plane **TAC** **(2 marks)**

Solution

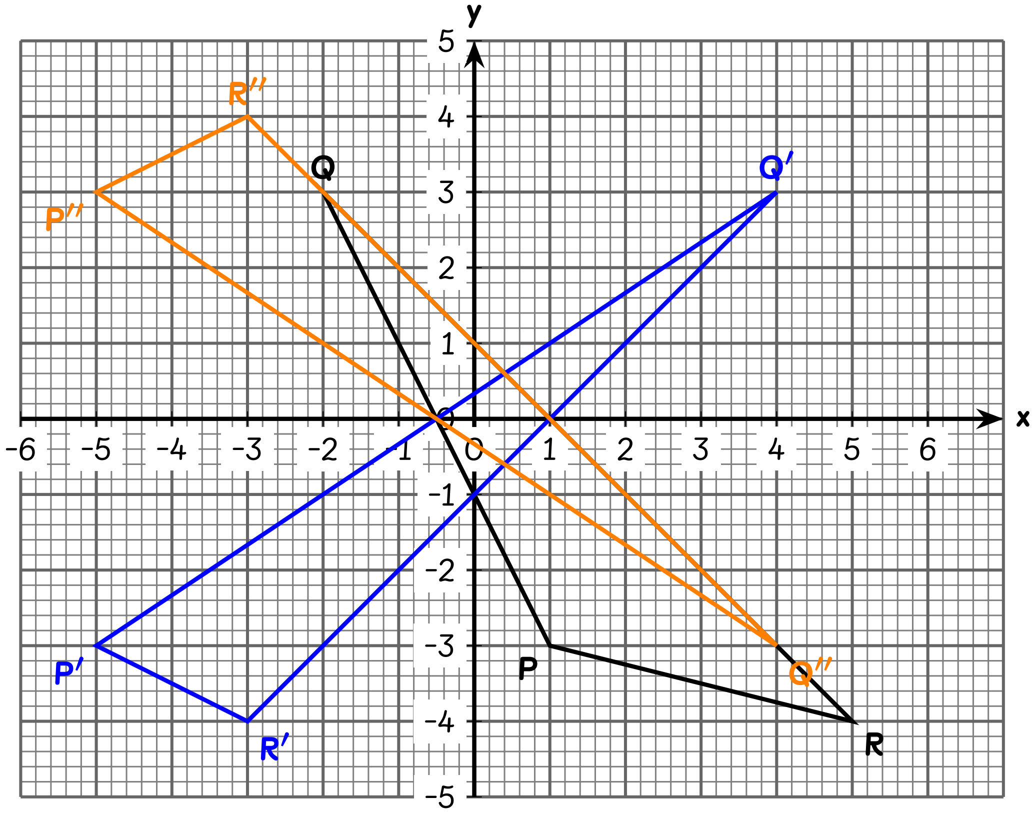
$$61.93^\circ + \angle DTO = 90^\circ \qquad \angle ODT + \angle DTO = 90^\circ$$
$$\implies \angle DTO = 28.07^\circ$$

(iv) The angle between **TAB** and **ABCD** **(2 marks)**

Solution

$$\tan \angle M = \frac{OT}{OM} = \frac{15}{3.2\sqrt{5}}$$
$$\implies \angle M = \tan^{-1} \frac{15}{3.2\sqrt{5}} = 64.50^\circ$$

22). (a) Given that point $\mathbf{Q}(-2,3)$ is mapped onto $\mathbf{Q'}(4,3)$ by a shear with $\mathbf{x}$ – **axis** invariant,



(i) Draw triangle $\mathbf{P'Q'R'}$, the image of **PQR** under the shear. **(3 marks)**

(ii) Determine the matrix representing the shear. **(2 marks)**

Solution

$$\text{shear factor} = \frac{4-(-2)}{3} = 2$$

$$\text{shear matrix} = \begin{pmatrix} 1 & k \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$$

(b) Triangle $\mathbf{P'Q'R'}$ is mapped onto triangle $\mathbf{P''Q''R''}$ by a transformation defined by the matrix $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

(i) Draw triangle $\mathbf{P''Q''R''}$ **(3 marks)**

Solution

$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} -5 & 4 & -3 \\ -3 & 3 & -4 \end{pmatrix} = \begin{pmatrix} -5 & 4 & -3 \\ 3 & -3 & 4 \end{pmatrix}$$

$\mathbf{P''}(-5,3),\ \mathbf{Q''}(4,-3),\ \mathbf{R''}(-3,4)$

(ii) Find a combined matrix that maps **PQR** onto $\mathbf{P''Q''R''}$ **(2 marks)**

Solution

$$\mathbf{M} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ 0 & -1 \end{pmatrix}$$

23). The table below shows the masses in kg of **50** animals selected at random in a farm.

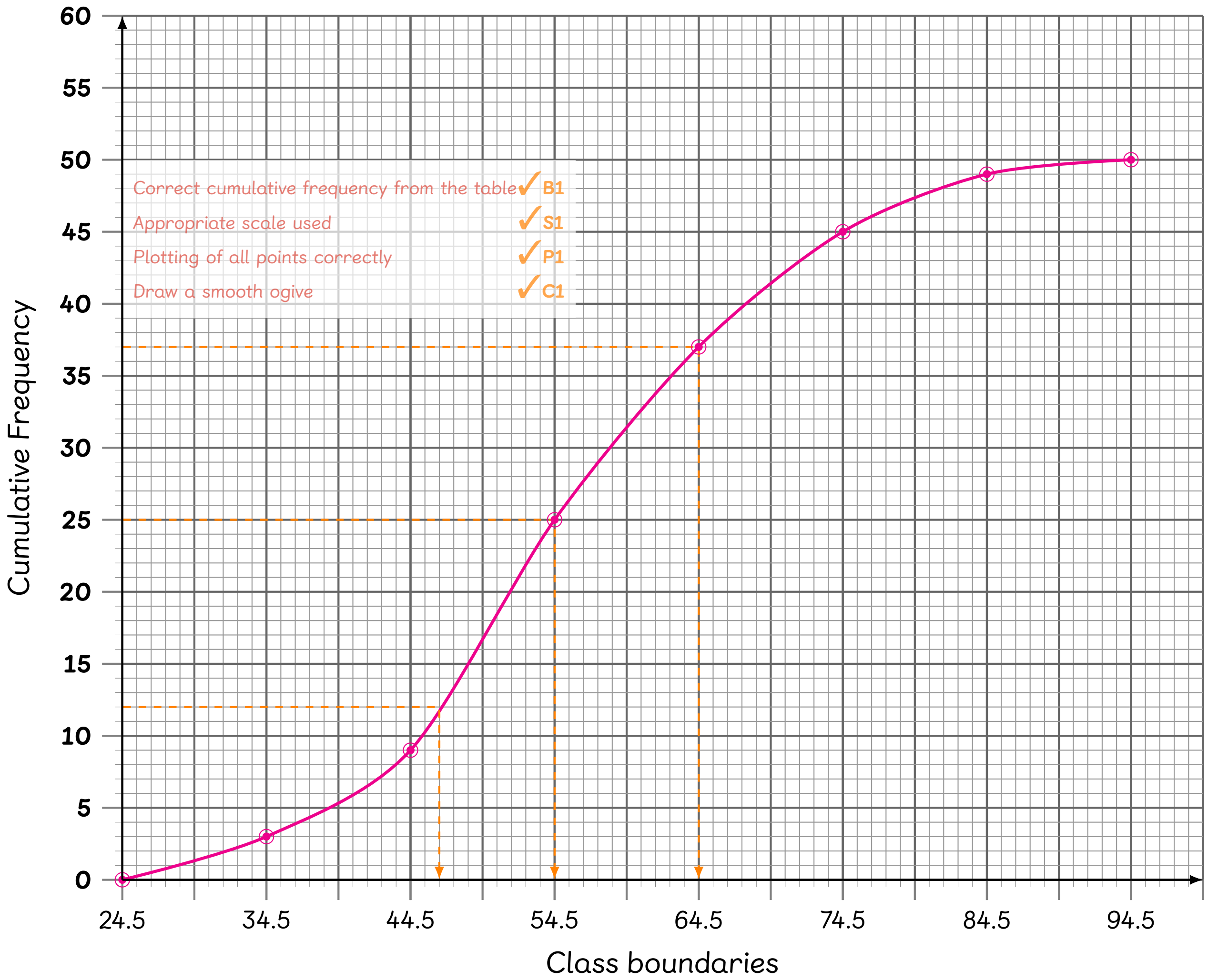
60	25	37	50	62	39	47	64	70	58
66	48	28	47	43	56	59	53	77	46
47	59	40	78	45	51	61	73	33	70
69	61	52	53	36	48	74	58	82	54
54	68	41	59	45	69	83	50	91	63

(a) Starting with the mass of **25** and using equal class intervals of **10**, make a frequency distribution table for the data. **(2 marks)**

Solution

Class	25 – 34	35 – 44	45 – 54	55 – 64	65 – 74	75 – 84	85 – 94
Tally	|||	~~||||~~ |	~~||||~~ ~~||||~~ ~~||||~~ |	~~||||~~ ~~||||~~ ||	~~||||~~ |||	||||	|
Frequency	3	6	16	12	8	4	1
cf	3	9	25	37	45	49	50

(b) On the grid provided draw a cumulative frequency curve for the data. **(4 marks)**



#23 continues on the next page...

(c) Use the graph in **(b)** above to determine:

(i) The median mass. **(2 marks)**

Solution

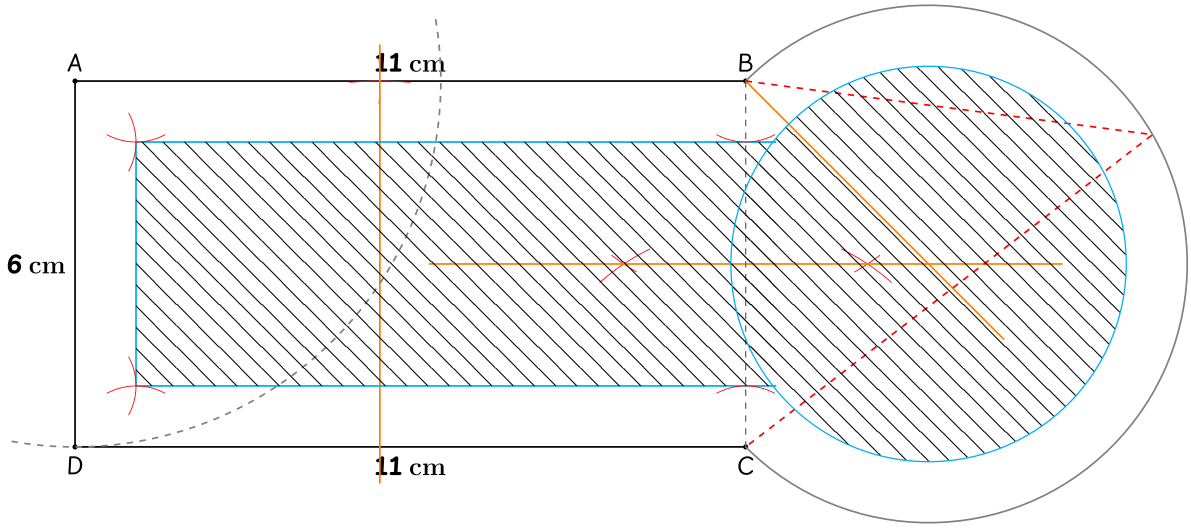
Median $= \mathbf{54.5}$ read from the graph

(ii) The quartile deviation. **(2 marks)**

Solution

$$\text{Lower quartile} = \mathbf{Q_1} = \mathbf{47.5}$$
$$\text{Upper quartile} = \mathbf{Q_3} = \mathbf{64.5}$$
$$\text{Quartile deviation} = \frac{\mathbf{Q_3 - Q_1}}{\mathbf{2}}$$
$$= \frac{\mathbf{64.5 - 47.5}}{\mathbf{2}} = \mathbf{8.5}$$

24). The diagram below is a scale drawing of a piece of land. Three boundaries **AB**, **AD** and **DC** of the land are given. The fourth boundary is not given but it is known that the area of the land if greater than that of rectangle **ABCD**.



Use a ruler and pair of compasses only in this question.

(a) Construct the locus of all points equidistant from points **B** and **C**. **(1 mark)**

Solution

Construct a perpendicular bisector of line **BC**

(b) The locus of any point **P** lying on the fourth boundary is such that $\angle$**BPC** $=$ **45**°. Draw the fourth boundary. **(4 marks)**

Solution

Construct **45**° at **B** to meet the bisector at **O**.

Draw an arc center **O** radius **BO**.

Locus of **P** lies on the arc above.

(c) Shade the region within the scale drawing in which a variable point **X** must lie giving that **X** satisfies the following conditions. **(5 marks)**

(i) **X** is at least **1** cm from each of the four boundaries

(ii) **X** is at least **6** cm from **A**

(iii) Area of $\triangle$**AXD** $\geq$ **15** cm^2

Solution

Construct lines parallel to the boundaries on the inner side.

Draw an arc center **O**, **1** cm away from the locus of **P**.

Draw an arc centre **A** radius **6** cm.

Draw a line parallel to **AD**, **5** cm away from **AD**.

Shade the required region as shown above.