## MATHEMATICS PAPER 1

FORM THREE
END OF TERM 2

## MARKING SCHEME.

1. Evaluate:
(3mks)

$$
\begin{aligned}
& =1 / 2\left(3 / 5+1 / 4\left(\frac{40}{21}\right) \text { of } 1 \frac{1}{2} \div 5\right) \\
& =1 / 2\left(3 / 5+1 / 4\left(\frac{7}{3}-\frac{3}{7}\right) \text { of } 1 \frac{1}{2} \div 5\right) \\
& \left.=1 / 2\left(3 / 5+1 / 4\left(\frac{40}{21} \times \frac{30}{7}\right) \div 5\right) \div 5\right) \\
& =1 / 2\left(3 / 5+\left(\frac{1}{4} \times \frac{20}{7} \times \frac{1}{5}\right)\right) \\
& =1 / 2\left(3 / 5+\frac{1}{7}\right) \\
& =1 / 2 \times 26 / 35 \\
& =13 / 35
\end{aligned}
$$

2. A triangle has vertices $A(2,5), B(1,-2)$ and $C(-5,1)$. Determine;
a) The equation of line BC .

$$
\begin{aligned}
& B(1,-2) \quad C(-5,1) \\
& \text { Gradient }=\frac{1--2}{-5-1} \\
& =\frac{-3}{6}=\frac{-1}{2} \\
& B(1,-2)(x-y) \\
& =\frac{y+2}{x-1}=\frac{-1}{2} \\
& y+2=-1 / 2 x+1 / 2 \\
& y=-1 / 2 x+-3 / 2
\end{aligned}
$$

b) The equation of perpendicular line from A to BC .

$$
\begin{aligned}
& A(2,5) \quad G=2 \\
& =\frac{y-5}{x-2}=2 \\
& y-5=2 x-4 \\
& y=2 x+1
\end{aligned}
$$

3. The shaded region in the figure below shows an area swept out on a flat windscreen by a wiper. Calculate the area of the region. Take $\pi=3.142$.

$3.142 \times \frac{120}{360} \times 20^{2}-3.142 \times \frac{120}{360} \times 16^{2}$
418.93-268.12
$=150.81 \mathrm{~cm}^{2}$
4. A piece of metal has a volume of $20 \mathrm{~cm}^{3}$ and a mass of 300 g . Calculate the density of the metal in $\mathrm{kg} / \mathrm{m}^{3}$.

D $=\frac{\text { mass }}{\text { volume }}$
$=\frac{20}{300} \times 1000$
$=66.67 \mathrm{~kg} / \mathrm{m}^{3}$
5. List the integral values of x which satisfy the inequalities below. (3mks)
$2 \boldsymbol{x}+\mathbf{2 1}>15-2 \boldsymbol{x} \geq \boldsymbol{x}+\mathbf{6}$
$\begin{array}{ll}2 x+21>15-2 x & 15-2 x \geq x+6 \\ 4 x>-6 & 9 \geq 3 x \\ x>-1.5 & 3 \geq x\end{array}$
$-1.5<x \leq 3$

## Integral values -1, 0, 1, 2, 3

6. Janet is a saleslady earning a basic salary of Kshs. 20,000 per month and a commission of 8\% for the sales in excess of Kshs. 100,000. If in January 2010 she earned a total of Kshs. 48,000 in salaries and commissions. Determine the amount of sales. She made in that month.
$48,000-20,000=28,000 /=$
$28000=\frac{8}{100} \times x$
$x=350,000+100,000$
= 450,000/=
7. The interior angle of a regular polygon is $108^{\circ}$ larger than the exterior angle.

Find the number of sides of the polygon.
(3mks)
$x+(x+108)=180$
$2 x+108=180$
$x=36$
Interior $=36+108 \quad$ Exterior $=36^{\circ}$

$$
=144
$$

No. of sides $=\frac{360}{36}$
$=10$ sides.
8. Given that $\operatorname{Cos} A=\frac{5}{13}$ and angle $A$ is acute. Find the value of $2 \tan A+3 \sin A$ without calculators.
(3mks)
12


$$
\begin{aligned}
& 2\left(\frac{12}{5}\right)+3\left(\frac{12}{13}\right) \\
& \frac{24}{5}+\frac{36}{13} \\
& =\frac{492}{65}
\end{aligned}
$$

9. Without using a calculator evaluate:

$$
\begin{array}{lc}
\frac{-9+(-7) \times(-8)-(-5)}{-2+(-6) \div 3 \times 6} & \\
-9+56+5 & -2+(-2) \times 6 \\
52 & -14 \\
=\frac{52}{-14}=\frac{-26}{7}= & \\
=-3 \frac{5}{7} &
\end{array}
$$

10. Solve for x in the equation below.

$$
\begin{aligned}
& \left.\left(\frac{6 x-4}{3}\right)^{t^{2}}-\left(\frac{2 x-1}{2}\right)^{t}=\frac{6-5 x}{-6}\right)^{/ 6} \\
& 2(6 x-4)-3(2 x-1)=6-5 x \\
& 12 x-8-6 x+3=6-5 x \\
& 11 x=11 \\
& x=1
\end{aligned}
$$

11. Calculate the area of the trapezium show below.

$A=1 / 2 x(15+20) \times 8 \sin 35$
$=80.3 \mathrm{~cm}^{2}$
12. Solve the simultaneous equation.
```
\(x^{2}+y^{2}=26\)
\(x+y=4\)
\(x=4-y\)
\((4-)^{2}+y^{2}=26\)
\(16-8 y+y^{2}+y^{2}=26\)
\(2 y^{2}-8 y-10=0\)
\(y^{2}-4 y-5=0\)
\(y=-1 \quad x=5\)
\(y=5 \quad x=-1\)
```

13. Express 72 and 125 as products of their prime factors.
(2mks)


2

$72=2 \times 2 \times 2 \times 3 \times 3$

$125=5 \times 5 \times 5$
14. A service vehicle left town $P$ for town $Q$ at 1000 hrs had a puncture after travelling for 4 hrs 20 mins . Fixing a new tyre took 33 minutes. The vehicle then travelled for 1 hr 20 mins to reach town Q. At what time did it arrive in 12 hour clock system.
1420 hrs
33
1453
120
1613

## 4:13p.m

15. A tourist visited Kenya with 2500 US dollars and changed the US dollars into Kenya shillings at a local bank in Kenya when the exchange rates at the time were as follows:

1 US dollar
1 Sterling Pound
a) How much did he get in Kenya shillings? $2500 \times 78.45$

Buying
shs. 78.45
shs. 120.25

Selling
shs. 78.55
shs. 120.45
b) While in Kenya he used shs. 80,000 and after his stay he converted the remaining amount into Sterling pounds. Calculate to 2 decimal places the Sterling pounds that he got.

| 196125 | $\underline{116123}$ |
| ---: | :--- |
| -80000 | 120.45 |

116125
$=964.09$ sterling pounds
16. Use logarithms tables to evaluate:

17. A motorist left Embu for Nairobi a distance of 240km at 8:00 a.m and travelled at average speed of $90 \mathrm{~km} / \mathrm{hr}$. Another motorist left Nairobi for Embu at 8:30a.m and travelled at $100 \mathrm{~km} / \mathrm{hr}$. Find;
a) The time they met.

$T=39 / 38$
8.30 am

62
9.32 am
b) How far they met from Nairobi.

```
\(\mathrm{T}=39 / \mathbf{3 8 h r}\)
\(\mathrm{S}=100 \mathrm{~km} / \mathrm{hr}\)
\(\mathrm{D}=102.63 \mathrm{~km}\)
```

c) The time of the day each motorist arrived at his destination.

| Embu $\longrightarrow$ Nairobi | Nairobi $\longrightarrow$ Embu |
| :--- | :--- |
| $\mathrm{D}=240 \mathrm{~km}$ | $\mathrm{D}=240 \mathrm{~km}$ |
| $\mathrm{~S}=90 \mathrm{~km} / \mathrm{hr}$ | $\mathrm{S}=100 \mathrm{~km} / \mathrm{hr}$ |
| $\mathrm{T}=2 \mathrm{hr} 40 \mathrm{mins}$ | $\mathrm{T}=2 \mathrm{~h} 24 \mathrm{mins}$ |
| 8.00 | 8.30 |
| $\mathbf{2 . 4 0}$ | $\underline{2.24}$ |
| $\mathbf{1 0 . 4 0 a m}$ | $\mathbf{1 0 . 5 4} \mathrm{am}$ |

18. A farmer has a rectangular farm which measures 100 m by 80 m . The farmer intends to fence the plot using post at intervals of 4 m apart leaving a gate of 4 m . Also he will use four strands of barbed wire. Each post cost shs. 125 and wire is sold at rolls of 60 m costing $1,500 /=$. Calculate;
a) The number of post he will use.

$$
\begin{aligned}
& P=180 \times 2 \\
& =\frac{360-4}{4}=89+1 \\
& =90 \text { posts }
\end{aligned}
$$

b) The total length of the barbed wire.
$(360-4) \times 4$
$=1424 \mathrm{~m}$
c) The total cost of fencing the farm if the cost of the gate is $8,000 /=$ and labour is shs. 1,500 .
(5mks)
$\left.(90 \times 125)+\frac{(1424}{60} \times 1500\right)+8000$
$11250+35600+8000+1500$
$=$ sh. 56350
d) The farmer wishes to subdivide further the farm into square plot. Find the maximum area of the plot.

| 10 | 100 | 80 |
| ---: | ---: | ---: |
| 2 | 10 | 8 |
|  | 5 | 4 |

$$
\text { G.C.D }=20
$$

```
AREA = 20 X 20
    = 400m}\mp@subsup{}{}{2
```

19. The parents of a certain mixed school decided to buy a school van worth Kshs. 900,000. Each student was to contribute the same amount of money. 50 students were transferred from the school; as a result each of the remaining students had to pay kshs. 600 more.
a) Find the original number of the students in the school.
```
900000-90-900000=600
(x-50)
900000x-9000000x + 45000000 = 600
    X (x -50)
45000000 = 600x}\mp@subsup{x}{}{2}-30000
    600x}\mp@subsup{x}{}{2}-30000x-45000000=
    X2}-50x-75000=
    50\pm\sqrt{}{2500-4 x 75000}
        2
    \frac{50+550}{2}}\frac{50-550}{2
X = 300 students
```

b) Find the percentage change in contributions per student.

```
Original \(=\underline{900000}\)
            300
            \(=3000\)
New \(=\underline{900000}\)
        250
        \(=3600\)
```

$=\underline{600 \times 100}$
3000
= 20\%
c) If the ratio of boys to girls in the school was 11:7, find the amount of money contributed by boys alone.

```
B G
11
    7
        = 11\times900000
        18
    = sh. 55,000
```

20. The figure below shows two circles of radii 8 cm and 6 cm with centres $\mathrm{O}_{1}$ and $\mathrm{O}_{2}$ respectively. The circles intersect at points A and B . The lines $\mathrm{O}_{1} \mathrm{O}_{2}$ and AB are perpendicular to each other. If the common chord is 9 cm ;
(Take $\quad \pi=3.142$.


Calculate to 4s.f
a) Angle $\mathrm{AO}_{1} \mathrm{~B}$

$$
\operatorname{Sin}^{-1} \theta=\frac{4.5}{8}
$$

$$
\theta=34.23
$$

$$
A O_{1} B=34.23 \times 2
$$

$$
=68.46^{\circ}
$$

b) Angle $\mathrm{AO}_{2} \mathrm{~B}$

$$
\begin{aligned}
& \operatorname{Sin}^{-1} \theta=\frac{4.5}{6} \\
& \theta=48.59 \\
& A O_{2} B=48.59 \times 2 \\
& =97.18^{\circ}
\end{aligned}
$$

c) Area of the shaded region.
$\frac{68.46}{360} \times 3.142 \times 8^{2}-1 / 2 \times 8 \times 8 \operatorname{Sin} 68.46$
38.24-29.76
$=8.48 \mathrm{~cm}^{2}$
$\frac{97.18}{360} \times 3.142 \times 6^{2}-1 / 2 \times 6 \times 6 \operatorname{Sin} 97.18$
30.53-17.86
= 12.67
Shaded Area $=8.48+12.67$

$$
=21.15 \mathrm{~cm}^{2}
$$

21. A village water tank is in the form of a frustum of a cone of height 3.2 m . The top and bottom radii of 18 m and 24 m respectively as shown below.

a. Calculate;
i. The surface area of the tank excluding the bottom.


$$
\begin{aligned}
& \frac{24}{18}=\frac{3.2+x}{x} \\
& 24 x=57.6+18 x \\
& 6 x=57.6 \\
& x=9.6 m
\end{aligned}
$$

24
S.A Big Cone $=\frac{22}{7} \times 24 \times 27.2$

$$
=2051.66 \mathrm{~cm}^{2}
$$

S.A Big Cone $=\frac{22}{7} \times 18 \times 20.4$
= 1154.06
S.A Big Cone $=2051.66-1154.06+\left(\frac{22}{7} \times 24 \times 27.2\right)^{2}$

$$
\begin{aligned}
& =897.6+1018.29 \\
& =1915.89 m^{2}
\end{aligned}
$$

ii. The capacity of the tank in litres.

V $=1 / 3 \times \frac{22}{7} \times 24 \times 24 \times 12.8$
Big cone
$=7723.89 m^{3}$
$V=1 / 3 \times \frac{22}{7} \times 18 \times 18 \times 9.6$
Small cone
= 3258.51
$V=7723.89-3258.51$
$4465.3757 \times 1000$
4465375.7 litres
b. 15 families each having 15 members use the water tank and each person uses 65 litres daily. How long will it take for the full tank to be emptied?
4465375.7
$\overline{15 \times 15 \times 65}$
$=305.3$ days.
22. Measurements of a maize field using baseline XY were recorded as shown below in metres

To R 160

| Y |  |
| :---: | :---: |
| 240 |  |
| 190 |  |
| 180 | 75 To Q |
| 150 | 50 To P |
| 120 |  |
| 100 | 100 To N |
| 50 |  |
| 20 | 20 To M |
| X |  |

a) Show the map of the maize field by scale drawing. Take 1 cm rep 20 m .
(4mks)


1. $\mathbf{A}=1 / 2(\mathbf{a}+\mathbf{b}) h$

$$
\begin{aligned}
& =1 / 2(20+100) 80 \\
& =4800 \mathrm{~m}^{2}
\end{aligned}
$$

2. $\mathbf{A}=1 / 2(\mathbf{a}+\mathbf{b}) \mathbf{h}$

$$
=1 / 2(20+100) 60
$$

$=4500 \mathrm{~m}^{2}$
3. $\mathbf{A}=1 / 2(\mathbf{a}+\mathbf{b}) \mathbf{h}$

$$
\begin{aligned}
& =1 / 2(50+75) 40 \\
& =2500 \mathrm{~m}^{2}
\end{aligned}
$$

4. $A=1 / 2 b h$

$$
\begin{aligned}
& =1 / 2 \times 75 \times 60 \\
& =2250 \mathrm{~m}^{2}
\end{aligned}
$$

5. $A=1 / 2$ bh

$$
\begin{aligned}
& =1 / 2 \times 160 \times 50 \\
& =4000 \mathrm{~m}^{2}
\end{aligned}
$$

6. $A=1 / 2(a+b) h$
$=1 / 2(100+160) 70$
$=4550 \mathrm{~m}^{2}$
7. $\mathbf{A}=1 / 2 \mathrm{bh}$
$=1 / 2 \times 30 \times 50$
$=750$
8. $A=1 / 2 b h$
$=1 / 2 \times 20 \times 20$
$=200 \mathrm{~m}^{2}$
b) Find the area of the field in hectares.
```
= 4800 + 4500 + 2500 + 2250 + 4000 + 5600 + 4550 + 750 + 200
= 29150m}\mp@subsup{}{}{2
    1 ha = 10,000m
        = 29,150m}\mp@subsup{}{}{2
        = 29150 x 1
        10000
        = 2.9150ha
```

c) If the cost of one hectare is Kshs. 65,000, find the total cost of the maize field.
1 ha = shs 5000
$2.9150 \mathrm{ha}=$
$=2.9150 \times 65000$
1
= ksh. 189475
23. Using a ruler and pair of compass only construct the following.
a) Triangle XYZ where XY is 6 cm and angle XYZ is $135^{\circ}$ and $\mathrm{YZ}=7 \mathrm{~cm}$. Measure XZ.
b) Drop a perpendicular from $Z$ to meet line XY at K . measure YK. (3mks)
c) Bisect line XY and let the bisector meet line XZ at Q .
d) Join Q to Y and measure angle XQY.

# a) Triangle XYZ where XY is 6 cm and angle XYZ is $135^{\circ}$ and $Y Z=7 \mathrm{~cm}$. Measure XZ . 


24. Complete the table for the function.
a) $y=1-2 x-3 x^{2}$ in the range $-3 \leq x \leq 3$

| x | -3 | -2 | -1 | 0 | 1 | 2 | 3 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $-3 x^{2} 2$ | -27 | $\mathbf{- 1 2}$ | -3 | 0 | $\mathbf{- 3}$ | -12 | $\mathbf{- 2 7}$ |
| -2 x | $\mathbf{6}$ | $\mathbf{4}$ | $\mathbf{2}$ | 0 | $\mathbf{- 2}$ | $\mathbf{- 4}$ | -6 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| y | -20 | $\mathbf{- 7}$ | $\mathbf{0}$ | 1 | $\mathbf{- 4}$ | -15 | $\mathbf{- 3 2}$ |

b) Use the table above to draw a graph of $y=1-2 x-3 x^{2}$ on the graph provided. (4mks)

c) Use the graph in (b) above to solve;
i. $\quad y=1-2 x-3 x^{2}=0$
$1-2 x-3 x^{2}=0$
-1, 0.2
ii. $\quad 2-5 x-3 x^{2}=0$

$$
1-2 x-3 x^{2}=0
$$

$$
2-5 x-3 x^{2}=0
$$

$$
-1+3 x=0
$$

$$
3 x=1
$$

$$
X=1 / 3
$$

