

# **F4 TOPICAL REVISION MATHEMATICS**

***A SERIES OF TOPICAL QUESTIONS IN FORM  
FOUR MATHEMATICS***

***FOR MARKING SCHEMES  
CALL/WHATSAPP 0705525657***

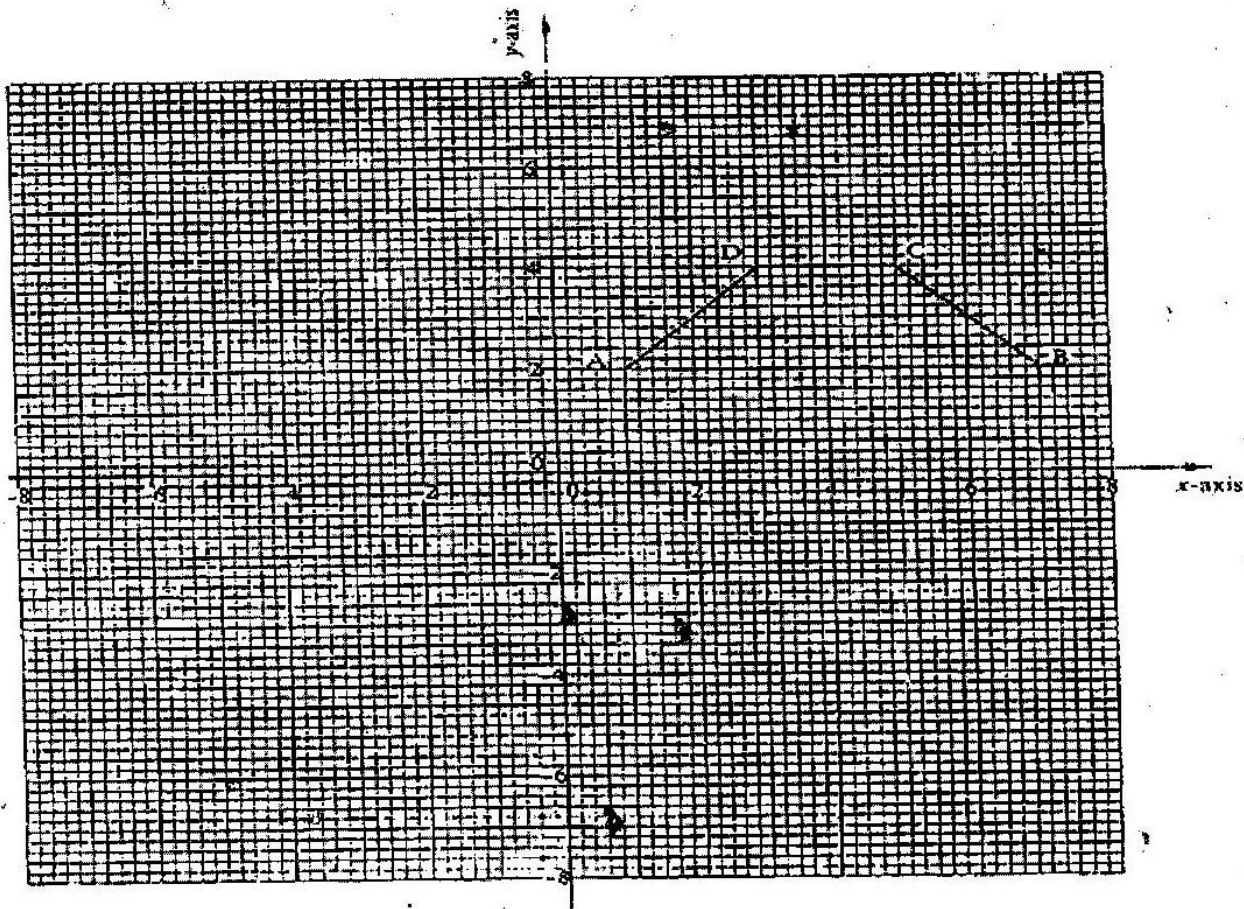
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## FORM FOUR WORK

### TOPIC 1

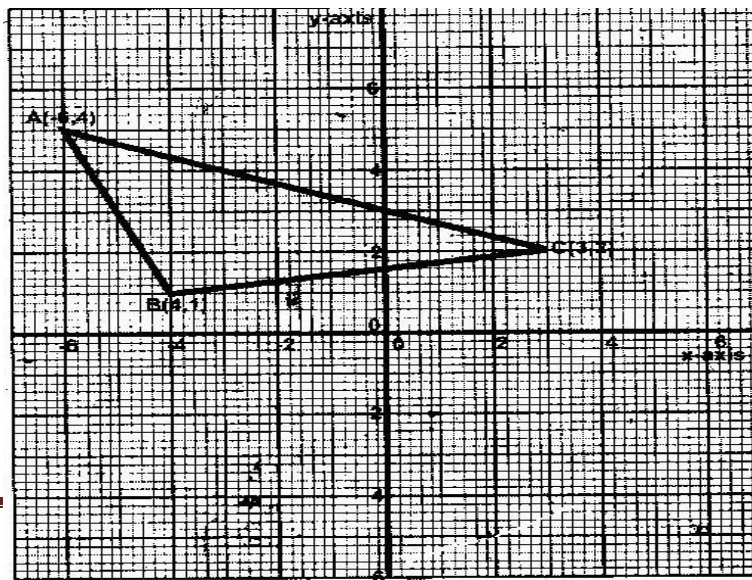
#### MATRICES AND TRANSFORMATIONS

1. Matrix  $P$  is given by 
$$P = \begin{pmatrix} 1 & 2 \\ 4 & 3 \end{pmatrix}$$
- (a) Find  $P^{-1}$
- (b) Two institutions, Elimu and Somo, purchase beans at Kshs.  $B$  per bag and maize at Kshs  $m$  per bag. Elimu purchased 8 bags of beans and 14 bags of maize for Kshs 47,600. Somo purchased 10 bags of beans and 16 of maize for Kshs. 57,400
- (c) The price of beans later went up by 5% and that of maize remained constant. Elimu bought the same quantity of beans but spent the same total amount of money as before on the two items. State the new ratio of beans to maize.
2. A triangle is formed by the coordinates A (2, 1) B (4, 1) and C (1, 6). It is rotated clockwise through  $90^\circ$  about the origin. Find the coordinates of this image.
3. On the grid provided on the opposite page A (1, 2) B (7, 2) C (4, 4) D (3, 4) is a trapezium



- (a) ABCD is mapped onto A'B'C'D' by a positive quarter turn. Draw the image A'B'C'D' on the grid
- (b) A transformation  $\begin{pmatrix} -2 & -1 \\ 0 & 1 \end{pmatrix}$  maps A'B'C'D' onto A''B''C''D'' Find the coordinates of A''B''C''D''
4. A triangle T whose vertices are A (2, 3) B (5, 3) and C (4, 1) is mapped onto triangle T<sup>1</sup> whose vertices are A<sup>1</sup> (-4, 3) B<sup>1</sup> (-1, 3) and C<sup>1</sup> (x, y) by a Transformation M =  $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$

- a) Find the: (i) Matrix  $M$  of the transformation  
(ii) Coordinates of  $C_1$
- b) Triangle  $T^2$  is the image of triangle  $T^1$  under a reflection in the line  $y = x$ .  
Find a single matrix that maps  $T$  and  $T_2$
5. Triangles  $ABC$  is such that  $A$  is  $(2, 0)$ ,  $B$   $(2, 4)$ ,  $C$   $(4, 4)$  and  $A''B''C''$  is such that  $A''$  is  $(0, 2)$ ,  $B''$   $(-4, -10)$  and  $C''$  is  $(-4, -12)$  are drawn on the Cartesian plane  
Triangle  $ABC$  is mapped onto  $A''B''C''$  by two successive transformations  
 $R = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$  Followed by  $P = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$
- (a) Find  $R$
- (b) Using the same scale and axes, draw triangles  $A'B'C'$ , the image of triangle  $ABC$  under transformation  $R$   
Describe fully, the transformation represented by matrix  $R$
6. Triangle  $ABC$  is shown on the coordinates plane below



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- (a) Given that A (-6, 5) is mapped onto A' (6,-4) by a shear with y- axis invariant
- (i) Draw triangle A'B'C', the image of triangle ABC under the shear
- (ii) Determine the matrix representing this shear
- (b) Triangle A B C is mapped on to A'' B'' C'' by a transformation defined by the matrix
- $$\begin{pmatrix} -1 & 0 \\ 1\frac{1}{2} & -1 \end{pmatrix}$$
- (i) Draw triangle A'' B'' C''
- (ii) Describe fully a single transformation that maps ABC onto A''B'' C''

7. Determine the inverse  $T^{-1}$  of the matrix
- $$\begin{pmatrix} 1 & 2 \\ 1 & -1 \end{pmatrix}$$

Hence find the coordinates to the point at which the two lines

$$x + 2y = 7 \text{ and } x - y = 1$$

8. Given that  $A = \begin{pmatrix} 0 & -1 \\ 3 & 2 \end{pmatrix}$  and  $B = \begin{pmatrix} -1 & 0 \\ 2 & -4 \end{pmatrix}$

Find the value of x if

- (i)  $A - 2x = 2B$
- (ii)  $3x - 2A = 3B$

(iii)  $2A - 3B = 2x$

9. The transformation R given by the matrix

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \text{ maps } \begin{pmatrix} 17 \\ 0 \end{pmatrix} \text{ to } \begin{pmatrix} 15 \\ 8 \end{pmatrix} \text{ and } \begin{pmatrix} 0 \\ 17 \end{pmatrix} \text{ to } \begin{pmatrix} -8 \\ 15 \end{pmatrix}$$

- (a) Determine the matrix A giving a, b, c and d as fractions
- (b) Given that A represents a rotation through the origin determine the angle of rotation.
- (c) S is a rotation through 180 about the point (2, 3). Determine the image of (1, 0) under S followed by R.

**TOPIC 2**  
**STATISTICS**

1. Every week the number of absentees in a school was recorded. This was done for 39 weeks these observations were tabulated as shown below

Number of absentees	0-3	4 -7	8 -11	12 - 15	16 - 19	20 - 23
(Number of weeks)	6	9	8	11	3	2

Estimate the median absentee rate per week in the school

2. The table below shows high altitude wind speeds recorded at a weather station in a period of 100 days.

Wind speed ( knots)	0 - 19	20 - 39	40 - 59	60-79	80- 99	100- 119	120-139	140-159	160-179
Frequency (days)	9	19	22	18	13	11	5	2	1

- (a) On the grid provided draw a cumulative frequency graph for the data
- (b) Use the graph to estimate
- (i) The interquartile range
- (ii) The number of days when the wind speed exceeded 125 knots
3. Five pupils A, B, C, D and E obtained the marks 53, 41, 60, 80 and 56 respectively. The table below shows part of the work to find the standard deviation.

Pupil	Mark x	x - a	( x-a) <sup>2</sup>
A	53	-5	
B	41	-17	
C	60	2	

D	80	22	
E	56	-2	

- (a) Complete the table
- (b) Find the standard deviation

4. In an agricultural research centre, the length of a sample of 50 maize cobs were measured and recorded as shown in the frequency distribution table below.

Length in cm	Number of cobs
8 – 10	4
11 – 13	7
14 – 16	11
17 – 19	15
20 – 22	8
23 - 25	5

Calculate

- (a) The mean
- (b) (i) The variance
- (ii) The standard deviation
5. The table below shows the frequency distribution of masses of 50 new- born calves in a ranch

Mass (kg)	Frequency
15 – 18	2
19- 22	3



23 – 26	10
27 – 30	14
31 – 34	13
35 – 38	6
39 – 42	2

- (a) On the grid provided draw a cumulative frequency graph for the data
- (b) Use the graph to estimate
- (i) The median mass
- (ii) The probability that a calf picked at random has a mass lying between 25 kg and 28 kg.

6. The table below shows the weight and price of three commodities in a given period

Commodity	Weight	Price Relatives
X	3	125
Y	4	164
Z	2	140

Calculate the retail index for the group of commodities.

7. The number of people who attended an agricultural show in one day was 510 men, 1080 women and some children. When the information was represented on a pie chart, the combined angle for the men and women was  $216^\circ$ . Find the angle representing the children.

8. The mass of 40 babies in a certain clinic were recorded as follows:

<u>Mass in Kg</u>	<u>No. of babies.</u>
1.0 – 1.9	6
2.0 – 2.9	14
3.0 -3.9	10
4.0 – 4.9	7
5.0 – 5.9	2
6.0 – 6.9	1

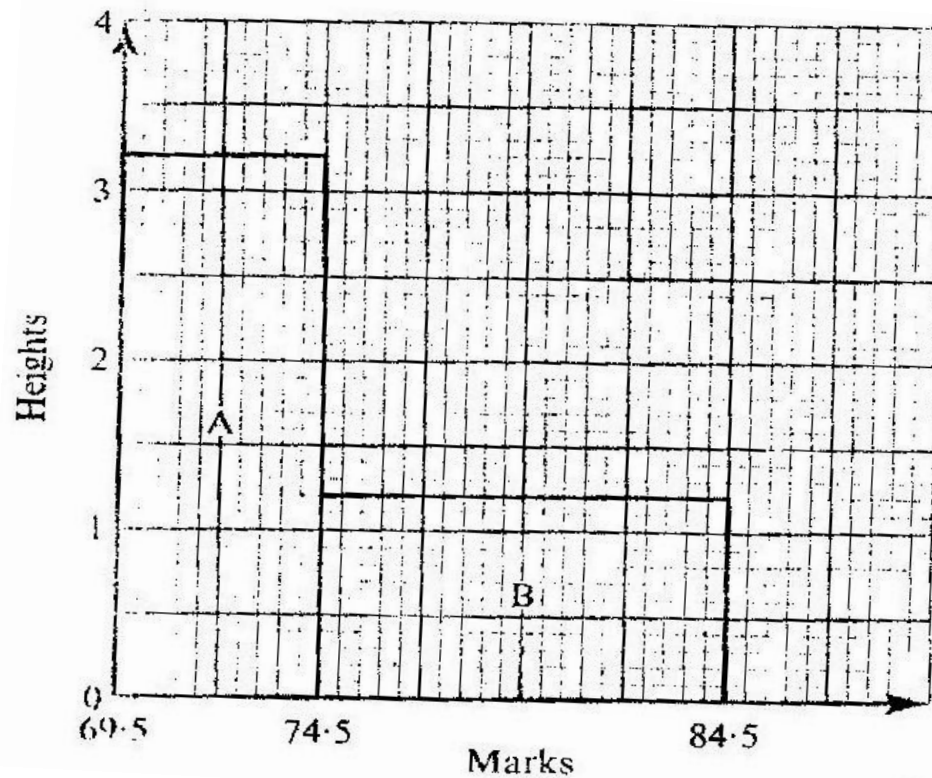
Calculate

- (a) The inter – quartile range of the data.
- (b) The standard deviation of the data using 3.45 as the assumed mean.
9. The data below shows the masses in grams of 50 potatoes

Mass (g)	25- 34	35-44	45 - 54	55- 64	65 - 74	75-84	85-94
No of potatoes	3	6	16	12	8	4	1

- (a) On the grid provide, draw a cumulative frequency curve for the data
- (b) Use the graph in (a) above to determine
- (i) The 60<sup>th</sup> percentile mass
- (ii) The percentage of potatoes whose masses lie in the range 53g to 68g
10. The histogram below represents the distribution of marks obtained in a test.

The bar marked A has a height of 3.2 units and a width of 5 units. The bar marked B has a height of 1.2 units and a width of 10 units



If the frequency of the class represented by bar B is 6, determine the frequency of the class represented by bar A.

11. A frequency distribution of marks obtained by 120 candidates is to be represented in a histogram. The table below shows the grouped marks. Frequencies for all the groups and also the area and height of the rectangle for the group 30 – 60 marks.

Marks	0-10	10-30	30-60	60-70	70-100
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Frequency	12	40	36	8	24
Area of rectangle			180		
Height of rectangle			6		

(a) (i) Complete the table

(ii) On the grid provided below, draw the histogram

(b) (i) State the group in which the median mark lies

(ii) A vertical line drawn through the median mark divides the total area of the histogram into two equal parts

Using this information or otherwise, estimate the median mark

12. In an agriculture research centre, the lengths of a sample of 50 maize cobs were measured and recorded as shown in the frequency distribution table below

Length in cm	Number of cobs
8 – 10	4
11- 13	7
14 – 16	11
17- 19	15
20 – 22	8
23- 25	5

Calculate

- (a) The mean
- (b) (i) The variance
- (ii) The standard deviation

11. The table below shows the frequency distribution of masses of 50 newborn calves in a ranch.

Mass (kg)	Frequency
15 – 18	2
19- 22	3
23 – 26	10
27 – 30	14
31- 34	13
35 – 38	6
39 - 42	2

- (a) On the grid provided draw a cumulative frequency graph for the data
- (b) Use the graph to estimate
  - (i) The median mass
  - (ii) The probability that a calf picked at random has a mass lying between 25 kg and 28 kg

14. The table shows the number of bags of sugar per week and their moving averages

Number of bags per week	340	330	x	343	350	345
Moving averages		331	332	y	346	

(a) Find the order of the moving average

(b) Find the value of X and Y axis

### TOPIC 3

#### LOC1

- Using ruler and compasses only, construct a parallelogram ABCD such that  $AB = 10\text{cm}$ ,  $BC = 7\text{ cm}$  and  $\angle ABC = 105^\circ$ . Also construct the loci of P and Q within the parallelogram such that  $AP \leq 4\text{ cm}$ , and  $BC \leq 6\text{ cm}$ . Calculate the area within the parallelogram and outside the regions bounded by the loci.

- Use ruler and compasses only in this question

The diagram below shows three points A, B and D

- Construct the angle bisector of acute angle BAD
- A point P, on the same side of AB and D, moves in such a way that  $\angle APB = 22\frac{1}{2}^\circ$  construct the locus of P
- The locus of P meets the angle bisector of  $\angle BAD$  at C measure  $\angle ABC$

- Use a ruler and a pair of compasses only for all constructions in this question.

- On the line BC given below, construct triangle  $\triangle ABC$  such that  $\angle ABC = 30^\circ$  and  $BA = 12\text{ cm}$
- Construct a perpendicular from A to meet BC produced at D. Measure CD

- (c) Construct triangle  $A'B'C'$  such that the area of triangle  $A'B'C'$  is the three quarters of the area of triangle  $ABC$  and on the same side of  $BC$  as triangle  $ABC$ .
- (d) Describe the locus of  $A'$
4. Use a ruler and compasses in this question. Draw a parallelogram  $ABCD$  in which  $AB = 8$  cm,  $BC = 6$  cm and  $\angle BAD = 75^\circ$ . By construction, determine the perpendicular distance between  $AB$  and  $CD$ .
5. In this question use a ruler and a pair of compasses.
- a) Line  $PQ$  drawn below is part of a triangle  $PQR$ . Construct the triangle  $PQR$  in which  $\angle QPR = 30^\circ$  and line  $PR = 8$  cm
- b) On the same diagram construct triangle  $PRS$  such that points  $S$  and  $Q$  are on the opposite sides of  $PR$  such that  $PS = PQ$  and  $QS = 8$  cm
- c) A point  $T$  is on the line passing through  $R$  and parallel to  $QS$ . If  $\angle QTS = 90^\circ$ , locate possible positions of  $T$  and label them  $T_1$  and  $T_2$ , Measure the length of  $T_1T_2$ .
6. (a)  $ABCD$  is a rectangle in which  $AB = 7.6$  cm and  $AD = 5.2$  cm. Draw the rectangle and construct the locus of a point  $P$  within the rectangle such that  $P$  is equidistant from  $CB$  and  $CD$  (3 marks)
- (b)  $Q$  is a variable point within the rectangle  $ABCD$  drawn in (a) above such that  $60^\circ \leq \angle AQB \leq 90^\circ$



On the same diagram, construct and show the locus of point Q, by leaving unshaded, the region in which point Q lies.

7. The figure below is drawn to scale. It represents a field in the shape of an equilateral triangle of side 80m

The owner wants to plant some flowers in the field. The flowers must be at most, 60m from A and nearer to B than to C. If no flower is to be more than 40m from BC, show by shading, the exact region where the flowers may be planted.

8. In this question use a ruler and a pair of compasses only  
In the figure below, AB and PQ are straight lines

(a) Use the figure to:

- (i) Find a point R on AB such that R is equidistant from P and Q
- (ii) Complete a polygon PQRST with AB as its line of symmetry and hence measure the distance of R from TS.

(b) Shade the region within the polygon in which a variable point X must lie given that X satisfies the following conditions

1. X is nearer to PT than to PQ
2. RX is not more than 4.5 cm
3.  $\angle PXT > 90^\circ$

9. Four points B, C, Q and D lie on same plane. Point B is 42 km due south – west of town Q. Point C is 50 km on a bearing of  $560^\circ$  from Q. Point D is equidistant from B, Q and C.

(a) Using the scale: 1 cm represents 10 km, construct a diagram showing the position of B, C, Q and D

(b) Determine the

- (i) Distance between B and C
- (ii) Bearing of D from B

10. The diagram below represents a field PQR

- (a) Draw the locus of point equidistant from sides PQ and PR
  - (b) Draw the locus of points equidistant from points P and R
  - (c) A coin is lost within a region which is near to point P than R and closer to side PR than to side PQ. Shade the region where the coin can be located.
12. In the figure below, a line XY and three point A,B and C are as given. On the figure construct
- (a) The perpendicular bisector of AB
  - (b) A point P on the line XY such that  $\angle APB = \angle ACB$

**TOPIC 4:**  
**TRIGONOMETRY**

1. (a) Complete the table for the function  $y = 2 \sin x$

x	0°	10°	20°	30°	40°	50°	60°	70°	80°	90°	100°	110°	120°
Sin 3x	0	0.5000							-0.8660				
y	0	1.00							-1.73				

- (b) (i) Using the values in the completed table, draw the graph of  $y = 2 \sin 3x$  for  $0^\circ \leq x \leq 120^\circ$  on the grid provided
- (ii) Hence solve the equation  $2 \sin 3x = -1.5$

2. Complete the table below by filling in the blank spaces

X°	0°	30°	60°	90°	120°	150°	180°	210°	240°	270°	300°	330°	360°
Cos x°	1.00		0.50			-0.87		-0.87					
2 cos ½ x°	2.00	1.93				0.52			-1.00				-2.00

Using the scale 1 cm to represent  $30^\circ$  on the horizontal axis and 4 cm to represent 1 unit on the vertical axis draw, on the grid provided, the graphs of  $y = \cos x^\circ$  and  $y = 2 \cos \frac{1}{2} x^\circ$  on the same axis.

- (a) Find the period and the amplitude of  $y = 2 \cos \frac{1}{2} x^\circ$

- (b) Describe the transformation that maps the graph of  $y = \cos x^0$  on the graph of  $y = 2 \cos \frac{1}{2} x^0$

1. (a) Complete the table below for the value of  $y = 2 \sin x + \cos x$ .

x	$0^0$	$30^0$	$45^0$	$60^0$	$90^0$	$120^0$	$135^0$	$150^0$	$180^0$	$225^0$	$270^0$	$315^0$	$360^0$
2 sin x	0		1.4	1.7	2	1.7	1.4	1	0		-2	-1.4	0
Cos x	1		0.7	0.5	0	-0.5	-0.7	-0.9	-1		0	0.7	1
y	1		2.1	2.2	2	1.2	0.7	0.1	-1		-2	-0.7	1

- (b) Using the grid provided draw the graph of  $y=2\sin x + \cos x$  for  $0^0$ . Take

1cm represent  $30^0$  on the x- axis and 2 cm to represent 1 unit on the axis.

- (c) Use the graph to find the range of x that satisfy the inequalities

$$2 \sin x \cos x > 0.5$$

4. (a) Complete the table below, giving your values correct to 2 decimal places.

x	0	10	20	30	40	50	60	70
Tan x	0							
2 x + 300	30	50	70	90	110	130	150	170
Sin ( 2x + 30 <sup>0</sup> )	0.50			1				

- b) On the grid provided, draw the graphs of  $y = \tan x$  and  $y = \sin ( 2x + 30^0)$

for  $0^0 \leq x 70^0$

Take scale: 2 cm for 100 on the x- axis

4 cm for unit on the y- axis

Use your graph to solve the equation  $\tan x - \sin (2x + 30^\circ) = 0$ .

5. (a) Complete the table below, giving your values correct to 2 decimal places

$X^\circ$	0	30	60	90	120	150	180
$2 \sin x^\circ$	0	1		2		1	
$1 - \cos x^\circ$			0.5	1			

- (b) On the grid provided, using the same scale and axes, draw the graphs of

$$y = \sin x^\circ \text{ and } y = 1 - \cos x^\circ \leq x \leq 180^\circ$$

Take the scale: 2 cm for  $30^\circ$  on the x- axis

2 cm for 1 unit on the y- axis

- (c) Use the graph in (b) above to

- (i) Solve equation

$$2 \sin x^\circ + \cos x^\circ = 1$$

- (ii) Determine the range of values  $x$  for which  $2 \sin x^\circ > 1 - \cos x^\circ$

6. (a) Given that  $y = 8 \sin 2x - 6 \cos x$ , complete the table below for the missing values of  $y$ , correct to 1 decimal place.

X	$0^\circ$	$15^\circ$	$30^\circ$	$45^\circ$	$60^\circ$	$75^\circ$	$90^\circ$	$105^\circ$	$120^\circ$
$Y = 8 \sin 2x - 6 \cos x$	-6	-1.8		3.8	3.9	2.4	0		-3.9

- (b) On the grid provided, below, draw the graph of  $y = 8 \sin 2x - 6 \cos$  for  $0^\circ \leq x \leq 120^\circ$

Take the scale 2 cm for  $15^\circ$  on the x- axis

2 cm for 2 units on the y – axis

- (c) Use the graph to estimate
- The maximum value of y
  - The value of x for which  $4 \sin 2x - 3 \cos x = 1$

7. Solve the equation  $4 \sin (x + 30^\circ) = 2$  for  $0 \leq x \leq 360^\circ$
8. Find all the positive angles not greater than  $180^\circ$  which satisfy the equation  
 $\sin^2 x - 2 \tan x = 0$   
 $\cos x$
9. Solve for values of x in the range  $0^\circ \leq x \leq 360^\circ$  if  $3 \cos^2 x - 7 \cos x = 6$
10. Simplify  $\frac{9 - y^2}{y}$  where  $y = 3 \cos \theta$
11. Find all the values of  $\theta$  between  $0^\circ$  and  $360^\circ$  satisfying the equation  $5 \sin \theta = -4$

12. Given that  $\sin(90 - x) = 0.8$ . Where  $x$  is an acute angle, find without using mathematical tables the value of  $\tan x^{\circ}$

13. Complete the table given below for the functions

$$y = -3 \cos 2x^{\circ} \text{ and } y = 2 \sin \left(\frac{3x}{2} + 30\right) \text{ for } 0 \leq x \leq 180^{\circ}$$

$x^{\circ}$	$0^{\circ}$	$20^{\circ}$	$40^{\circ}$	$60^{\circ}$	$80^{\circ}$	$100^{\circ}$	$120^{\circ}$	$140^{\circ}$	$160^{\circ}$	$180^{\circ}$
$-3\cos 2x^{\circ}$	-3.00	-2.30	-0.52	1.50	2.82	2.82	1.50	-0.52	-2.30	-3.00
$2 \sin (3 x^{\circ} + 30^{\circ})$	1.00	1.73	2.00	1.73	1.00	0.00	-1.00	-1.73	-2.00	-1.73

Using the graph paper draw the graphs of  $y = -3 \cos 2x^{\circ}$  and  $y = 2 \sin \left(\frac{3x}{2} + 30^{\circ}\right)$

(a) On the same axis. Take 2 cm to represent  $20^{\circ}$  on the x- axis and 2 cm to represent one unit on the y – axis

(b) From your graphs. Find the roots of  $3 \cos 2 x^{\circ} + 2 \sin \left(\frac{3x}{2} + 30^{\circ}\right) = 0$

14. Solve the values of  $x$  in the range  $0^{\circ} \leq x \leq 360^{\circ}$  if  $3 \cos^2 x - 7 \cos x = 6$

15. Complete the table below by filling in the blank spaces

$x^{\circ}$	$0^{\circ}$	$30^{\circ}$	$60^{\circ}$	$90$	$1^{\circ}$	$150^{\circ}$	$180$	$210$	$240$	$270$	$300$	$330$	$360$
$\cos x^{\circ}$	1.00		0.50			-0.87		-0.87					
$2 \cos \frac{1}{2} x^{\circ}$	2.00	1.93					0.5						



Using the scale 1 cm to represent  $30^\circ$  on the horizontal axis and 4 cm to represent 1 unit on the vertical axis draw on the grid provided, the graphs of  $y = \cos x^\circ$  and  $y = 2 \cos \frac{1}{2} x^\circ$  on the same axis

(a) Find the period and the amplitude of  $y = 2 \cos \frac{1}{2} x^\circ$

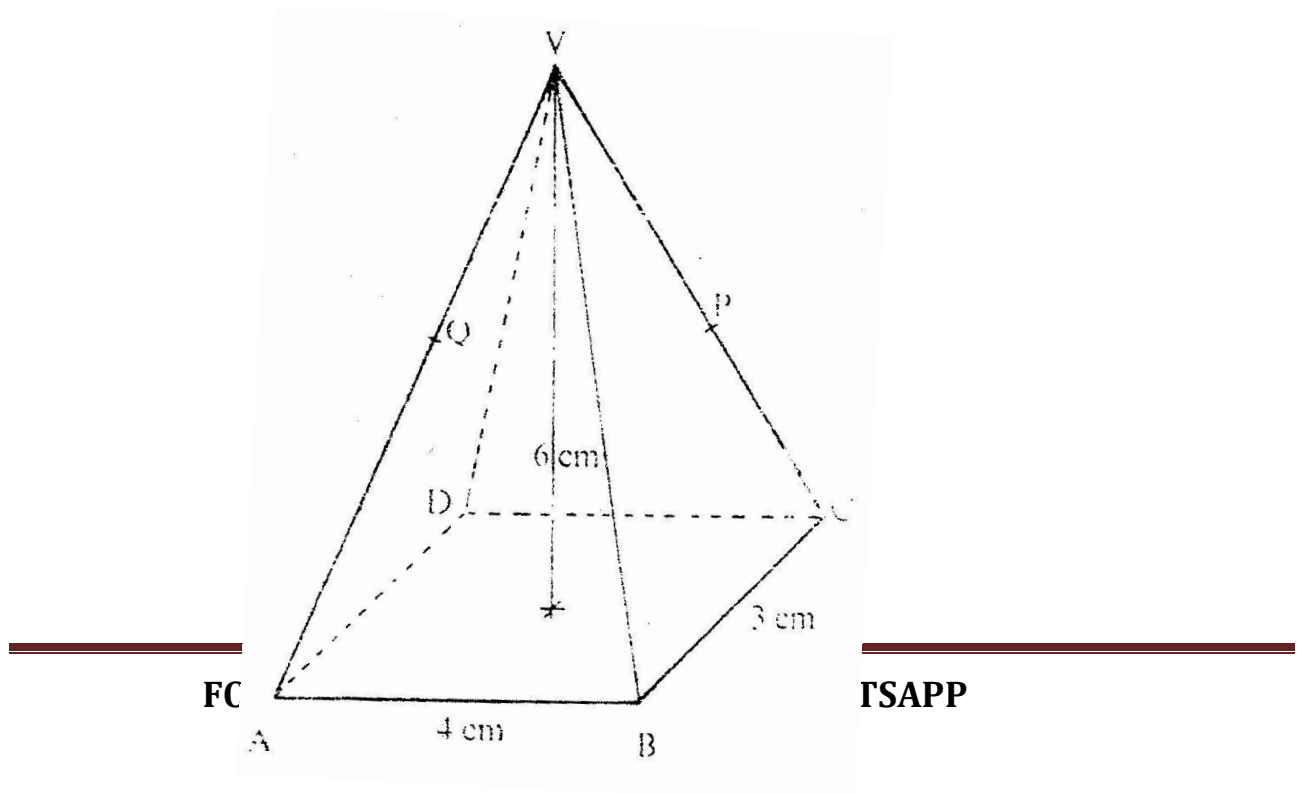
Ans. Period =  $720^\circ$ . Amplitude = 2

(b) Describe the transformation that maps the graph of  $y = \cos x^\circ$  on the graph of  $y = 2 \cos \frac{1}{2} x^\circ$

## TOPIC 5

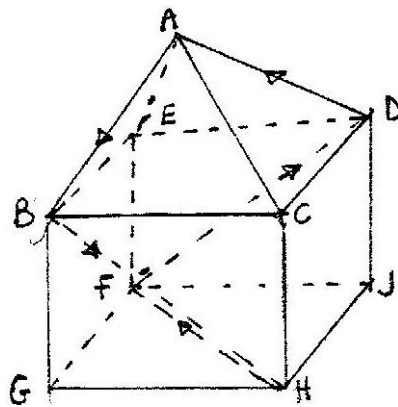
### THREE DIMENSIONAL GEOMETRY

- The diagram below shows a right pyramid VABCD with V as the vertex. The base of the pyramid is rectangle ABCD, WITH  $ab = 4$  cm and  $BC = 3$  cm. The height of the pyramid is 6 cm.



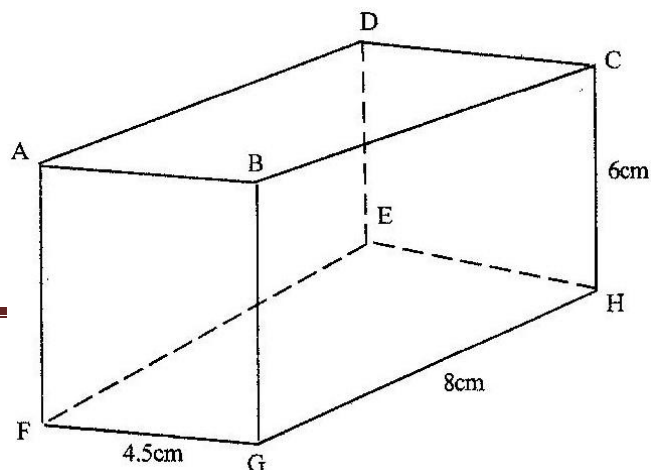
- (a) Calculate the
- Length of the projection of VA on the base
  - Angle between the face VAB and the base
- (b) P is the mid-point of VC and Q is the mid-point of VD.  
Find the angle between the planes VAB and the plane ABPQ

2. The figure below represents a square based solid with a path marked on it.



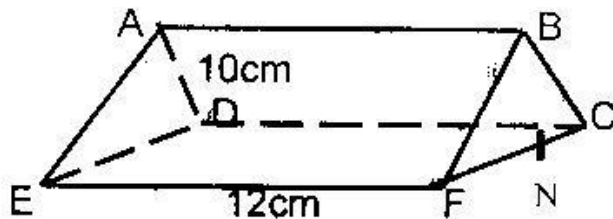
Sketch and label the net of the solid.

3. The diagram below represents a cuboid ABCDEFGH in which  $FG = 4.5$  cm,  $GH = 8$  cm and  $HC = 6$  cm



Calculate:

- (a) The length of FC
  - (b) (i) The size of the angle between the lines FC and FH  
(ii) The size of the angle between the lines AB and FH
  - (c) The size of the angle between the planes ABHE and the plane FGHE
4. The base of a right pyramid is a square ABCD of side  $2a$  cm. The slant edges VA, VB, VC and VD are each of length  $3a$  cm.
- (a) Sketch and label the pyramid
  - (b) Find the angle between a slanting edge and the base
5. The triangular prism shown below has the sides  $AB = DC = EF = 12$  cm. the ends are equilateral triangles of sides 10cm. The point N is the mid point of FC.



Find the length of:

(a) (i) BN

(ii) EN

(b) Find the angle between the line EB and the plane CDEF

**TOPIC 6:**

**LATITUDES AND LONGITUDES**

1. An aeroplane flies from point A ( $1^{\circ} 15' S, 37^{\circ} E$ ) to a point B directly North of A. the arc AB subtends an angle of  $45^{\circ}$  at the center of the earth. From B, aeroplanes flies due west two a point C on longitude  $23^{\circ} W$ .)  
(Take the value of  $\pi^{22/7}$  as and radius of the earth as 6370km)
  - (a)
    - (i) Find the latitude of B
    - (ii) Find the distance traveled by the aeroplane between B and C
  - (b) The aeroplane left at 1.00 a.m local time. When the aeroplane was leaving B, hat was the local time at C?
  
2. The position of two towns X and Y are given to the nearest degree as X ( $45^{\circ} N, 10^{\circ} W$ ) and Y ( $45^{\circ} N, 70^{\circ} W$ )  
Find
  - (a) The distance between the two towns in
    - (i) Kilometers (take the radius of the earth as 6371)
    - (ii) Nautical miles (take 1 nautical mile to be 1.85 km)
  - (b) The local time at X when the local time at Y is 2.00 pm.
  
3. A plane leaves an airport A ( $38.5^{\circ} N, 37.05^{\circ} W$ ) and flies dues North to a point B on latitude  $52^{\circ} N$ .
  - (a) Find the distance covered by the plane
  - (b) The plane then flies due east to a point C, 2400 km from B. Determine the position of C

Take the value  $\pi$  of as  $\frac{22}{7}$  and radius of the earth as 6370 km

4. A plane flying at 200 knots left an airport A ( $30^{\circ}\text{S}$ ,  $31^{\circ}\text{E}$ ) and flew due North to an airport B ( $30^{\circ}\text{N}$ ,  $31^{\circ}\text{E}$ )
- (a) Calculate the distance covered by the plane, in nautical miles
- (b) After a 15 minutes stop over at B, the plane flew west to an airport C ( $30^{\circ}\text{N}$ ,  $13^{\circ}\text{E}$ ) at the same speed.
- Calculate the total time to complete the journey from airport C, though airport B.
5. Two towns A and B lie on the same latitude in the northern hemisphere.
- When its 8 am at A, the time at B is 11.00 am.
- a) Given that the longitude of A is  $15^{\circ}\text{E}$  find the longitude of B.
- b) A plane leaves A for B and takes  $3\frac{1}{2}$  hours to arrive at B traveling along a parallel of latitude at 850 km/h. Find:
- (i) The radius of the circle of latitude on which towns A and B lie.
- (ii) The latitude of the two towns (take radius of the earth to be 6371 km)
6. Two places A and B are on the same circle of latitude north of the equator. The longitude of A is  $118^{\circ}\text{W}$  and the longitude of B is  $133^{\circ}\text{E}$ . The shorter distance between A and B measured along the circle of latitude is 5422 nautical miles.
- Find, to the nearest degree, the latitude on which A and B lie
7. (a) A plane flies by the short estimate route from P ( $10^{\circ}\text{S}$ ,  $60^{\circ}\text{W}$ ) to Q ( $70^{\circ}\text{N}$ ,

- 120° E) Find the distance flown in km and the time taken if the average speed is 800 km/h.
- (b) Calculate the distance in km between two towns on latitude 50°S with longitudes 20° W and 120° W. (take the radius of the earth to be 6370 km)
8. Calculate the distance between M (30°N, 36°E) and N (30° N, 144° W) in nautical miles.
- (i) Over the North Pole
- (ii) Along the parallel of latitude 30° N
9. (a) A ship sailed due south along a meridian from 12° N to 10°30' S. Taking the earth to be a sphere with a circumference of  $4 \times 10^4$  km, calculate in km the distance traveled by the ship.
- (b) If a ship sails due west from San Francisco (37° 47'N, 122° 26'W) for distance of 1320 km. Calculate the longitude of its new position (take the radius of the earth to be 6370 km and  $\pi = 22/7$ ).

## TOPIC 7

### LINEAR PROGRAMMING

1. A school has to take 384 people for a tour. There are two types of buses available, type X and type Y. Type X can carry 64 passengers and type Y can carry 48 passengers. They have to use at least 7 buses.
  - (a) Form all the linear inequalities which will represent the above information.
  - (b) On the grid [provide, draw the inequalities and shade the unwanted region.
  - (c) The charges for hiring the buses are  
Type X: Kshs 25,000  
Type Y Kshs 20,000  
Use your graph to determine the number of buses of each type that should be hired to minimize the cost.
  
2. An institute offers two types of courses technical and business courses. The institute has a capacity of 500 students. There must be more business students than technical students but at least 200 students must take technical courses. Let  $x$  represent the number of technical students and  $y$  the number of business students.
  - (a) Write down three inequalities that describe the given conditions
  - (b) On the grid provided, draw the three inequalities
  - (c) If the institute makes a profit of Kshs 2, 500 to train one technical students and Kshs 1,000 to train one business student, determine



- (i) The number of students that must be enrolled in each course to maximize the profit
- (ii) The maximum profit.
3. A draper is required to supply two types of shirts A and type B. The total number of shirts must not be more than 400. He has to supply more type A than of type B however the number of types A shirts must be more than 300 and the number of type B shirts not be less than 80.
- Let  $x$  be the number of type A shirts and  $y$  be the number of types B shirts.
- (a) Write down in terms of  $x$  and  $y$  all the linear inequalities representing the information above.
- (b) On the grid provided, draw the inequalities and shade the unwanted regions
- (c) The profits were as follows
- Type A: Kshs 600 per shirt
- Type B: Kshs 400 per shirt
- (i) Use the graph to determine the number of shirts of each type that should be made to maximize the profit.
- (ii) Calculate the maximum possible profit.
4. A diet expert makes up a food production for sale by mixing two ingredients N and S. One kilogram of N contains 25 units of protein and 30 units of vitamins. One kilogram of S contains 50 units of protein and 45 units of vitamins. The food is sold in small bags each containing at least 175 units of protein and at least 180 units of vitamins. The mass of the food product in each bag must not exceed 6kg.

If one bag of the mixture contains  $x$  kg of N and  $y$  kg of S

(a) Write down all the inequalities, in terms of  $x$  and representing the information above ( 2 marks)

(b) On the grid provided draw the inequalities by shading the unwanted regions ( 2 marks)

(c) If one kilogram of N costs Kshs 20 and one kilogram of S costs Kshs 50, use the graph to determine the lowest cost of one bag of the mixture.

5. Mwanjoki flying company operates a flying service. It has two types of aeroplanes. The smaller one uses 180 litres of fuel per hour while the bigger one uses 300 litres per hour.

The fuel available per week is 18,000 litres. The company is allowed 80 flying hours per week.

(a) Write down all the inequalities representing the above information

(b) On the grid provided on page 21, draw all the inequalities in (a) above by shading the unwanted regions

(c) The profits on the smaller aeroplane is Kshs 4000 per hour while that on the bigger one is Kshs. 6000 per hour. Use your graph to determine the maximum profit that the company made per week.

6. A company is considering installing two types of machines. A and B. The information about each type of machine is given in the table below.

Machine	Number of operators	Floor space	Daily profit
A	2	5m <sup>2</sup>	Kshs 1,500
B	5	8m <sup>2</sup>	Kshs 2,500

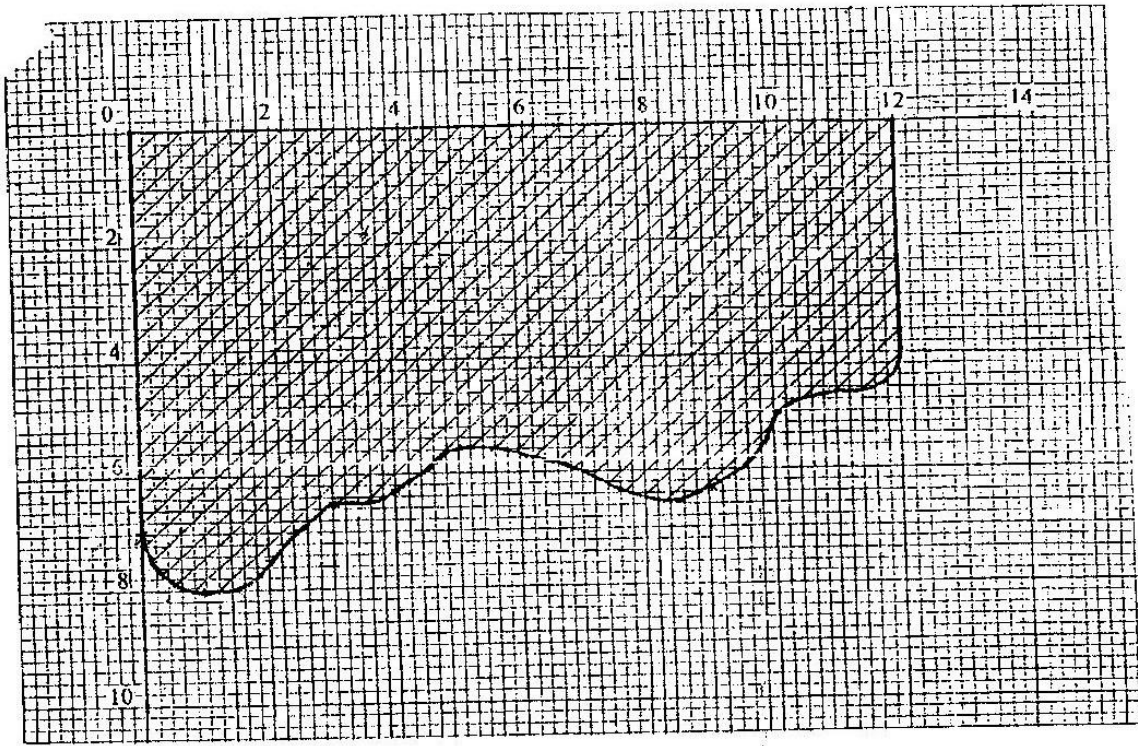
The company decided to install  $x$  machines of types A and  $y$  machines of type B

- (a) Write down the inequalities that express the following conditions
- i. The number of operators available is 40
  - ii. The floor space available is 80m<sup>2</sup>
  - iii. The company is to install not less than 3 type of A machine
  - iv. The number of type B machines must be more than one third the number of type A machines
- (b) On the grid provided, draw the inequalities in part (a) above and shade the unwanted region.
- (c) Draw a search line and use it to determine the number of machines of each type that should be installed to maximize the daily profit.

**TOPIC 8:**

**CALCULUS**

1. The shaded region below represents a forest. The region has been drawn to scale where 1 cm represents 5 km. Use the mid – ordinate rule with six strips to estimate the area of forest in hectares. (4 marks)



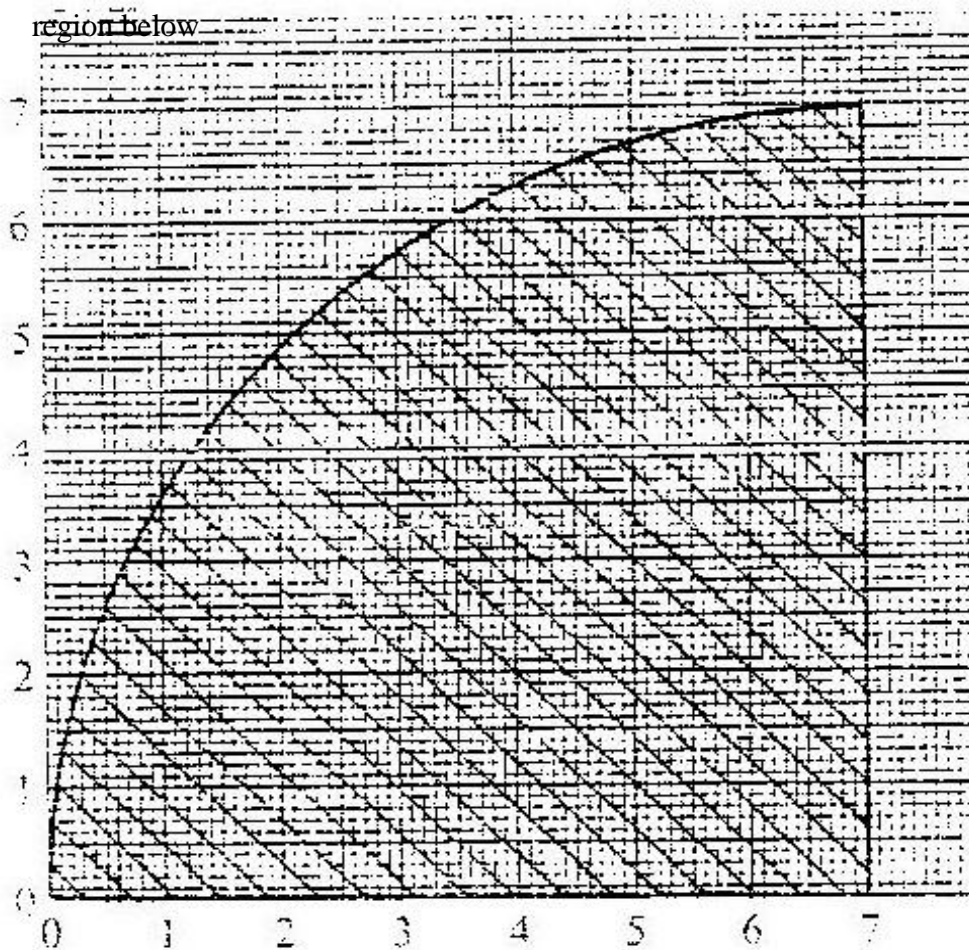
2. Find the area bounded by the curve  $y=2x^3 - 5$ , the x-axis and the lines  $x=2$  and  $x=4$ .
3. Complete the table below for the function  $y=3x^2 - 8x + 10$  (1 mk)

x	0	2	4	6	8	10
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y	10	6		70		230
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Using the values in the table and the trapezoidal rule, estimate the area bounded by the curve  $y = 3x^2 - 8x + 10$  and the lines  $y=0$ ,  $x=0$  and  $x=10$ .

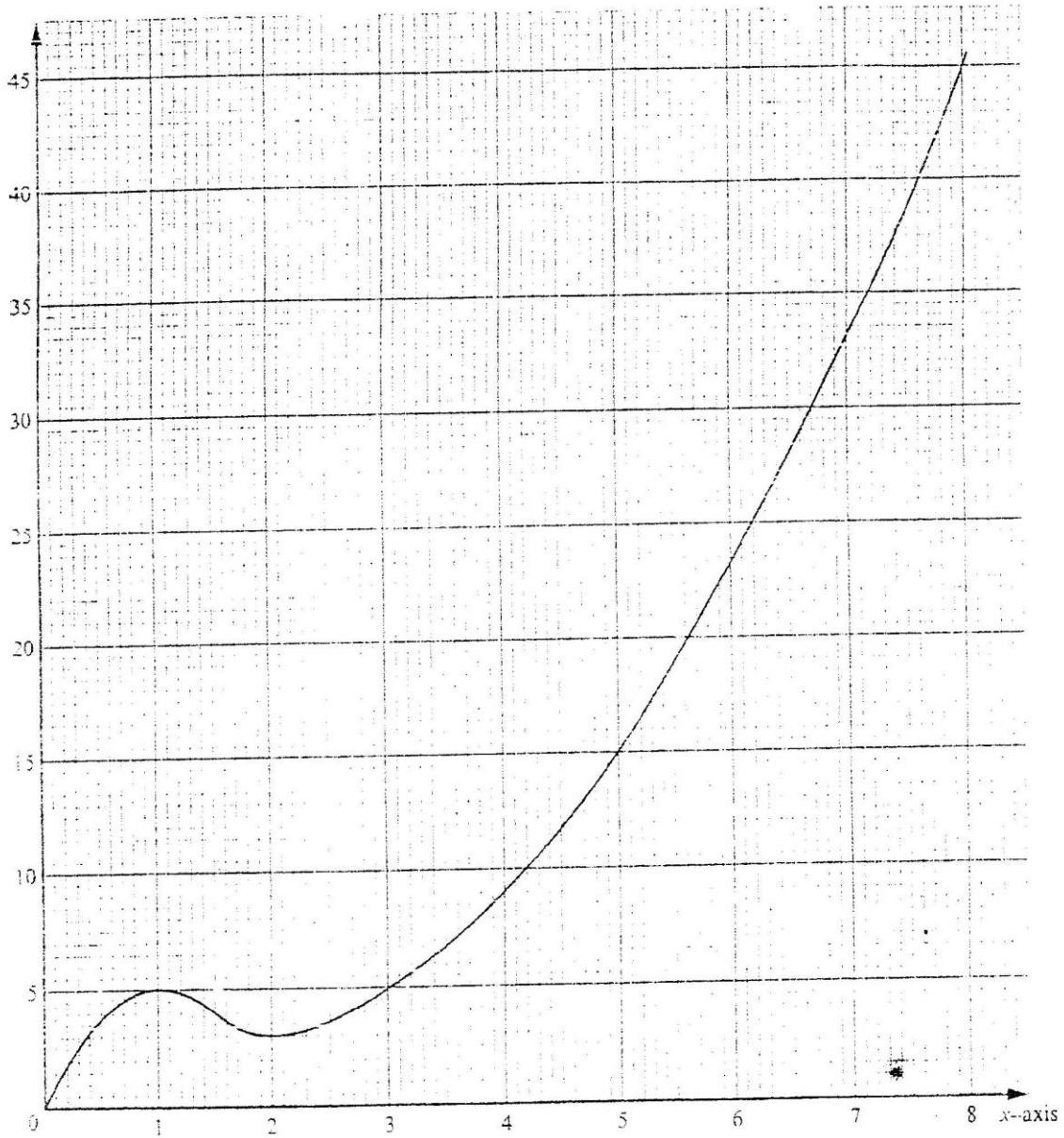
4. Use the trapezoidal rule with intervals of 1 cm to estimate the area of the shaded region below



5. (a) Find the value of  $x$  at which the curve  $y = x^2 - 2x^2 - 3$  crosses the  $x$ -axis
- (b) Find  $\int (x^2 - 2x - 3) dx$
- (c) Find the area bounded by the curve  $y = x^2 - 2x - 3$ , the axis and the lines  $x = 2$  and  $x = 4$ .



6. The graph below consists of a non-quadratic part ( $0 \leq x \leq 2$ ) and a quadratic part ( $2 \leq x \leq 8$ ). The quadratic part is  $y = x^2 - 3x + 5$ ,  $2 \leq x \leq 8$



(a) Complete the table below

x	2	3	4	5	6	7	8
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y	3						
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(1mk)

(b) Use the trapezoidal rule with six strips to estimate the area enclosed by the curve, x = axis and the line x = 2 and x = 8 (3mks)

(c) Find the exact area of the region given in (b) (3mks)

(d) If the trapezoidal rule is used to estimate the area under the curve between x = 0 and x = 2, state whether it would give an under- estimate or an over- estimate. Give a reason for your answer.

7. Find the equation of the gradient to the curve  $Y = (x^2 + 1)(x - 2)$  when  $x = 2$

8. The distance from a fixed point of a particular in motion at any time t seconds is given by

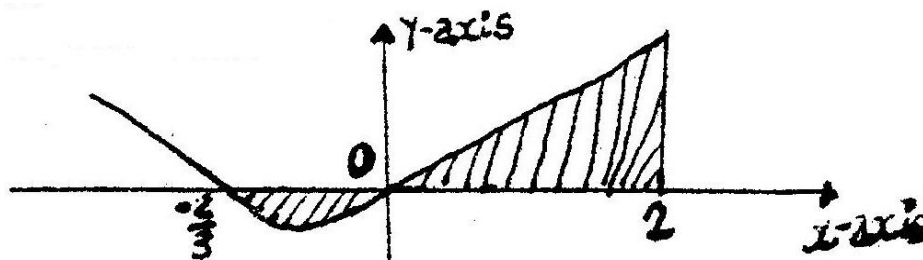
$$S = t^3 - 5t^2 + 2t + 5$$

$$2t^2$$

Find its:

- (a) Acceleration after 1 second
  - (b) Velocity when acceleration is Zero
9. The curve of the equation  $y = 2x + 3x^2$ , has x = -2/3 and x = 0 and x intercepts.

The area bounded by the axis x = -2/3 and x = 2 is shown by the sketch below.





Find:

(a)  $(2x + 3x^2) dx$

(b) The area bounded by the curve  $x - \text{axis}$ ,  $x = -\frac{2}{3}$  and  $x = 2$

10. A particle is projected from the origin. Its speed was recorded as shown in the table below

Time (sec)	0	5	10	15	20	25	39	35
Speed (m/s)	0	2.1	5.3	5.1	6.8	6.7	4.7	2.6

Use the trapezoidal rule to estimate the distance covered by the particle within the 35 seconds.

11. (a) The gradient function of a curve is given by  $\frac{dy}{dx} = 2x^2 - 5$

Find the equation of the curve, given that  $y = 3$ , when  $x = 2$

- (b) The velocity,  $v$  m/s of a moving particle after seconds is given:

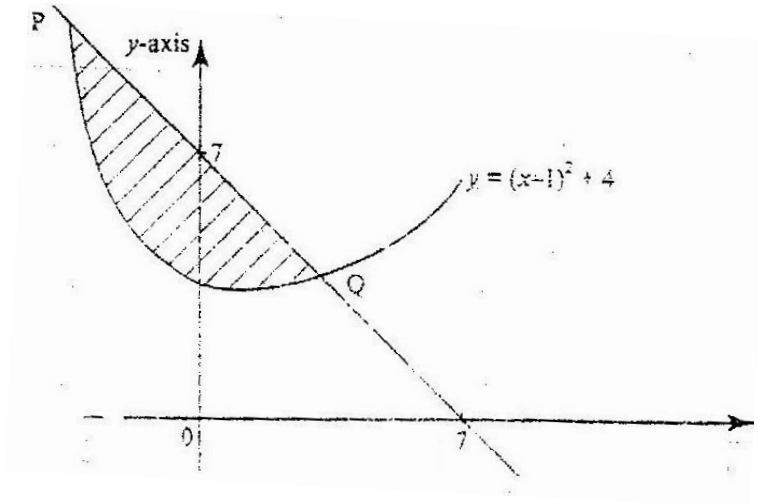
$v = 2t^3 + t^2 - 1$ . Find the distance covered by the particle in the interval  $1 \leq t \leq 3$

12. Given the curve  $y = 2x^3 + \frac{1}{2}x^2 - 4x + 1$ . Find the:

- i) Gradient of curve at  $\{1, -\frac{1}{2}\}$

ii) Equation of the tangent to the curve at  $\{1, -\frac{1}{2}\}$

13. The diagram below shows a straight line intersecting the curve  $y = (x-1)^2 + 4$  at the points P and Q. The line also cuts x-axis at (7, 0) and y axis at (0, 7)



- Find the equation of the straight line in the form  $y = mx + c$ .
  - Find the coordinates of p and Q.
  - Calculate the area of the shaded region.
14. The acceleration,  $a \text{ ms}^{-2}$ , of a particle is given by  $a = 25 - 9t^2$ , where t in seconds after the particle passes fixed point O.
- If the particle passes O, with velocity of  $4 \text{ ms}^{-1}$ , find
- An expression of velocity V, in terms of t
  - The velocity of the particle when  $t = 2$  seconds
15. A curve is represented by the function  $y = \frac{1}{3}x^3 + x^2 - 3x + 2$
- Find:  $\frac{dy}{dx}$
  - Determine the values of y at the turning points of the curve

$$y = \frac{1}{3}x^3 + x^2 - 3x + 2$$

(c) In the space provided below, sketch the curve of  $y = \frac{1}{3}x^3 + x^2 - 3x + 2$

16. A circle centre O, has the equation  $x^2 + y^2 = 4$ . The area of the circle in the first quadrant is divided into 5 vertical strips of width 0.4 cm

(a) Use the equation of the circle to complete the table below for values of  $y$  correct to 2 decimal places

X	0	0.4	0.8	1.2	1.6	2.0
Y	2.00			1.60		0

(b) Use the trapezium rule to estimate the area of the circle

17. A particle moves along straight line such that its displacement  $S$  metres from a given point is  $S = t^3 - 5t^2 + 4$  where  $t$  is time in seconds

Find

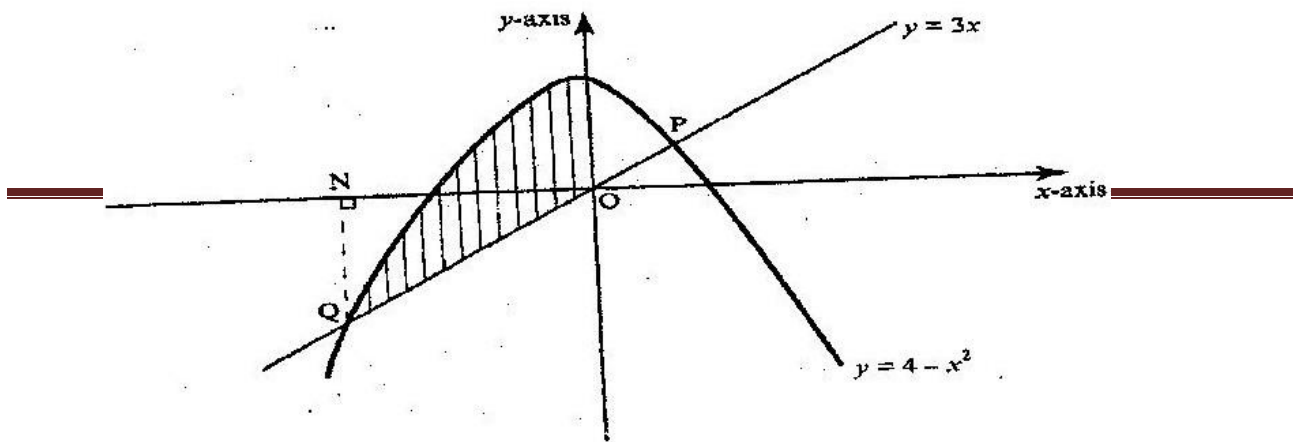
(a) The displacement of particle at  $t = 5$

(b) The velocity of the particle when  $t = 5$

(c) The values of  $t$  when the particle is momentarily at rest

(d) The acceleration of the particle when  $t = 2$

18. The diagram below shows a sketch of the line  $y = 3x$  and the curve  $y = 4 - x^2$  intersecting at points P and Q.



- (a) Find the coordinates of P and Q
- (b) Given that QN is perpendicular to the x- axis at N, calculate
- (i) The area bounded by the curve  $y = 4 - x^2$ , the x- axis and the line QN (2 marks)
- (ii) The area of the shaded region that lies below the x- axis
- (iii) The area of the region enclosed by the curve  $y = 4 - x^2$ , the line  $y - 3x$  and the y-axis.

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19. The gradient of the tangent to the curve  $y = ax^3 + bx$  at the point (1, 1) is -5  
Calculate the values of a and b.

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20. The diagram on the grid below represents as extract of a survey map showing two adjacent plots belonging to Kazungu and Ndoe.

The two dispute the common boundary with each claiming boundary along different smooth curves coordinates (x, y) and (x, y<sub>2</sub>) in the table below, represents points on the boundaries as claimed by Kazungu Ndoe respectively.

x	0	1	2	3	4	5	6	7	8	9
y <sub>1</sub>	0	4	5.7	6.9	8	9	9.8	10.6	11.3	12

$y_2$	0	0.2	0.6	1.3	2.4	3.7	5.3	7.3	9.5	12
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- (a) On the grid provided above draw and label the boundaries as claimed by Kazungu and Ndoe.
- (b) (i) Use the trapezium rule with 9 strips to estimate the area of the section of the land in dispute
- (ii) Express the area found in b (i) above, in hectares, given that 1 unit on each axis represents 20 metres
21. The gradient function of a curve is given by the expression  $2x + 1$ . If the curve passes through the point  $(-4, 6)$ ;
- (a) Find:
- (i) The equation of the curve
- (ii) The values of  $x$ , at which the curve cuts the  $x$ -axis
- (b) Determine the area enclosed by the curve and the  $x$ -axis
22. A particle moves in a straight line through a point P. Its velocity  $v$  m/s is given by  $v = 2 - t$ , where  $t$  is time in seconds, after passing P. The distance  $s$  of the particle from P when  $t = 2$  is 5 metres. Find the expression for  $s$  in terms of  $t$ .
23. Find the area bounded by the curve  $y = 2x - 5$  the  $x$ -axis and the lines  $x = 2$  and  $x = 4$ .
23. Complete the table below for the function

$$Y = 3x^2 - 8x + 10$$

X	0	2	4	6	8	10
Y	10	6	-	70	-	230

Using the values in the table and the trapezoidal rule, estimate the area bounded

by the curve  $y = 3x^2 - 8x + 10$  and the lines  $y = 0$ ,  $x = 0$  and  $x = 10$

24. (a) Find the values of  $x$  which the curve  $y = x^2 - 2x - 3$  crosses the axis
- (b) Find  $(x^2 - 2x - 3) dx$
- (c) Find the area bounded by the curve  $Y = x^2 - 2x - 3$ . The  $x$  - axis and the lines  $x = 2$  and  $x = 4$
25. Find the equation of the tangent to the curve  $y = (x + 1)(x - 2)$  when  $x = 2$
26. The distance from a fixed point of a particle in motion at any time  $t$  seconds is given by  $s = t - \frac{5}{2}t^2 + 2t + s$  metres
- Find its
- (a) Acceleration after  $t$  seconds
- (b) Velocity when acceleration is zero
27. The curve of the equation  $y = 2x + 3x^2$ , has  $x = -\frac{2}{3}$  and  $x = 0$ , as  $x$  intercepts. The area bounded by the curve,  $x$  - axis,  $x = -\frac{2}{3}$  and  $x = 2$  is shown by the sketch below.

- (a) Find  $\int(2x + 3x^2) dx$
- (b) The area bounded by the curve, x axis  $x = -\frac{2}{3}$  and  $x = 2$
28. A curve is given by the equation  $y = 5x^3 - 7x^2 + 3x + 2$
- Find the
- (a) Gradient of the curve at  $x = 1$
- (b) Equation of the tangent to the curve at the point  $(1, 3)$
29. The displacement  $x$  metres of a particle after  $t$  seconds is given by  $x = t^2 - 2t + 6$ ,  
 $t > 0$
- (a) Calculate the velocity of the particle in m/s when  $t = 2s$
- (b) When the velocity of the particle is zero,  
Calculate its
- (i) Displacement
- (ii) Acceleration
30. The displacement  $s$  metres of a particle moving along a straight line after  $t$  seconds is given by  $s = 3t + \frac{3}{2}t^2 - 2t^3$



- (a) Find its initial acceleration
- (b) Calculate
- (i) The time when the particle was momentarily at rest.
- (ii) Its displacement by the time it comes to rest momentarily when  
 $t = 1$  second,  $s = 1 \frac{1}{2}$  metres when  $t = \frac{1}{2}$  seconds
- (c) Calculate the maximum speed attained

