JUNIOR SCHOOL EDUCATION

MATHEMATICS

FOR GRADE 9

2025

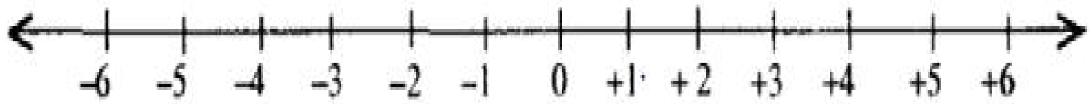
CBC

STRAND 1: NUMBERS
SUBSTRAND: Integers
Introduction to Integers

- Meaning of integers: An integer is a positive whole number, negative whole numbers and including zero.
- Examples of integers include 2, -3, 5, 0, 4 , 7, etc.

The number line

- A number line is a pictorial representation of numbers on a straight line. It's a reference for comparing and ordering numbers. It can be used to represent any integer.
- Integers can be illustrated on a number line as shown below.



- Any integer is less than all other integers to the right of it. Thus, -2 is less than -1 but greater than -3.
- The symbol < and > are used to denote 'less than greater than 'respectively.
- Thus, -2 < -1 and -2> -3

ASSIGNMENT

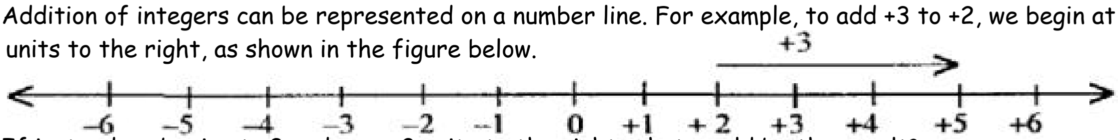
Use < or to compare the following pairs of numbers;

1. -5 and + 1
2. -3 and +4
3. -5 and +5
4. -10 and +1
5. -7 and -9
6. -20 and -36
7. 1 and -25
8. 15 and -30
9. 25 and -38

Addition of Integers

Addition of integers

- Addition of integers can be represented on a number line. For example, to add +3 to +2, we begin at +2 and move 3 units to the right, as shown in the figure below.



- If instead we begin at +3 and move 2 units to the right, what would be the result?
- Similarly, to add -1 and +5, we begin at +5 and move 1 unit to the left, as shown in the figure below.

EXERCISE

Show how the following additions can be done using a number line and give the results.

1.
2. (+2) + (+3)
3. (+8) + (+7)
4. (+12) + (+9)
5. (+7) + (+7)

6.

7. (+7) + (-4)
8. (-8) + (+5)
9. (+15) + (-14)
10. (-9) + (+2)

11.

12. (+8) + (-4)
13. (-13) + (+13)
14. (+4) + (-13)
15. (-11) + (+5)

16.

17. (-3) + (-4)
18. (-7) + (+2)
19. (-15) + (+12)
20. (-6) + (-6)

21.

22. (+2) + (+3) + (+5)
23. (+4) + (-2) + (-3)
24. (+6) + (-2) + (+6)
25. (-7) + (-2) + (+6)

26.

27. (-4) + (-3) + (-2)
28. (-1) + (-7) + (0)
29. (+6) + (+2) + (-5)
30. (+8) + (-3) + (+12)

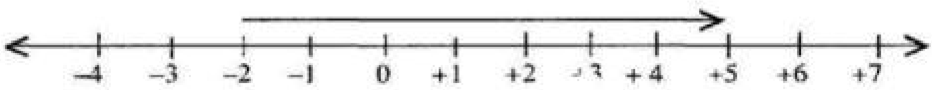
Subtraction of Integers

Steps for Subtracting Integers

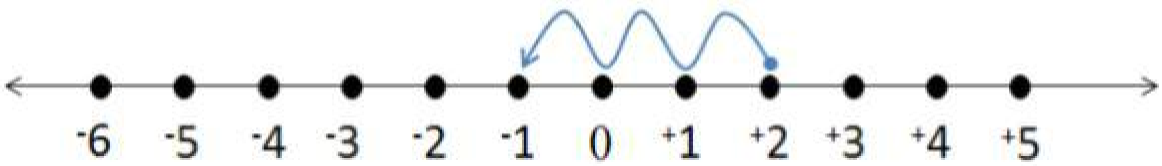
- To subtract integers on a number line, we need to move towards the left side when subtracting a positive number from a given number. On the other hand, when we subtract a negative integer from a given number, we move towards the right side of the number line.

Examples

(1). -2 - (-7)



(2).



$$2 - 3 = -1$$

<u>**ASSIGNMENT**</u>

<u>**Evaluate the following;**</u>

1.
2. 45-15
3. 35-16
4. 17-42
5. 19-70

6.

7. 12 - (-7)
8. 25 -(-36)
9. 30 - (-50)
10. 55 - (-28)

11.

12. (-5) - (+16)
13. (-11) - (+18)
14. (-40) - (20)
15. (-36) - (+52)

16.

17. (-15) - (-22)
18. (-33) - (-23)
19. (-26) - (-19

20. (-76) - (-58)

Multiplication of Integers

Steps for Multiplying Integers

Multiplication of integers follows the following rules.

- A positive number times a positive number equals a positive number.
  - A positive number times a negative number equals a negative number. A negative number times a positive number equals a negative number. A negative number times a negative number equals a positive number.

EXAMPLES

5 x -6 = -30 -2 x 5 = -10

4 x -6 = -24 -2 x 4 = -8

3 x -6 = -18 -2 x 3 = -6

2 x -6 = -12 -2 x 2 = -4

1 x -6 = -6 -2 x 1 = -2

0 x -6 = 0 -2 x 0 = 0

-1 x -6 = 6 -2 x -1 = 2

-2 x -6 = 12 -2 x -2 = 4

-3 x -6 = 18 -2 x -3 = 6

-4 x -6 = 24 -2 x -4 = 8

-5 x -6 = 30 -2 x -5 = 10

EXERCISE

Evaluate:

1.
2. -3 x -7 =
3. -8 x -10 =
4. -13 x -3 =
5. -16 x -2 =
6.
7. -60 x -4 =
8. -16 x -8 =
9. -33 x 3 =
10. -45 x -20 =
11.
12. -56 x -2 =
13. -5 x 8 x -2 =
14. -7 x -3 x 10 =
15.
16. -4 x -4 x -4 x -4 =
17. -10 x 2 x 10 =

Division of Integers

Steps for Dividing Integers

Division of integers follows the following rules.

(i) (a positive number) ÷ (a positive number) = (a positive number)

(ii) (a positive number) ÷ (a negative number) = (a negative number)

(iii) (a negative number) ÷ (a positive number) = (a negative number)

(iv) (a negative number) ÷ (a negative number) = (a positive number) In

general, for multiplication and division of integers:

- Two like signs give positive sign,
- Two unlike signs give negative sign

EXAMPLES

1. **-4÷-2=2**
2. **-2÷-2=1**
3. **-4÷-4=1**
4. **-8÷4=2**
5. **-6÷3=-2**
6. **4÷-2=-2**

EXERCISE

Evaluate

1.
2. 10÷2
3. 50 ÷ -25
4. 98 ÷ -14
5. 126÷9

67.288 ÷ -24

8. 42÷6
9. -90 ÷ 10
10. -125 ÷ 5

112. -615 ÷ 15

13. -1080 ÷ 90
14. -140 ÷ -20

15. -256 ÷ 16

167. -289 ÷ 17

18. -560 ÷ 16
19. -912 ÷ 19
20. -570 ÷ 19

Combined Operations on Integers

Steps for Combined Operations on Integers

The order in which operations are performed can be shown by use of brackets.

(1). 'Subtract 8 from 18 and then subtract 5 from the result' can be written as (18 - 8) -5 = 10 - 5 = 5

(2) 'Subtract 5 from 8 and then subtract the result from 18' can be written as 18 - (8 - 5) = 18 - 3 = 15

Note that;

- $(18-8)-5 \neq 18-(8-5)$

At times, more than two operations may occur in one expression, e.g, 6 x 3 - 4 ÷ 2 + 5. In such a case, we begin by brackets, then division, followed by multiplication, addition and finally subtraction, in that order. This can be shown by the use of the brackets, as below:

(6 x 3) - (4 ÷ 2) + 5 = 18 - 2 + 5 = 21

The BODMAS rule is applicable when more than operation appears in the same question.

EXERCISE

(1).

a. (18 - 24) + 30 = b. 32 + (17 + 30) =

c. 13 - (18 +7) = d. (62 - 94) + 20 =

e. 84 - (100 + 2) = f. (74 -24) + 30 =

g. (77 + 54) - 110 = h. (100 - 150) + 180

i. 222 - (158 + 90) = j. 1120 - (1450 + 120) =

(2).

a. 72 - 30 + 25 = b. 86 - 109 + 4 =

c. 209 + 43 - 300 = d. 348 + 60 - 510 =

e. 890 - 100 + 23 = f. 989 + 100 - 1470 =

g. 763 - 26 + 471 = h. 1190 + 340 + 670 =

i. 666 - 892 + 238 = j. 3004 - 563 + 1044 =

(3).

a. 2 x (10 ÷ 5) = b. (6 x18) ÷ 9 =

c. 90 ÷ (10 x 3) = d. -84 ÷ (7 x 4) =

e. (-39 ÷ 13) -8 = f. 21 x (14 ÷ 7) =

g. 1320 ÷ (11 x 5) = h. (-420 ÷ 28) x 5 =

i. 20 x (525 ÷ 21) = j. (1125 ÷ 15) x 19 =

k. 11 x 12 ÷ 4 = l. 19 x 8 ÷ 2 =

m. 64 ÷ 16 x 9 = n. -256 ÷ 64 x 10 =

o. 3 x 68 ÷ 17 = p. 91 ÷ 13 x 5 =

q. -11 x 125 ÷ 5 = r. -369 ÷ 123 x 8 =

Word Problems on Integers

1. If x = -2, y = -6 and z = 4, find the values of each of the following:
2. 4z + 2y - x
3. 2y - 3x + z
4. 4xy/z
5. 3yz/x
6. On a certain day, a student measured the temperature inside a deep freezer and found that it was -3°C while the room temperature was 24°C. What was the temperature difference between the room and the deep freezer?
7. Rhoda walked four floors down from the tenth floor and then took a lift to the eighteenth floor. How many floors did she go through while in the lift?
8. Kericho is a town on Kisumu-Nakuru road. The distance between Kisumu and Kericho is 85km, while that between Kericho and Nakuru is 105km. What is the distance between Nakuru and Kisumu?
9. A man was born in 1966. His father was born in 1928 and the mother three years later. If the man's daughter was born in 1992 and the son 5 years earlier, find the difference between the age of the man's mother and that of his son.
10. The temperature of a patient admitted to a hospital with fever was 42°C. After treatment, his temperature settled at 36.8°C. Find the change in temperature.

Cubes and Cube Root

Cubes from the Tables

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
1.0	1.000	1.030	1.061	1.093	1.125	1.158	1.191	1.225	1.260	1.295	3	7	10	13	16	20	23	26	30
1.1	1.331	1.368	1.405	1.443	1.482	1.521	1.561	1.602	1.643	1.685	4	8	12	16	20	24	28	31	35
1.2	1.728	1.772	1.816	1.861	1.907	1.953	2.000	2.048	2.097	2.147	5	9	14	19	23	28	33	37	42
1.3	2.197	2.248	2.300	2.353	2.406	2.460	2.515	2.571	2.628	2.686	5	11	16	22	27	33	38	43	49
1.4	2.744	2.803	2.863	2.924	2.986	3.049	3.112	3.177	3.242	3.308	6	13	19	25	31	38	44	50	56
1.5	3.375	3.443	3.512	3.582	3.652	3.724	3.796	3.870	3.944	4.020	7	14	21	29	36	43	50	57	64
1.6	4.096	4.173	4.252	4.331	4.411	4.492	4.574	4.657	4.742	4.827	8	16	24	32	41	49	57	65	73
1.7	4.913	5.000	5.088	5.178	5.268	5.359	5.452	5.545	5.640	5.735	9	18	27	37	46	55	64	73	82
1.8	5.832	5.930	6.029	6.128	6.230	6.332	6.435	6.539	6.645	6.751	10	20	31	41	51	61	71	82	92
1.9	6.859	6.968	7.078	7.189	7.301	7.415	7.530	7.645	7.762	7.881	11	23	34	45	57	68	79	91	102
2.0	8.000	8.121	8.242	8.365	8.490	8.615	8.742	8.870	8.999	9.129	13	25	38	50	63	75	88	100	113
2.1	9.261	9.394	9.528	9.664	9.800	9.938	10.078	10.218	10.360	10.503	14	28	41	55	69	83	97	110	124
2.2	10.648	10.794	10.941	11.090	11.239	11.391	11.543	11.697	11.852	12.009	15	30	45	60	76	91	106	121	136
2.3	12.167	12.326	12.487	12.649	12.813	12.978	13.144	13.312	13.481	13.652	16	33	49	66	82	99	115	132	148
2.4	13.824	13.998	14.172	14.349	14.527	14.706	14.887	15.069	15.253	15.438	18	36	54	72	90	108	126	143	161
2.5	15.625	15.813	16.003	16.194	16.387	16.581	16.777	16.975	17.174	17.374	19	39	58	78	97	117	136	155	175
2.6	17.576	17.780	17.985	18.191	18.400	18.610	18.821	19.034	19.249	19.465	21	42	63	84	105	126	147	168	189
2.7	19.683	19.903	20.124	20.346	20.571	20.797	21.025	21.254	21.485	21.718	23	45	68	90	113	136	158	181	203
2.8	21.952	22.188	22.426	22.665	22.906	23.149	23.394	23.640	23.888	24.138	24	49	73	97	121	146	170	194	219
2.9	24.389	24.642	24.897	25.154	25.412	25.672	25.934	26.198	26.464	26.731	26	52	78	104	130	156	182	208	234
3.0	27.000	27.271	27.544	27.818	28.094	28.373	28.653	28.934	29.218	29.504	28	56	83	111	139	167	195	223	250
3.1	29.791	30.080	30.371	30.664	30.959	31.256	31.554	31.855	32.157	32.462	30	59	89	119	148	178	208	237	267
3.2	32.768	33.076	33.386	33.698	34.012	34.328	34.646	34.966	35.288	35.611	32	63	95	126	158	190	221	253	284
3.3	35.937	36.265	36.594	36.926	37.260	37.595	37.933	38.273	38.614	38.958	34	67	101	134	163	201	235	269	302
3.4	39.304	39.652	40.002	40.354	40.708	41.064	41.422	41.782	42.144	42.509	36	71	107	142	178	214	249	285	320
3.5	42.87	43.24	43.61	43.99	44.36	44.74	45.12	45.50	45.88	46.27	4	8	11	15	19	23	26	30	34
3.6	46.66	47.05	47.44	47.83	48.23	48.63	49.03	49.43	49.84	50.24	4	8	12	16	20	24	28	32	36
3.7	50.65	51.06	51.48	51.90	52.31	52.73	53.16	53.58	54.01	54.44	4	8	13	17	21	25	29	34	38
3.8	54.87	55.31	55.74	56.18	56.62	57.07	57.51	57.96	58.41	58.86	4	9	13	18	22	27	31	35	40
3.9	59.32	59.78	60.24	60.70	61.16	61.63	62.10	62.57	63.04	63.52	5	9	14	19	23	28	33	37	42
4.0	64.00	64.48	64.96	65.45	65.94	66.43	66.92	67.42	67.92	68.42	5	10	15	20	25	29	34	39	44
4.1	68.92	69.43	69.93	70.44	70.96	71.47	71.99	72.51	73.03	73.56	5	10	15	21	26	31	36	41	46
4.2	74.09	74.62	75.15	75.69	76.23	76.77	77.31	77.85	78.40	78.95	5	11	16	22	27	32	38	43	49
4.3	79.51	80.06	80.62	81.18	81.75	82.31	82.88	83.45	84.03	84.60	6	11	17	23	28	34	40	45	51
4.4	85.18	85.77	86.35	86.94	87.53	88.12	88.72	89.31	89.92	90.52	6	12	18	24	30	36	41	47	53
4.5	91.12	91.73	92.35	92.96	93.58	94.20	94.82	95.44	96.07	96.70	6	12	19	25	31	37	43	50	56
4.6	97.34	97.97	98.61	99.25	99.90	100.54	101.19	101.85	102.50	103.16	6	13	19	26	32	39	45	52	58
4.7	103.82	104.49	105.15	105.82	106.50	107.17	107.85	108.53	109.22	109.90	7	14	20	27	34	41	47	54	61
4.8	110.59	111.28	111.98	112.68	113.38	114.08	114.79	115.50	116.21	116.93	7	14	21	28	35	42	49	56	63
4.9	117.65	118.37	119.10	119.82	120.55	121.29	122.02	122.76	123.51	124.25	7	15	22	29	37	44	51	59	66
5.0	125.000	125.75	126.51	127.26	128.02	128.79	129.55	130.32	131.10	131.87	8	15	23	31	38	46	53	61	69
5.1	132.65	133.43	134.22	135.01	135.80	136.59	137.39	138.19	138.99	139.80	8	16	24	32	40	48	56	64	71
5.2	140.61	141.42	142.24	143.06	143.88	144.70	145.53	146.36	147.20	148.04	8	17	25	33	41	50	58	66	74
5.3	148.88	149.72	150.57	151.42	152.27	153.13	153.99	154.85	155.72	156.59	9	17	26	34	43	51	60	69	77
5.4	157.46	158.34	159.22	160.10	160.99	161.88	162.77	163.67	164.57	165.47	9	18	27	36	44	53	62	71	80

Cubes

- The cube of a number is simply a number multiplied by itself three times e.g.

a × a × a=a3

1×1×1=1

3;8=2×2×2=23;27=3×3×3=33;

Example 1

What is the value of 63?

Solution

6 3 = 6 x 6 x 6

10. 36 x 6

11. 216

Example 2

Find the cube of 1.4

Solution

10. 1.4 x 1.4 x 1.4

11. 1 .96 x 1 .4

12. 2.744

Use of Tables to Find Roots

- The cubes can be read directly from the tables just like squares and square root.

a. $(1.8)^3$ = 5.832

b. $(2.12)^3$ = 9.528

c. $(3.254)^3$ = 34.454

= 34.45 (**4 s.f)**

QUESTIONS

(1). Use mathematical tables to find the cube of each of the following:

(a) 8.3 (b) 1.01 (c) 2.504 (d) 0.87 (e) 15.45

(2). Use tables to find:

31. (4.06)

32. (6.312)

33. (0.0912)

34. (381.7)

35. (2.1534)

36. (5.3679)

(3). A cubic building block measures 21 cm. Find its volume.

(4). A cubic water tank has sides of length 2.143 m. What is the capacity of the tank in litres?

Cube Root by Factor Method

Cube Roots Using Factor Methods

- Cubes and cubes roots are opposite. The cube root of a number is the number that is multiplied by itself three times to get the given number

Example

The cube root of 64 is written as;

= 4 Because 4× 4 × 4 = 64

$\sqrt[3]{27}$ = 3 Because 3 ×3 × 3 = 27

Example

Evaluate: $\sqrt[3]{216}$

Solution

10. $\sqrt[3]{(2 \times 2 \times 2 \times 3 \times 3 \times 3)}$

11. 2×3

12. 6

The volume of a cube is 1000 cm

What is the length of the cube?

Solution

Volume of the cube, V = ***P***

P = 1000

I = 3/1000

20. 3/l0 x 10 x 10

21. 10

.-. The length of the cube is 10 cm.

QUESTIONS

1. The volume of a sphere is given by $^4/_3\ \pi r^3$ Find the radius of a sphere whose volume is 104.816 cm^3. (Take π to be $^{22}/_7$)

2. The volume of material used to make a cube is 1 728 cm^3. What is the length of the side of the cube.

3. The volume of water in a measuring cylinder reads 200cm^3. When a cube is immersed into the water, the cylinder reads 543 cm^3. Find:

a. The volume of the cube.

b. The length of the side of the cube.

4. A metallic cuboid measuring 16 cm by 8 cm by 4 cm was melted. The material was then used to make a cube. What was the length of the cube?

Indices and Logarithms

Law of indices

Indices

Index and Base Form

The power to which a number is raised is called index or indices in plural.

$2^5=2\times2\times2\times2\times2$

5 is called the power or index while 2 two is the base.

$100 = 10^2$

2 is called the index and 1 0 is the base.

Laws of Indices

- For the laws to hold the base must be the same.

Rule 1

Any number, except zero whose index is 0 is always equal to 1

Example

$5^0=1$

100000000000000000=1

Rule 2

To multiply an expression with the same base, copy the base and add the indices.

a m × an = am + n

Example

$5^2\times 5^3 = 5^5$

= 3125

Rule 3

To divide an expression with the same base, copy the base and subtract the powers.

$a^m \div a^n = a^{m-n}$

Example

$9^5 \div 9^2 = 9^3$

Rule 4

To raise an expression to the nth index, copy the base and multiply the indices a $^{m\times n}$

$= a^{mn}$

Example

$(5^3)^2$

$= 5^{3\times 2} = 5^6$

QUESTIONS

Solve for x in $5^{2x-1} = 12^x$ **(3mks)**

Solve for x in $4^{x+1} = 32$ **(3mks)**

Find the value of x which satisfies the equation $16^{x^2} = 8^{4x-3}$

Solve for x in the equation $32^{(x-3)} \div 8^{(x+4)} = 64 \div 2^x$

$$\frac{81^{2x} \times 27^x}{9^x} = 729$$

Solve each of the following equations:

a. $(3^{2x})^3 = 3^4 \times 3^8$

b. $(7^5) = (7^4) \times 7^2$

c. $(3^{2x})4 = 81$

d. $4^{5x} \div (2^{3x})^2 = 256$

e. $9^{2x} = 729$

f. $2^{8x} = 512$

g. $(7^4)^{2x} = (7^4)^3$

h. $(5^3)^a (5^8)^a = 5^{72}$

i. $94^x \div 3^{2x} = 2187$

Negative Indices

Rule 5

- When dealing with a negative power, you simply change the power to positive by changing it into a

fraction with 1 as the numerator.

$a^{-m} = 1$

a^{m}

Example

$2^{-2} = 1$

2^{2}

$= {}^{1}/_{4}$

Example

Evaluate:

a. $2^{3} \times 2^{-3} = 2^{(3+-3)}$

$=2^{0} = 1$

b. $({}^{2}/_{3})^{-2} = (1)^{2}$

$({}^{2}/_{3})^{2}$

1

${}^{4}/_{9}$

$=1 \div {}^{4}/_{9}$

$=1 \times {}^{9}/_{4} = 2\ {}^{1}/_{4}$ or $({}^{2}/_{3})^{-2} = ({}^{3}/_{2})^{2} = {}^{9}/_{4}$

QUESTIONS

$6^{2} \times 7^{-4} \times (8^{-2})^{2} \times 6^{3} \times 7^{2} \times 8^{4}$

$7^{-3} \times 8^{4} \times 7^{2} \times 8^{-3}$

$20^{-3} \times 25^{2} \times 20^{3} \times 25^{-4}$

$$\frac{2^{-2} \times (2^{2})^{-6}}{2^{-4} \times 2^{-6}}$$

$$\frac{20^{-2} \times 20^{-2} \times 20^{-2}}{20}$$

$$\frac{a^{-a}b^{2} \times b^{-a} \times c^{-a}}{a^{-2a} \times b^{-2a} \times c^{-2a}}$$

Fractional Indices

Fractional Indices

- Fractional indices are written in fraction form. In summary if $a^n = b$. a is called the nth root of b written as $n\sqrt{b}$.

Example

$27^{1/3} = \sqrt[3]{27} = 3$

$16^{3/4}$

$= (4\sqrt{16})^3 = 2^3 = 8$

$4^{-½} = 1$

$4^{½}$

$=1$

$\sqrt{4}$

$= ½$

QUESTIONS

a. $$\frac{27^{\frac{2}{3}} \div 2^4}{32^{-\frac{3}{5}}}$$

b. $$27^{\frac{2}{3}} \times \left(\frac{81}{16}\right)^{-\frac{1}{4}}$$

c. $$\frac{64^{-\frac{1}{2}} \times 27000^{\frac{2}{3}}}{2^{-4} \times 3^0 \times 5^2}$$

d. $$\frac{243^{\frac{-2}{5}} \times 125^{\frac{2}{3}}}{9^{\frac{-3}{2}}}$$

$$\frac{\left(9a^2\right)^{\frac{1}{3}}}{6^4} \times \frac{\left(4b^2\right)^{\frac{1}{3}}}{5a}$$

Introduction to Logarithms

- The indices 0, 1, 2, 3, 4 ... are called the **logarithms** of the corresponding numbers to **base** 3.
- For example; logarithm of 9 to **base** 3 is 2.
- Logarithm of 81 to **base** 3 is 4.

These are usually written in short form as;

$\log_3 9 = 2$

$\log_3 81 = 4$

Examples

Index notation	Logarithm form
$2^2 = 4$	$\log_2 4 = 2$
$9^{\frac{1}{2}} = 3$	$\log_9 3 = \frac{1}{2}$
$b^n = m$	$\log_b m = n$

Generally,

$a^m = n$ is the index notation while $\log_a n = m$ is the logarithmic notation.

Write in logarithmic form:

a. $2^4 = 16$

b. $9^{\frac{1}{2}} = 3$

c. $b^n = m$

Solution

a. If $2^4 = 16$, then $\log_2 16 = 4$

b. If $9^{\frac{1}{2}} = 3$, then $\log_9 3 = \frac{1}{2}$

c. If $b^n = m$, then $\log_b m = n$

<u>QUESTIONS</u>

(1). write in logarithmic form

a. $3^2 = 9$

b. $2^4 = 16$

c. $3^3 = 27$

d. $2^5 = 32$

e. $3^4 = 81$

f. $5^3 = 125$

g. $10^0 = 1$

h. $2^{10} = 1024$

i. $a^n = b$

(2). Write each of the following in index from.

a. $\log_2 8 = 3$

b. $\log_4 16 = 2$

c. $\log_5 125 = 3$

d. $\log_{10} 8 = x$

e.

f. $\log_3 27 = 3$

g. $\log_6 216 = 3$

h. $\log_x 40 = y$

i. $\log_4 6 = y$

j

k. $\log_{10} 10\,000 = 4$

l. $\log_2 16 = 4$

Standard form of Logarithms

Standard Form
Consider the following:

$12 = 1.2 \times 10^1$

$120 = 1.2 \times 10^2$

$1200 = 1.2 \times 10^3$

$0.12 = 1.2 \times {}^1/_{10}$

$=1.2 \times 10^{-1}$

$0.286 = 2.86 \times {}^1/_{10}$

$=2.86 \times 10^{-1}$

$0.0074 = 7.4 \times {}^1/_{1000}$

$=7.4\times 10^{-3}$

Any number can be written in the form $A \times 10^n$, Where A is a number between 1 and 10 (10 not included) or $1 \leq A < 10$, and n is an integer. When written in this way, a number is said to be in **standard form.**

Write each of the following numbers in standard from.

12. 26

13. 357

14. 4068 iv.15

000 000

13. 0.031

14. 0.00215

15. 0.005012

16. 0.000000152

17. 100 000

18. ${}^{46}/_{100000}$

Common Logarithms

Reading logarithms from the tables is the same as reading squares square roots and reciprocals.

We can read logarithms of numbers between 1 and 10 directly from the table. For numbers greater than 10 we proceed as follows:

- Express the number in standard form, $A \times 10n$ .Then n will be the whole number part of the logarithms.
- Read the logarithm of A from the tables, which gives the decimal part of the logarithm.
- Then add it to n which is the power of 10 to form the positive part of the logarithm.

Logarithms to Base 10

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
1.0	.0000	0043	0086	0128	0170	0212	0253	0294	0334	0374	4	8	12	17	21	25	29	33	37
1.1	.0414	0453	0492	0531	0569	0607	0645	0682	0719	0755	4	8	11	15	19	23	27	30	34
1.2	.0792	0828	0864	0899	0934	0969	1004	1038	1072	1106	3	7	10	14	17	21	24	28	31
1.3	.1139	1173	1206	1239	1271	1303	1335	1367	1399	1430	3	6	10	13	16	19	23	26	29
1.4	.1461	1492	1523	1553	1584	1614	1644	1673	1703	1732	3	6	9	12	15	18	21	24	27
1.5	.1761	1790	1818	1847	1875	1903	1931	1959	1987	2014	3	6	8	11	14	17	20	22	25
1.6	.2041	2068	2095	2122	2148	2175	2201	2227	2253	2279	3	5	8	11	13	16	18	21	24
1.7	.2304	2330	2355	2380	2405	2430	2455	2480	2504	2529	2	5	7	10	12	15	17	20	22
1.8	.2553	2577	2601	2625	2648	2672	2695	2718	2742	2765	2	5	7	9	12	14	16	19	21
1.9	.2788	2810	2833	2856	2878	2900	2923	2945	2967	2989	2	4	7	9	11	13	16	18	20
2.0	.3010	3032	3054	3075	3096	3118	3139	3160	3181	3201	2	4	6	8	11	13	15	17	19
2.1	.3222	3243	3263	3284	3304	3324	3345	3365	3385	3404	2	4	6	8	10	12	14	16	18
2.2	.3424	3444	3464	3483	3502	3522	3541	3560	3579	3598	2	4	6	8	10	12	14	15	17
2.3	.3617	3636	3655	3674	3692	3711	3729	3747	3766	3784	2	4	6	7	9	11	13	15	17
2.4	.3802	3820	3838	3856	3874	3892	3909	3927	3945	3962	2	4	5	7	9	11	12	14	16
2.5	.3979	3997	4014	4031	4048	4065	4082	4099	4116	4133	2	3	5	7	9	10	12	14	15
2.6	.4150	4166	4183	4200	4216	4232	4249	4265	4281	4298	2	3	5	7	8	10	11	13	15
2.7	.4314	4330	4346	4362	4378	4393	4409	4425	4440	4456	2	3	5	6	8	9	11	13	14
2.8	.4472	4487	4502	4518	4533	4548	4564	4579	4594	4609	2	3	5	6	8	9	11	12	14
2.9	.4624	4639	4654	4669	4683	4698	4713	4728	4742	4757	1	3	4	6	7	9	10	12	13
3.0	.4771	4786	4800	4814	4829	4843	4857	4871	4886	4900	1	3	4	6	7	9	10	11	13

Example

Find the logarithm of: 379

Solution

In standard form

= 3.79 x 102

Log 3.79 = 0.5786

Therefore the logarithm of 379 is 2 + 0.5786 = 2.5786

Note: The whole number part of the logarithm is called **the characteristic** and the decimal part is the **mantissa**.

QUESTIONS

Use logarithm tables to express each of the following numbers in the form 10^x.

37. 8.2

38. 1.47

39. 4.73

40. 7.25

41. 9.89

42. 5.672

43. 8.137

h. 3.142

13. 2.718

14. 3.333

15. 12.3

16. 59.7

17. 82.9

18. 72

19. 96.1

20. 431.5

21. 7924

22. 1025

23. 1913

24. 4937

25. 237.7

26. 475 000

27. 3 910 000

28. 958 312

Logarithm of Positive Numbers Less Than 1

Logarithms of Positive Numbers Less Than 1

Example

Log to base 10 of 0.034

Solution

We proceed as follows:

Express 0.034 in standard form, i.e., A X10n .

Read the logarithm of A and add to n

Thus 0.034 = 3.4 x 10-2

Log 3.4 from the tables is 0.531 5

Hence 3.4 x 10-2 = 100.5315 × 10-2

Using laws of indices add 0.531 5 + -2 which is written as $\bar{2}.5315$.

It reads bar two point five three one five. The negative sign is written directly above two to show

that it's only the characteristic is negative.

Example

Find the logarithm of: 0.00063

Solution

5. 6.3×10^{-3} (Find the logarithm of 6.3)
6. $10^{0.6993} \times 10^{-3}$
7. -3 + 0.7993
8. $\bar{3}.7993$

<u>QUESTIONS</u>

Find the algorithm of;

9. 0.57
10. 0.00063
11. 0.0029
12. 0.0765
13. 0.82

Antilogarithms

<u>Antilogarithms</u>

- Finding antilogarithm is the reverse of finding the logarithms of a number.

For example;

The logarithm of 1000 to base 10 is 3. So the antilogarithm of 3 is 1000.

In algebraic notation, if

Log x = y then antilog of y = x.

Example

Find the antilogarithm of $\bar{2}$ .3031

Solution

Let the number be x

x = 10$\bar{2}$ .3031

14. 10-2+0.3031
15. 10-2 x 100.3031(Find the antilog, press shift and log then key in the number)
16. 10-2 x 2.01
17. 1 /100 x 2.01
18. 2.01/100
19. 0.0201

<u>**QUESTIONS**</u>

(1). Read from the table

18. 0.1461

19. 0.2487

20. 1.4900

21. 2.4835

(2). Find the number whose logarithm is;

(a) $\bar{2}.3031$ (b) $\bar{4}.5441$

Applications of Logarithms

Example

Use logarithm tables to evaluate:

$$\frac{456 \times 398}{271}$$

Number	Standard form	logarithm
456	4.56×10^2	2.6590
398	3.98×10^2	2.5999

5.2589

+

271	2.71×10^2	2.4330 -

$6.697 \times 10^2 \leftarrow$ 2.8259

Evaluate $\frac{415.2 \times 0.0761}{135}$

Solution

Number	logarithm
415.2 0.0761 135	2.6182 2.8814 + 1.4996 2.1303 -
2.341×10^{-1}	1.3693
0.2341	

a.

b. $3.14^2 = 3.14 \times 3.14$

No.	log
3.14^2	0.4969×2
9.858×10^0	0.9938

Therefore, $3.14^2 = 9.858 \times 10^0$

(v) 9

.858 c.

8.36^3

No.	log
8.36^3	0.9222×3
5.842×10^2	2.7666

Therefore, $8.36^3 = 5.842 \times 10^2$

$= 584.2$

Questions

Use algorithms to evaluate.

a.

$$\frac{1.34}{(5.24)^{0.8} \times 0.0029}$$

b. 21.47 x 362.1

c.

$$\frac{18^2 \times 391^4}{15^3 \times 56^4}$$

d.

$$\frac{94.7 \times 16.45}{12.5 \times 8.93}$$

e.

$$\frac{48.35 \times 125.3}{39.3 \times 50.4}$$

Roots

Example

$\sqrt{0.945} = (9.45 \text{ x } 10^{-1})^{\frac{1}{2}}$

$= (10^{1.9754 \text{ x } \frac{1}{2}})$

Note;

- In order to divide 1.9754 by 2, We write the logarithm in such a way that the characteristic is exactly divisible by 2. if we are looking for the n^{th} root, we arrange the characteristic to be exactly divisible by n)

1.9754 = -1 + 0.9754

= -2 + 1.9754

Therefore, $^1/_2$ (1.9754) = $^1/_2$(-2 + 1.9754)

=-1 + 0.9877

= 1.9877

Find the antilog of 1.9877 by writing the mantissa as power of 10 and then find the antilog characteristic = 9.720×10^{-1} = 0.9720

$\sqrt[3]{0.0618}$

Solution

Number	Logarithm
$\sqrt[3]{0.0618} = 0.0618^{1/3}$	$\bar{2}.7910 \times {}^{1}/_{3}$
	$(\bar{3} + 1.7910) \times {}^{1}/_{3}$
3.954×10^{-1} ←antilog	$= \bar{1}.5970$

__Questions__

Use logarithm tables to evaluate:

(1).

$$\sqrt[3]{(35.6 \times 0.0613^2)}$$

(2).

$$\sqrt{\left[\frac{2.935 \times 0.0765}{32.74}\right]}$$

(3).

$$\sqrt[4]{\frac{4.562 \times 0.038}{0.82}}$$

(4).

$$\frac{(0.07284)^2}{3\sqrt{0.06195}}$$

(5).

$$3\sqrt{\frac{36.15 \times 0.02573}{1.938}}$$

Compound Proportions and Rates of Work

Compound Proportions

- The proportion involving two or more quantities is called compound proportion. Any four quantities a , b , c and d are in proportion if;
  $\frac{a}{b} = \frac{c}{d}$

Example

Find the value of a that makes 2, 5, a and 25 to be in proportion;

Solution

Since 2, 5, a, and 25 are in proportion

$\frac{2}{5} = \frac{a}{25}$

$5a = 2 \times 25$

$a = \frac{2 \times 25}{5}$

$a = 10$

Continued Proportions

- In continued proportion, all the ratios between different quantities are the same; but always remember that the relationship exists between two quantities for example:

P:Q	Q:R	R:S
10:5	16:8	4:2

- Note that in the example, the ratio between different quantities i.e. P:Q, Q:R and R:S are the same i.e. 2:1 when simplified.
- Continued proportion is very important when determining the net worth of individuals who own the same business or even calculating the amounts of profit that different individual owners of a company or business should take home.

Proportional Parts

In general, if n is to be divided in the ratio a: b: c, then the parts of n proportional to a, b, c are $^{a}/_{a+b+c} \times n$, $^{b}/_{a+b+c} \times n$ and $^{c}/_{a+b+c} \times n$ respectively

Example

Omondi, Joel, cheroot shared sh 27,000 in the ratio 2:3:4 respectively. How much did each get?

Solution

The parts of sh 27,000 proportional to 2, 3, 4 are

$^{2}/_{9} \times 27,000$ = sh 6000 →Omondi

$^{3}/_{9} \times 27,000$ = sh 6000 →Joel

$^{4}/_{9} \times 27,000$ = sh 6000 →Cheroot

Example

Three people – John, Debby and Dave contributed ksh 119, 000 to start a company. If the ratio of the contribution of John to Debby was 12:6 and the contribution of Debby to Dave was 8:4, determine the amount in dollars that every partner contributed.

Solution

Ratio of John to Debbys contribution = 12:6 = 2:1

Ratio of Debby to Daves contribution = 8:4 = 2:1

As you can see, the ratio of the contribution of John to Debby and that of Debby to Dave is in continued proportion.

Hence $\frac{\text{John}}{\text{Debby}} = \frac{\text{Debby}}{\text{Dave}} = \frac{2}{1}$

To determine the ratio of the contribution between the three members, we do the calculation as follows:

John : Debby : Dave

12 :6

8 : 4

We multiply the upper ratio by 8 and the lower ratio by 6, thus the resulting ratio will be:

John : Debby : Dave

96 : 48 :24

=4:2:1

The total ratio = 7

The contribution of the different members can then be found as follows:

John $^4/_7$ x ksh 119, 000 = ksh 68,000

Debby $^2/_7$ x ksh 119, 000 = ksh 34,000

Dave $^1/_7$ x ksh 119, 000 = ksh 17,000

John contributed ksh 68, 000 to the company while Debby contributed ksh 34, 000 and Dave contributed ksh 17, 000

Example 2

You are presented with three numbers which are in continued proportion. If the sum of the three numbers is 38 and the product of the first number and the third number is 144, find the three numbers.

Solution

Let us assume that the three numbers in continued proportion or Geometric Proportion are a, ar and ar^2 where a is the first number and r is the rate.

a+ar+ar2 = 38 .. (1)

The product of the 1 st and 3rd is

a × ar2 = 144

Or

$(ar)^2$ = 144..(2)

If we find the square root of (ar) 2, then we will have found the second number:

√[(ar)2]= √144

ar = 12

Since the value of the second number is 12, it then implies that the sum of the first and the third number is 26.

We now proceed and look for two numbers whose sum is 26 and product is 144.

Clearly, the numbers are 8 and 18.

Thus, the three numbers that we were looking for are 8, 12 and 18.

Let us work backwards and try to prove whether this is actually true:

8+12+18=18

What about the product of the first and the third number?

8×18=144

What about the continued proportion

$\frac{a}{ar} = \frac{ar}{ar^2} = \frac{2}{3}$

The numbers are in continued proportion

Example

Given that x: y =2:3, Find the ratio (5x 4y): (x + y).

Solution

Since x: y =2: 3

$^x/_2 = {^y/_3} = k,$

x = 2k and y = 3k

(5x 4y): (x + y) = (10k 12k) : (2k + 3 k)

15. −2k: 5k

16. −2:5

Example

If $^a/_b = {^c/_d}$,show that $\frac{a - 3b}{b-3a} = \frac{c - 3d}{d - 3c}$.

Solution

$^a/_b = {^c/_d} \rightarrow {^a/_c} = {^b/_d}$

$^a/_b = {^b/_d} = k$

a = kc and b = kd

Substituting kc for a and kd for b in the expression $\frac{a - 3b}{b - 3a}$

$\frac{kc - 3kd}{kd - 3kd} = \frac{k(c - 3d)}{k(d - 3c)}$

Therefore expression $\frac{a - 3b}{b-3a} = \frac{c - 3d}{d - 3c}$

Rates of Work and Mixtures

Examples

195 men working 10 hour a day can finish a job in 20 days. How many men employed to finish the job in 15 days if they work 13 hours a day.

Solution:

Let x be the no. of men required

Days	hours	Men
20	10	195
15	13	x

$20 \times 10 \times 195 = 15 \times 13 \times x$

$x = \frac{20 \times 10 \times 195}{15 \times 13} = 200$ men

Example

Tap P can fill a tank in 2 hrs, and tap Q can fill the same tank in 4 hrs. Tap R can empty the tank in 3 hrs.

19. If tap R is closed, how long would it take taps P and Q to fill the tank?
20. Calculate how long it would take to fill the tank when the three taps P, Q and R. are left running?

Solution

44. Tap P fills $^1/_2$ of the tank in 1 h. Tap Q fills $^1/_4$ of the tank in 1 h. Tap R empties $^1/_3$ of the tank in 1 h.
    In one hour, P and Q fill $^1/_2 + {}^1/_4 = {}^3/_4$ of the tank
    Therefore $^3/_4$ of the tank is filled in 1 h.
    Time taken to fill the tank$(^4/_4) = (^4/_4 \div {}^3/_4)$h
    $^4/_3$ h
45. In 1h, P and Q fill $^3/_4$ of tank while R empties $^1/_3$ of the tank.
    When all taps are open, $(^1/_2 + {}^1/_4 - {}^1/_3 = {}^5/_{12})$ of the tank is filled in 1 hour.
    $^5/_{12}$ of tank is filled in 1 hour.
    Therefore time required to fill the tank $^{12}/_{12} = (^{12}/_{12} \div {}^5/_{12}) \times 1$ h
    $2^2/_5$ h

Example

In what proportion should grades of sugars costing sh. 45 and sh. 50 per kilogram be mixed in order to produce a blend worth sh. 48 per kilogram?

Solution

Method 1

Let n kilograms of the grade costing sh. 45 per kg be mixed with 1 kilogram of grade costing sh. 50 per kg.

Total cost of the two blends is sh. b(45n+50)

The mass of the mixture is (n +1) kg

Therefore total cost of the mixture is (n +1)48

$45n + 50 = 48 (n +1)$

$45n + 50 = 48 n + 48$

$50 = 3n + 48$

$2 = 3n$

$n = {}^{2}/_{3}$

The two grades are mixed in the proportion ${}^{2}/_{3} : 1 = 2 : 3$

Method 2

Let x kg of grade costing sh 45 per kg be mixed with y kg of grade costing sh.50 per kg. The total cost will be sh. (45x + 50 y)

Cost per kg of the mixture is sh. $\frac{45x+50y}{x+y}$

$$\frac{45x+50y}{x+y} = 48$$

$45x + 50y = 48(x + y)$

$45x + 50y = 48x + 48y$

$2y = 3x$

$= {}^{x}/_{y} = {}^{2}/_{3}$

The proportion is x : y = 2:3

Questions

1. Akinyi bought and beans from a wholesaler. She then mixed the maize and beans the ratio 4:3 she brought the maize as Kshs. 12 per kg and the beans 4 per kg. If she was to make a profit of 30% what should be the selling price of 1 kg of the mixture?
2. A rectangular tank of base 2.4 m by 2.8 m and a height of 3m contains 3,600 liters of water initially. Water flows into the tank at the rate of 0.5 litres per second

   Calculate the time in hours and minutes, required to fill the tank
3. A company is to construct a parking bay whose area is 135m2. It is to be covered with concrete slab of uniform thickness of 0.15. To make the slab cement. Ballast and sand are to be mixed so that their masses are in the ratio 1 : 4 : 4. The mass of m3 of dry slab is 2, 500kg.

   Calculate
   a.
      i. The volume of the slab
      ii. The mass of the dry slab
      iii. The mass of cement to be used
   b. If one bag of the cement is 50 kg, find the number of bags to be purchased
   c. If a lorry carries 7 tonnes of sand, calculate the number of lorries of sand to be purchased.
4. The mass of a mixture A of beans and maize is 72 kg. The ratio of beans to maize is 3:5 respectively
   a. Find the mass of maize in the mixture
   b. A second mixture of B of beans and maize of mass 98 kg in mixed with A. The final ratio of beans to maize is 8:9 respectively. Find the ratio of beans to maize in B
5. A retailer bought 49 kg of grade 1 rice at Kshs. 65 per kilogram and 60 kg of grade II rice at Kshs 27.50 per kilogram. He mixed the tow types of rice.
   a. Find the buying price of one kilogram of the mixture
   b. He packed the mixture into 2 kg packets
      i. If he intends to make a 20% profit find the selling price per packet
      ii. He sold 8 packets and then reduced the price by 10% in order to attract customers. Find the new selling price per packet.
      iii. After selling 1/3 of the remainder at reduced price, he raised the price so as to realize the original goal of 20% profit overall. Find the selling price per packet of the remaining rice.
6. A trader sells a bag of beans for Kshs 1, 200. He mixed beans and maize in the ration 3 : 2. Find how much the trader should he sell a bag of the mixture to realize the same profit?
7. Pipe A can fill an empty water tank in 3 hours while, pipe B can fill the same tank in 6 hours, when the tank is full it can be emptied by pipe C in 8 hours. Pipes A and B are opened at the same time when the tank is empty.

   If one hour later, pipe C is also opened, find the total time taken to fill the tank
8. A solution whose volume is 80 litres is made 40% of water and 60% of alcohol. When litres of water are added, the percentage of alcohol drops to 40%
   a. Find the value of x
   b. Thirty litres of water is added to the new solution. Calculate the percentage
   c. If 5 litres of the solution in (b) is added to 2 litres of the original solution, calculate in the simplest form, the ratio of water to that of alcohol in the resulting solution

STRAND 2 ALGEBRA
Matrices

Introduction

- A matrix is a rectangular arrangement of numbers in rows and columns.
- For instance, matrix A below has two rows and three columns. The dimensions of this matrix are 2 x 3 (read 2 by 3).
- The numbers in a matrix are its entries. In matrix A, the entry in the second row and third column is 5.
- Some matrices (the plural of matrix) have special names because of their dimensions or entries.

Order of Matrix

- Matrix consist of rows and columns. Rows are the horizontal arrangement while columns are the vertical arrangement.
- Order of matrix is being determined by the number of rows and columns. The order is given by stating the number of rows followed by columns.

Note;

- If the number of rows is m and the number of columns n, the matrix is of order m×n.

  E.g. If a matrix has m rows and n columns, it is said to be order m×n.

e.g. $\begin{bmatrix} 2 & 0 & 3 & 6 \\ 3 & 4 & 7 & 0 \\ 1 & 9 & 2 & 5 \end{bmatrix}$ is a matrix of order 3×4.

e.g. $\begin{bmatrix} 1 & 0 & -2 \\ 2 & 1 & 5 \\ -1 & 3 & 0 \end{bmatrix}$ is a matrix of order 3.

e.g. $\begin{bmatrix} 2 & 3 & 4 \\ 1 & -8 & 5 \end{bmatrix}$ is a 2×3 matrix.

e.g. $\begin{bmatrix} 2 \\ 7 \\ -3 \end{bmatrix}$ is a 3×1 matrix.

Elements of Matrix

- The element of a matrix is each number or letter in the matrix. Each element is locating by stating its position in the row and the column.
- For example, given the 3 x 4 matrix

$$\begin{bmatrix} 2 & 0 & 3 & 6 \\ 3 & 4 & 7 & 0 \\ 1 & 9 & 2 & 5 \end{bmatrix}$$

    - The element 1 is in the third row and first column.
    - The element 6 is in the first row and forth column.

Note;

- A matrix in which the number of rows is equal to the number of columns is called a **square matrix**.

$$\begin{bmatrix} 1 & 0 & -2 \\ 2 & 1 & 5 \\ -1 & 3 & 0 \end{bmatrix}$$

- $[\, a_1, a_2 \ldots a_n]$ Is called a row matrix or row vector.
- $\begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix}$ Is called a *column matrix* or *column vector.*
- $\begin{bmatrix} 2 \\ 7 \\ -3 \end{bmatrix}$ Is a column vector of order 3×1.
- [−2 −3 −4] is a row vector of order 1 × 3.
- Two or more matrices re equal if they are of the same order and their corresponding elements are equal.
- Thus, if $\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 3 & 4 \\ 1 & 5 \end{bmatrix}$ then, a = 3, b = 4 and d = 5.

Addition and Subtraction of Matrices

- Matrices can be added or subtracted if they are of the same order. The sum of two or more matrices is obtained by adding corresponding elements. Subtraction is also done in the same way.

Example

if $A = \begin{bmatrix} 2 & 5 \\ 0 & 7 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 3 \\ 6 & 2 \end{bmatrix}$ find :

17. A + B
18. A B

Solution

a.

$$A+B = \begin{bmatrix} 2 & 5 \\ 0 & 7 \end{bmatrix} + \begin{bmatrix} 1 & 3 \\ 6 & 2 \end{bmatrix} = \begin{bmatrix} 2+1 & 5+3 \\ 0+6 & 7+2 \end{bmatrix} = \begin{bmatrix} 3 & 8 \\ 6 & 9 \end{bmatrix}$$

b.

$$A - B = \begin{bmatrix} 2 & 5 \\ 0 & 7 \end{bmatrix} + \begin{bmatrix} 1 & 3 \\ 6 & 2 \end{bmatrix} = \begin{bmatrix} 2-1 & 5-3 \\ 0-6 & 7-2 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ -6 & 5 \end{bmatrix}$$

Example

$$\begin{bmatrix} 3 & 2 & 1 \\ 0 & 4 & 5 \\ 1 & 3 & 2 \end{bmatrix} - \begin{bmatrix} 2 & 4 & 1 \\ 1 & 2 & 0 \\ 5 & 9 & 6 \end{bmatrix} + \begin{bmatrix} 8 & 0 & 2 \\ 1 & 3 & 5 \\ 2 & 1 & 6 \end{bmatrix} = \begin{bmatrix} 3-2+8 & 2-4+0 & 1-1+2 \\ 0-1+1 & 4-2+3 & 5-0+5 \\ 1-5+2 & 3-9+1 & 2-6+6 \end{bmatrix}$$

$$= \begin{bmatrix} 9 & -2 & 2 \\ 0 & 5 & 10 \\ -2 & -5 & 2 \end{bmatrix}$$

Note;

After arranging the matrices you must use BODMAS

$$\begin{bmatrix} 2 & 7 \\ 4 & 9 \end{bmatrix} + \begin{bmatrix} 1 \\ 5 \\ 6 \end{bmatrix}$$

The matrix above cannot be added because they are not of the same order. $\begin{bmatrix} 2 & 7 \\ 4 & 9 \end{bmatrix}$ is of order 2 x 2 while $\begin{bmatrix} 1 \\ 5 \\ 6 \end{bmatrix}$ is of order 3 x1

Matrix Multiplication

To multiply a matrix by a number, you multiply each element in the matrix by the number.

Example

$$3\begin{bmatrix} -2 & 0 \\ 4 & -7 \end{bmatrix}$$

Solution

$$= \begin{bmatrix} -2(3) & 0(3) \\ 4\,(3) & -7(3) \end{bmatrix} = \begin{bmatrix} -6 & 0 \\ 12 & -21 \end{bmatrix}$$

Example

$$-2\begin{bmatrix} 1 & -2 \\ 0 & 3 \\ -4 & 5 \end{bmatrix} + \begin{bmatrix} -4 & 5 \\ 6 & -8 \\ -2 & 6 \end{bmatrix}$$

Solution

$$\begin{bmatrix} -2 & 4 \\ 0 & -6 \\ 8 & -10 \end{bmatrix} + \begin{bmatrix} -4 & 5 \\ 6 & -8 \\ -2 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} -6 & 9 \\ 6 & -14 \\ 6 & -4 \end{bmatrix}$$

Example

A woman wanted to buy one sack of potatoes, three bunches of bananas and two basket of onion. She went to kikuyu market and found the prices as sh 280 for the sack of potatoes ,sh 50 for a bunch of bananas and sh 100 for a basket of onions. At kondelee market the corresponding prices were sh 300, sh 48 and sh 80.

2. Express the woman's requirements as a row matrix
3. Express the prices in each market as a column matrix
4. Use the matrices in (a) and (b) to find the total cost in each market

Solution

a. Requirements in matrix form is [1 3 2]

b.

Price matrix for Kikuyu market is $\begin{bmatrix}280\\50\\100\end{bmatrix}$

Price matrix for kondelee market $\begin{bmatrix}300\\48\\80\end{bmatrix}$

c. Total cost in shillings at Kikuyu Market is

$$(1\ 3\ 2)\begin{bmatrix}280\\50\\100\end{bmatrix} = (1 \times 280 + 3 \times 50 + 2 \times 100) = (630)$$

Total cost in shillings at Kondelee Market is;

$$(1\ 3\ 2)\begin{bmatrix}300\\48\\80\end{bmatrix} = (1 \times 300 + 3 \times 48 + 2 \times 80) = (604)$$

The two results can be combined into one as shown below

$$(1\ 3\ 2)\begin{bmatrix}280 & 300\\50 & 48\\100 & 80\end{bmatrix} = (630\ \ 604)$$

Note;

- The product of two matrices A and B is defined provided the number of columns in A is equal to the number of rows in B.
- If A is an m x n matrix and B is an n x p matrix, then the product AB is an m a p matrix.

  AXB=AB

  m X n n X p = m X p
- Each time a row is multiplied by a column

Example

Find AB if A = $\begin{bmatrix}-2 & 3\\1 & -4\\6 & 0\end{bmatrix}$ and B= $\begin{bmatrix}-1 & 3\\-2 & 4\end{bmatrix}$

Solution

Because A is a 3 x 2 matrix and B is a 2 x 2 matrix, the product AB is defined and is a 3 x 2 matrix. To write the elements in the first row and first column of AB, multiply corresponding elements in the first row of A and the first column of B. Then add. Use a similar procedure to write the other entries of the product.

$$AB=\begin{bmatrix}-2 & 3\\1 & -4\\6 & 0\end{bmatrix}\begin{bmatrix}-1 & 3\\-2 & 4\end{bmatrix}$$

$$=\begin{bmatrix}(-2)(-1) + (3)(-2) & (-2)(3) + (3)(4)\\(1)(-1) + (-4)(-2) & (1)(3) + (-4)(4)\\(-6)(-1) + (0)(-2) & (6)(3) + (0)(4)\end{bmatrix}$$

$$=\begin{bmatrix}-4 & 6\\7 & -13\\-6 & 18\end{bmatrix}$$

Identity Matrix

- For matrices, the identity matrix or a unit matrix is the matrix that has 1's on the main diagonal and 0's elsewhere.
- The main diagonal is the one running from top left to bottom right .It is also called leading or principle diagonal. Examples are;

$$I=\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix} \qquad\qquad I=\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}$$

2 X 2 identity matrix 3 x 3 identity matrix

- If A is any n x n matrix and I is the n x n identity matrix, then IA = A and AI = A.

Determinant Matrix

- The determinant of a matrix is the difference of the products of the elements on the diagonals.

Examples

The determinant of A, det A or |A| is defined as follows:

$$\text{If } n=2, \ \det A=\begin{vmatrix}a_{11} & b_{12}\\b_{21} & a_{22}\end{vmatrix}=a_{11}a_{22}-b_{12}b_{21}$$

Example

Find the determinant of $\begin{bmatrix}1 & 3\\2 & 5\end{bmatrix}$

Solution

Subtract the product of the diagonals

$1 \times 5 - 2 \times 3 = 5 - 6 = -1$

Determinant is -1

Inverse of a Matrix

Two matrices of order n x n are inverse of each other if their product (in both orders) is theidentity matrix of the same order n x n. The inverse of A is written as A-1

Example

Show that

$$B=\begin{bmatrix}3 & -1\\-5 & 2\end{bmatrix} \ A=\begin{bmatrix}2 & 1\\5 & 3\end{bmatrix}$$

Solution

$$AB=\begin{bmatrix}3 & -1\\-5 & 2\end{bmatrix}\begin{bmatrix}2 & 1\\5 & 3\end{bmatrix}$$

$$=\begin{bmatrix}2 \times 3+1 \times -5 & 2 \times -1+1+2\\5 \times 3+3 \times -5 & 5 \times -1+3 \times 2\end{bmatrix}$$

$$=\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}=I$$

$$BA=\begin{bmatrix}3 & -1\\-5 & 2\end{bmatrix}\begin{bmatrix}2 & 1\\5 & 3\end{bmatrix}$$

Note;

To get the inverse matrix

- Find the determinant of the matrix. If it is zero, then there is no inverse
- If it is non zero, then;
- Interchange the elements in the main diagonal
- Reverse the signs of the element in the other diagonals
- Divide the matrix obtained by the determinant of the given matrix

In summary

The inverse of the matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is

$$A^{-1} = \frac{1}{|A|}\begin{bmatrix} d & -b \\ -e & a \end{bmatrix} = \frac{1}{ad-cb} = \begin{bmatrix} d & -b \\ -e & a \end{bmatrix} \text{ provided } ad - cb \neq 0$$

Example

Find the inverse of

$A = \begin{bmatrix} 3 & 1 \\ 4 & 2 \end{bmatrix}$

Solution

$$A^{-1} = \frac{1}{6-4}\begin{bmatrix} 2 & -1 \\ -4 & 3 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 2 & -1 \\ -4 & 3 \end{bmatrix} = \begin{bmatrix} 1 & -\frac{1}{2} \\ -2 & \frac{3}{2} \end{bmatrix}$$

Check

You can check the inverse by showing that AA^{-1} = identity matrix

$$\begin{bmatrix} 3 & 1 \\ 4 & 2 \end{bmatrix}\begin{bmatrix} 1 & -\frac{1}{2} \\ -2 & \frac{3}{2} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \text{ And } \begin{bmatrix} 1 & -\frac{1}{2} \\ -2 & \frac{3}{2} \end{bmatrix}\begin{bmatrix} 3 & 1 \\ 4 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Solutions of Simultaneous Linear Equations Using Matrix Method

Using matrix method solve the following pairs of simultaneous equation

$x + 2y = 4$

$3x - 5y = 1$

$$\begin{pmatrix} 1 & 2 \\ 3 & -5 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 4 \\ 1 \end{pmatrix}$$

Solution

$\begin{pmatrix} 1 & 2 \\ 3 & -5 \end{pmatrix}$ is the cooeffients matrix of the simulteneou equations

$\begin{pmatrix} 4 \\ y \end{pmatrix}$ is the constants matrix

We need to calculate the inverse of $A = \begin{pmatrix} 1 & 2 \\ 3 & -5 \end{pmatrix}$

$$A^{-} = \frac{1}{(1)(-5)-(2)(3)}\begin{pmatrix} -5 & -2 \\ -3 & 1 \end{pmatrix} = -\frac{1}{11}\begin{pmatrix} -5 & -2 \\ -3 & 1 \end{pmatrix}$$

$$\text{Hence } A^{-}B = -\frac{1}{11}\begin{pmatrix} -5 & -2 \\ -3 & 1 \end{pmatrix}\begin{pmatrix} 4 \\ 1 \end{pmatrix}$$

$$= -\frac{1}{11}\begin{pmatrix} -22 \\ 11 \end{pmatrix}$$

$$= \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

Hence the value of x = 2 and the value of y = 1 is the solution of the simultaneous equation

$A=\begin{pmatrix}1 & 3\\ 5 & 3\end{pmatrix}$ $B=\begin{pmatrix}3 & 1\\ 5 & -1\end{pmatrix}$, $C=\begin{pmatrix}p & 0\\ 0 & q\end{pmatrix}$

$$\begin{pmatrix}x & 0\\ 5 & y\end{pmatrix}$$

Questions

1. A and B are two matrices. If $A=\begin{pmatrix}1 & 2\\ 4 & 3\end{pmatrix}$ find B given that A2 = A + B
2. Given that and AB =BC, determine the value of P

3. A matrix A is given

   by A = a. Determine

   A2

   b. If $A^2=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$ determine the possible pairs of values of x and y

4.

   a. Find the inverse of the matrix $\begin{pmatrix}9 & 8\\ 7 & 6\end{pmatrix}$

   b. In a certain week a businessman bought 36 bicycles and 32 radios for total of Kshs 227 280. In the following week, he bought 28 bicycles and 24 radios for a total of Kshs 1 74 960. Using matrix method, find the price of each bicycle and each radio that he bought

   c. In the third week, the price of each bicycle was reduced by 1 0% while the price of each radio was raised by 1 0%. The businessman bought as many bicycles and as many radios as he had bought in the first two weeks. Find by matrix method, the total cost of the bicycles and radios that the businessman bought in the third week.

5. Determine the inverse T-1 of the matrix $\begin{pmatrix}1 & 2\\ 1 & -1\end{pmatrix}$ Hence find the coordinates to the point at which the two lines x + 2y=7 and x-y=1

6. Given that $A=\begin{pmatrix}0 & -1\\ 3 & 2\end{pmatrix}$ and $B=\begin{pmatrix}1 & 0\\ 2 & -4\end{pmatrix}$ Find the value of x of

   i. A □ 2x = 2B

   ii. 3x □ 2A = 3B

   iii. 2A □ 3B = 2x

7. Find the non- zero value of k for which k $\begin{pmatrix}k+1 & 2\\ 4k & 2k\end{pmatrix}$ is an inverse.

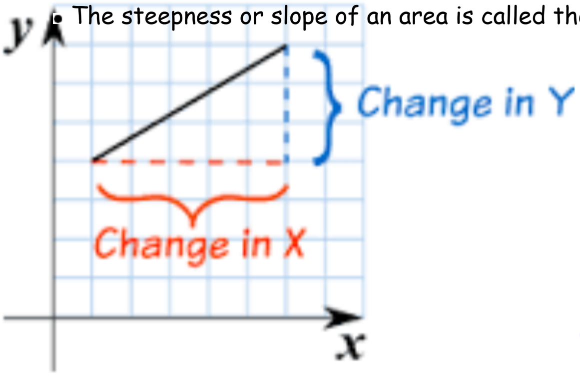
9. A clothes dealer sold 3 shirts and 2 trousers for Kshs. 840 and 4 shirts and 5 trousers for Kshs 1680. Form a matrix equation to represent the above information. Hence find the cost of 1 shirt and the cost of 1 trouser.

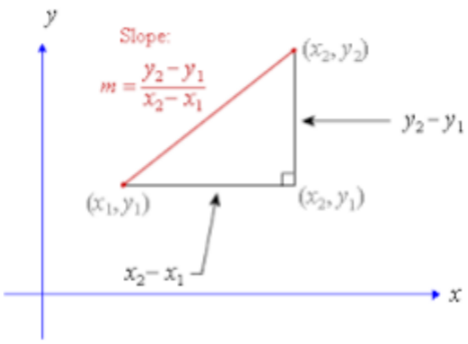
Equation of a Straight Line

Introduction

Gradient

- The steepness or slope of an area is called the gradient. Gradient is the change in y axis over the change in x axis.



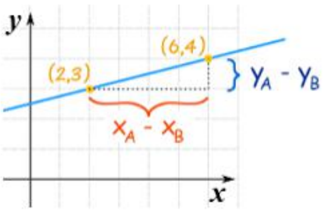


$$\frac{\text{change in y co-ordinates}}{\text{change in x co-ordinates}} = \frac{y_2 - y_1}{x_2 - x_1}$$

Note:

- If an increase in the x coordinates also causes an increase in the y coordinates the gradient is positive.
- If an increase in the x coordinates causes a decrease in the value of the y coordinate, the gradient is negative.
- If, for an increase in the x coordinate, there is no change in the value of the y coordinate, the gradient is zero.

For vertical line, the gradient is not defined



Solution

$$\text{Gradient} = \frac{\text{change in y axis}}{\text{change in x axis}}$$

$$= \frac{4-3}{6-2}$$

$$= {}^{1}/_{4}$$

QUESTIONS

(1). For each of the following pairs of points. find the change in the y coordinate and the corresponding change in the x coordinate. Hence, find the gradients of the lines passing through them;

a. A(2, 3), B(5, 6)

b. C(5,10), D(12, 20)

c. E(-5, 60), F(2,1)

d. G(4, 5), H(6, 5)

e. I(8, 0), J(12, -6)

f. K(5, -2), L(6,2)

g. M(6, 3), N(-6, +2)

h. P(2, -5), Q(2, 3)

(2). Find the gradients of the lines passing through the following pairs of points:

a. A(3, 2), B(-1,1)

b. C(7, 2), D(4, 3)

c. E(-1, -3), F(-2,-2)

d. G(0, 5), H(2, 5)

e. I($^1/_4$, $^1/_3$), J($^1/_3$, $^1/_4$)

f. K(0.5, 0.3), (L-0.2, -0.7)

(3). Find the gradient of each of the following lines:

a. $y = {}^1/_2 x + 3$

b. $3y - 4x = 5$

c. $y = -2x + 2$

d. $y + 2x - 3 = 0$

e. ${}^1/_3 x + {}^1/_4 y = {}^1/_{12}$

f. $y = -10$

Equation of a Line

Equation of a Straight Line.

Given Two Points

Example.

Find the equation of the line through the points A (1, 3) and B (2, 8)

Solution

The gradient of the required line is 8 – 3 = 5

2 – 1

Take any point p (x, y) on the line. Using... points P and A, the gradient is y – 3/ x – 1

Therefore y – 3 =5

x – 1

Hence y = 5x – 2

Given the Gradient and One Point on the Line

Example

Determine the equation of a line with gradient 3, passing through the point (1, 5).

Solution

Let the line pass through a general point (x, y).The gradient of the line is y – 5= 3 x – 1

Hence the equation of the line is y = 3x + 2

QUESTIONS

(1). Find the equations of lines with the given gradients and passing through the given points:

a. 4; (2, 5)

b. $^3/_4$; (-1, 3)

c. -2; (7, 2)

d. -$^1/_3$; (6, 2)

e. 0; (-3, -5)

f. $-{}^{3}/_{2}$; (0, 7)

g. m; (1, 2)

h. m; (a, b)

(2). Find the equation of the following line passing through the given points.

a. (0, 0) and (1, 3)

b. (0, -4) and (1, 2)

c. (0, 4) and (-1, -2)

d. (1,0) and (, 1)

e. (3, 7) and (5, 7)

f. (-1, 7) and (3, 3)

g. (11, 1) and (14, 4)

h. (5, -2.5) and (3.5, -2)

i. (a, b) and (c, d)

j. (x_1, y_1) and (x_2, y_2)

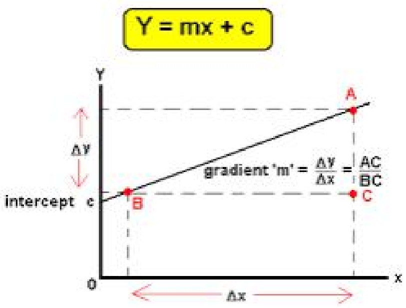
Linear Equation y = mx+c

We can express linear equation in the form y = mx + c.

Illustrations.

For example 4x + 3 y = −8 is equivalent to y= -4/3x - 8/3

In the linear equation below gradient is equal to m while c is the y intercept.



Using the above statement we can easily get the gradient.

Example

Find the gradient of the line whose equation is $3y - 6x + 7 = 0$

Solution

Write the equation in the form of $y = mx + c$

$3y = 6x - 7$

$y = 2x - {}^{7}/_{3}$

$m = 2$ and also gradient is 2.

<u>QUESTIONS</u>

(1). For each of the following straight lines, determine the gradient and the y-intercept. Do not draw the line:

a. $3y = 7x$

b. $2y = 6x + 1$

c. $7 - 2x = 4y$

d. $3y = 7$

e. $2y - 3x + 4 = 0$

f. $3(2x - 1) = 5y$

g. $y + 3x + 7 = 0$

h. $5x - 3y + 6 = 0$

i. ${}^{3}/_{2}y - 15 = {}^{2}/_{3}x$

j. $2(x + y) = 4$

k. ${}^{1}/_{3}x + {}^{2}/_{5}y + {}^{1}/_{6} = 0$

l. $-10(x+3) = 0.5y$

m. $ax + by + c = 0$

Graph of a Straight Line

Draw the graph of a line passing through (0, 4) and has a gradient of 2.

Solution

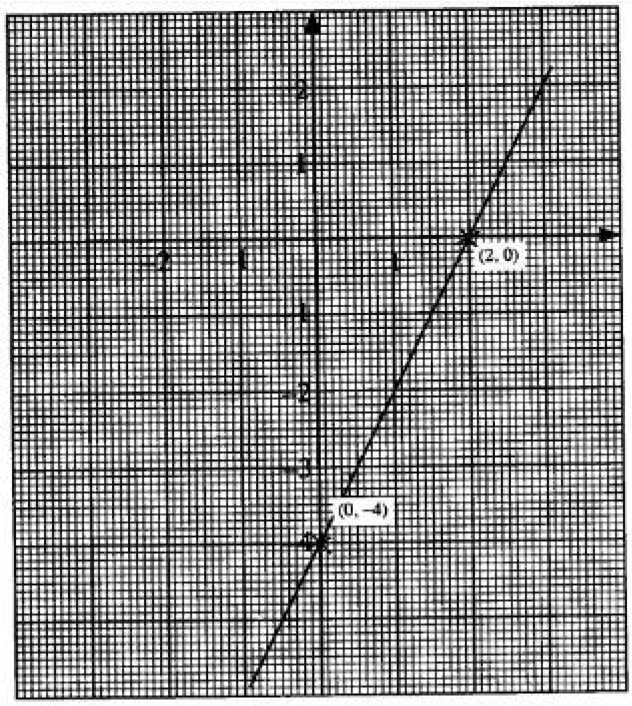
The equation of the line is;

$$\frac{y + 4}{x} = 2$$

y + 4 = 2x

y = 2x - 4

The x-intercept is 2, and the y-intercept is -4. the line cuts the x-axis at (2, 0). and the y-axis at (0, -4).



Questions

(1). Draw the lines passing through the given points and having the given gradients:

a. (0, 3); 3

b. (0, 2); 5

c. (4, 3); 2

(2). Draw the graph of the line passing through:

a. (5, 0) and the gradient is 2

b. (3,0); g=5

c. (2, 0); g= $^1/_3$

(3). Draw graphs of the lines represented by the following equations using the x and y-intercepts.

a. y = $^1/_2$ x + 3

b. 3y - 4x = 5

c. y + 2x - 3 = 0

d. y = -2x + 2

Perpendicular Lines

Perpendicular Lines

- If the products of the gradient of the two lines is equal to – 1, then the two lines are perpendicular to each other.

Example

Find if the two lines are perpendicular

$y = {}^{1}/_{3}\, x + 1;\ y = -3x - 2$

Solution

The gradients are

$M = {}^{1}/_{3}$ and $M = -3$

The product is

${}^{1}/_{3} \times -3 = -1$

The answer is -1 hence they are perpendicular.

Example

$Y = 2x + 7$

$Y = -2x + 5$

The products of their gradients is $2 \times -2 = -4$ hence the two lines are not perpendicular.

QUESTIONS

(1). A line L_1 passes through point (1, 2) and has a gradient of 5. Another line L_2, is perpendicular to L_1 and meets it at a point where $x = 4$. Find the equation for L_2 in the form of $y = mx + c$

(2). P (5, -4) and Q (-1, 2) are points on a straight line. Find the equation of the perpendicular bisector of PQ: giving the answer in the form $y = mx+c$.

(3). The equation of a line ${}^{-3}/_{5}x + 3y = 6$. Find the:

a. Gradient of the line

b. Equation of a line passing through point (1, 2) and perpendicular to the given line b

(5). Find the equation of the perpendicular to the line $x + 2y = 4$ and passes through point (2,1)

Parallel Lines

Parallel Lines

Parallel lines have the same gradients e.g.

- $y = 2x + 7$
- $y = 2x - 9$

Both lines have the same gradient which is 2 hence they are parallel

QUESTIONS

(1). In each of the following, find the equation of the line through the given point and parallel to the given line:

a. (0, 0); $y = {}^{2}/_{7}x + 1$

b. (3.5, 0); $x + y = 10$

c. (5, 2); $5y - 2x - 115 = 0$

d. (-3, 5); $7y = 3x$

e. $(^{-7}/_{3}, {}^{3}/_{4})$; $2(y - 2x) = 1.1$

f. (0, -3); $2x + y = 3$

g. $(3\ {}^{1}/_{7}, -1\ {}^{1}/_{7})$; $15(1 - x) = 22y$

h. (-3, 4); $x = 101$

i. (2, 3); $y = 0$

Inequalities

Representation

Introduction

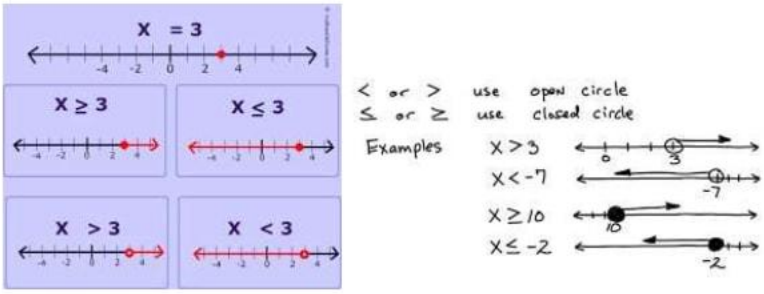
Inequality symbols

- $>$ Greater Than
- $\geq$ Greater Than or Equal To
- $<$ Less Than
- $\leq$ Less Than or Equal To

Statements connected by these symbols are called inequalities

Simple Statements

- Simple statements represents only one condition as follows



- x = 3 represents specific point which is number 3
- x >3 does not include 3 it represents all numbers to the right of 3 meaning all the numbers greater than 3 as illustrated above.
- x < 3 represents all numbers to left of 3 meaning all the numbers less than 3.
- The empty circle means that 3 is not included in the list of numbers to greater or less than 3.
- The expression x ≥ 3 or x ≤ 3 means that means that 3 is included in the list and the circle is shaded to show that 3 is included.

QUESTIONS

Illustrate each of the following inequalities on the number line:

(1).

a. x < 7

b. x > -3

c. $x \leq 0$

d. $x \leq -5$

(2).

a. $x < -10$

b. $x < -4$

c. $x \geq -6$

d. $x < 2.5$

(3).

a. $x \leq -{}^{1}/_{2}$

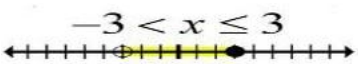
b. $x \leq -2.3$

Compound Statements

Compound Statements

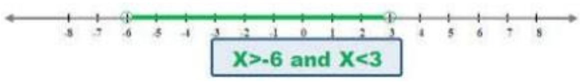
- A compound statement is a two simple inequalities joined by "and" or "or." Here are two

Examples.



- $3 \geq x$ and $x > -3$ Combined into one to form $-3 < x \leq 3$
- All real numbers that are greater than – 3 but less or equal to 3 • $x >$

-6 and $x < 3$ forms $-6 < x < 3$



- All real numbers that greater than - 6 but less than 3

QUESTIONS

(1). Write each of the following pairs of simple statements into compound statements and illustrate them on a number line.

a. $x > -4$, $x \leq 2$

b. $x < 0.5$, $x > 0$

(2). Write each of the following pairs of simple statements as a compound statement:

a. $x > 2$, $x < 5$

b. $x \geq 3$, $x < 6$

c. $x \geq 1$, $x \geq 7$

d. $x > -4$, $x \leq 0$

e. $x \geq -3$, $x \leq -1$

Solving Simple Inequalities

Solution to Simple Inequalities

Example

Solve the inequality

$x - 1 > 2$

Solution

Adding 1 to both sides gives;

$x - 1 + 1 > 2 + 1$

Therefore, $x > 3$

QUESTIONS

Solve each of the following inequalities and represent your solutions on a number line.

(1).

a. $2x + 4 > 10$

b. $3x - 5 < 2$

(2).

a. $5x + 3 > 4$

19. $3x - 4 \leq -13$

(3).

a. $3x - 7 \geq 5$

b. $1 - 4x \geq 9$

Multiple Inequalities with a Negative Number

Multiplication and Division by a Negative Number

• Multiplying or dividing both sides of an inequality by positive number leaves the inequality sign unchanged • Multiplying or dividing both sides of an inequality by negative number reverses the sense of the inequality sign.

Example

Solve the inequality $1 - 3x < 4$

Solution

$-3x - 1 < 4 - 1$

$-3x < 3$

$-3x > 3$

$-3 \; -3$

Note that the sign is reversed $x > -1$

QUESTIONS

Solve the inequalities below

a. $6 - {}^{1}/_{2}x > 12$

b. $3 - 2x < 17$

c. ${}^{1}/_{3} - 2x \leq -8\,{}^{1}/_{3}$

d. $3(1 - x) + 4(x + 3) \geq 30$

e. $2x + 3 < -1$

f. $-3x - 4 \geq 2$

Simple Simultaneous Inequalities

Simultaneous Inequalities

Example

Solve the following

$3x - 1 > -4$

$2x + 1 \leq 7$

Solution

Solving the first inequality

$3x - 1 > -4$

$3x > -3$

$x > -1$

Solving the second inequality

$2x + 1 \leq 7$

$2x \leq 6$ Therefore $x \leq 3$ The combined inequality is $-1 < x \leq 3$

<u>QUESTIONS</u>

Solve the following inequalities

(1).

a.

$x + 3 > 5$

$x - 4 < 4$

b.

$x + 10 \geq 6$

$x - 2 \leq 3$

(2).

a.

$-{}^{1}/_{2}x - 2 \leq 1$

$-3x - 9 > -6$

b.

$-5x + 7 < 12$

$^1/_3x + 2 \leq 5$

(3).

a.

$-7x - 1 < 6$

$^x/_3+1<^4/_3$

b.

$3x - ^1/_2 > 4$

$x - ^1/_5 < ^2/_5\, x + 1$

(4).

a.

$^x/_5+^1/_3<1$

$x - ^4/_5 > ^1/_8x$

b.

$5 \leq 3x + 2$

$3x - 14 < -2$

Compound Simultaneous Inequalities

Compound Simultaneous Inequalities

Simultaneous inequalities can as well be written as a compound statement.

For instance, $x - 4 < 4x < 4$ should be interpreted as:

i. $x - 4 < 4x$

ii. $4x < 4$

Solving the first inequality,

$x < 4x + 4$

$-3x < 4$

$x > ^4/_3$

Solving the second inequality

$4x < 4$

$x < {}^{4}/_{4}$

$x < 1$

The solution which satisfies both inequalities is, therefore; -

${}^{4}/_{3} < x < 1$

<u>QUESTIONS</u>

Solve the following inequalities;

(1).

a. ${}^{1}/_{2} - {}^{1}/_{4}x \leq x \leq 2$

b. $12 - x \geq 5 \geq 2x - 2$

(2).

a. $-4x < 6 \leq 180x$

b. $3x - 2 \geq -4 < -1 - 2x$

(3).

a. $6x - 13 \leq 17 < 8x - 7$

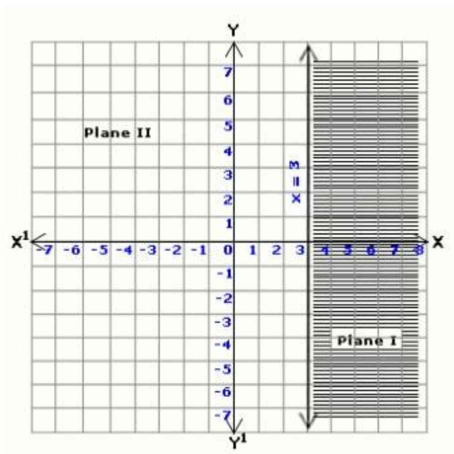
b. $2x + 3 > 5x - 3 > -8$

Graphs of Simple Inequalities

<u>Graphical Representation of Inequality</u>

Consider the following;

$x \leq 3$

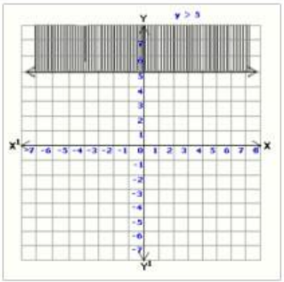


- The line $x = 3$ satisfy the inequality ≤ 3, the points on the left of the line satisfy the inequality. • We

don't need the points to the right hence we shade it

Note:

- We shade the unwanted region
- The line is continues because it forms part of the region e.g it starts at 3. For $\leq$ or $\geq$ inequalities the line must be continuous For < or > the line is not continuous, its dotted. This is because the value on the line **does not** satisfy the inequality.



<u>QUESTIONS</u>

Show the regions that satisfy each of the following inequalities on a squared paper:

(1).

a. $x \leq 4$

b. $x > -2$

(2).

a. $x < -1$

b. $y \leq 3$

(3).

a. $y \geq -4$

b. $y \leq 0$

(4).

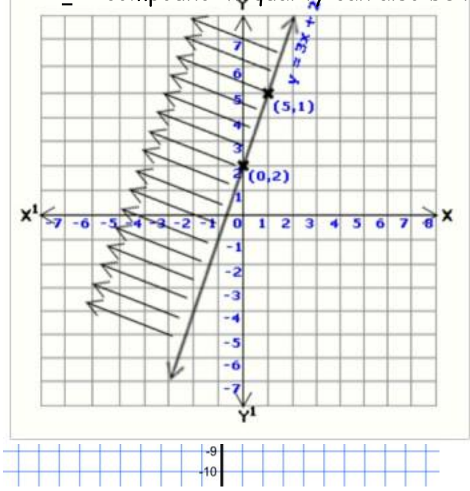
a. $y + 2 < -5$

b. $x + 2 \geq -1$

Graphs of Compound Inequalities

<u>Graphical Representation of Inequality</u>

- A compound inequality can also be represented graphically



QUESTIONS

Represent the following inequalities on the graph

(1).

a. $-1 \leq 3x - 1 < 5$

b. $\frac{x-3}{4} > \frac{x+5}{2}$

(2).

a. $\frac{1}{3} - \frac{1}{5}x < \frac{1}{2}x + 1$

b. $\frac{2}{3}x - 7 + \frac{1}{5}x \geq -1$

(3).

a. $\frac{1}{2}x + \frac{1}{2} > \frac{1}{4}$

b. $x^2 - 4x \geq x(x - 1) - 18$

Linear Inequality of Two Unknowns

Linear Inequality of Two Unknown

- Consider the inequality $y \leq 3x + 2$ the boundary line is $y = 3x + 2$

- If we pick any point above the line eg (-3, 3) then substitute in the equation $y - 3x \le 2$ we get $12 \le 2$ which is not true so the values lies in the unwanted region hence we shade that region .

QUESTIONS

Graph each of the following inequalities:

(1).

a. $2x + y > 3$

b. $x - y < 4$

(2).

a. $3x + 2y > 12$

b. $3y + x \le -5$

(3).

a. $y + 4x < 3$

b. $y - {}^{1}/{}_{2}x \ge 1$

(4).

a. $2x > y + 4$

b. ${}^{1}/{}_{6}x + {}^{1}/{}_{3}y \le {}^{1}/{}_{4}$

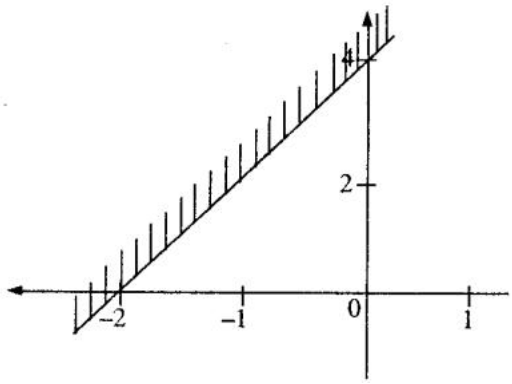
(5).

a. $2y - 3x - 5 \le 0$

b. ${}^{1}/{}_{6}x + {}^{1}/{}_{3}y \le -1$

Reading Inequalities from the Graph

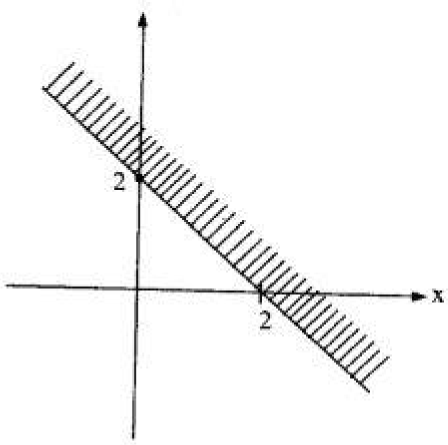
- Given a region satisfied by an inequality, the inequality can be found.

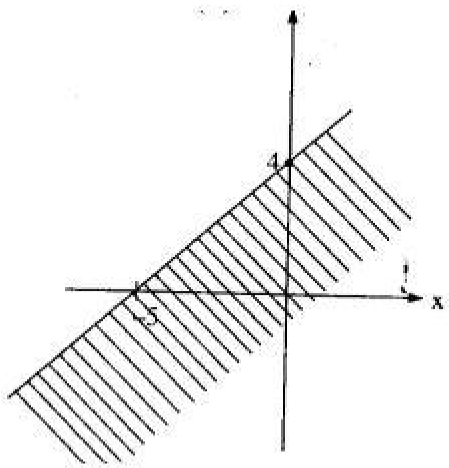


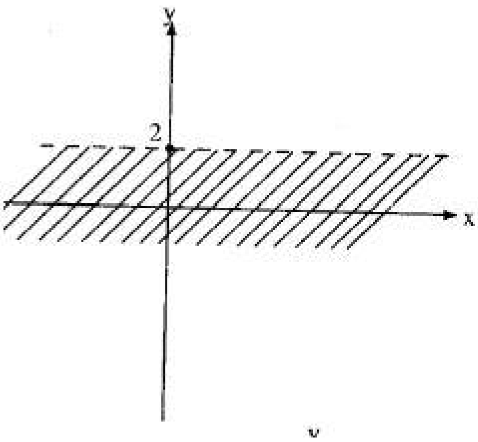
- The equation of the line is y = 2x + 4.
- Consider a point in the wanted region such as the origin (0, 0). The x co-ordinate and y co-ordinate are both zero. Substituting these values in the equation y = 2x + 4, we get zero on the left hand side and four on the right.
- Since 0 < 4 and the line y = 2x + 4 is continuous, the required inequality is y<2 x + 4

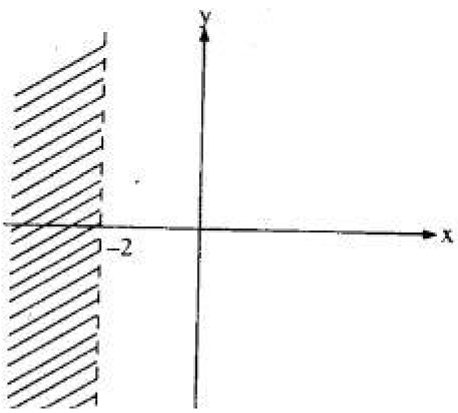
QUESTIONS

(1). Find the inequalities shown









Watch the video for this lesson on the link below.

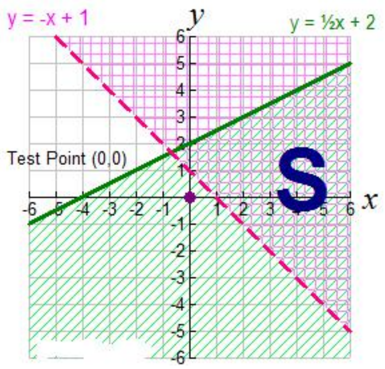
Intersecting Regions

Intersecting Regions

- These are identities regions which satisfy more than one inequality simultaneously.

Example

Draw a region which satisfy the following inequalities $y + x \geq 1$ and $y - 1/2x \geq 2$



QUESTIONS

In each of the questions, draw the regions which satisfy all the inequalities.

(1).

a.

$x + y \geq 0$

$x < 2$

$y > 0$

b.

$2x + y \geq 6$

$x < 3$

$y < 6$

(2).

a.

$4x - 3y \leq 12$

$x > 0$

$y > 0$

b.

$4x - 3y < 12$

$y \geq 0$

$y \leq 6$

STRAND 3: MEASUREMENT

Area

Unit Conversion of Area

Meaning of of area

- The area of a plane shape is the amount of the surface enclosed within its boundaries.
- It is normally measured in square units. For example, a square of sides 5 cm has an area of $5 \times 5 = 25$ cm^2 • A square of sides 1m has an area of 1m^2, while a square of side 1km has an area of 1km^2

Conversion of units of area

$$\begin{aligned} 1\ m^2 &= 1m \times 1m \\ &= 100\ cm \times 100\ cm \\ &= 10\ 000\ cm^2 \end{aligned}$$

$$\begin{aligned} 1\ km^2 &= 1\ km \times 1\ km \\ &= 1\ 000\ m \times 1\ 000\ m \\ &= 1\ 000\ 000\ m^2 \end{aligned}$$

$$\begin{aligned} 1\ area &= 10\ m \times 10\ m \\ &= 100\ m^2 \end{aligned}$$

$$1\ hectare\ (ha) = 100\ Ares = 10\ 000\ m^2$$

Example

Convert the following into the unit stated in bracket

2 m^2 (cm^2)

Solution

$$\begin{aligned} 2\ m^2 &= 2m \times 2m \\ &= 200\ cm \times 200\ cm \\ &= 40\ 000\ cm^2 \end{aligned}$$

Assignments

Convert each of the following into the units stated in the brackets:

(1). 6.8 ha (cm^2)

(2). 4 km^2 (m^2)

(3). 9000m^2 (ha)

(4). 300 cm^2 (m^2)

(5). 0.45 ha (m^2)

(6). 120 are (ha)

(7). 5000m^2 (km^2)

(8). 560 m^2 (cm^2)

(9). 0.02 km^2(m^2)

(10). 120,000 cm^2 (km^2)

Area of a Rectangle

AREA OF A RECTANGLE

- A rectangle is closed flat shape, having four sides, and each angle equal to 90 degrees. The opposite sides of the rectangle are equal and parallel.

Consider the figure below:

Width (W)

Length (L)

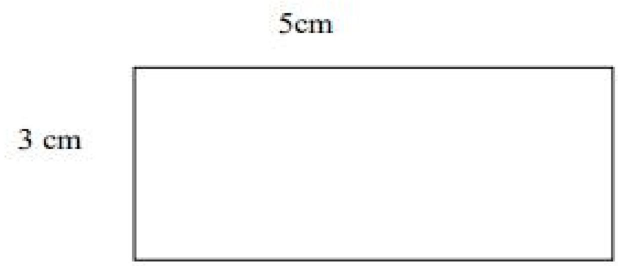
Formula

The area of a rectangle is given by:

Area = Length x Width

= L x W

Example



Area,

A = 5 x 3 cm

= $15c^2$

ASSIGNMENT

(1). The length of a rectangle is three times its breadth finds its area.

(2). A flower-bed measuring 3 m by 1.5 m is surrounded by a path 1 m wide. Find the area of the path.

(3). The length of a rectangle is twice its width. If its perimeter is 24 cm, what is the area of the rectangle?

(4). The length of a rectangle is three times its breadth. Find its area.

(5). What is the area in hectares of a rectangular ranch which is 50 km long and 15 km wide?

(6). A rectangular plot measures 100 m by 200 m find its area.

(7). A photograph measuring 14 cm by 10 cm is fixed inside a rectangular frame of dimensions 24 cm by 18 cm. What is the background area not covered by the mat.

(8). What is the area of a rectangular table top with a length of 130 cm and a width of 110 cm?

(9). A flower-bed measuring 3 m by 1.5 m is surrounded by a path 1m wide. Find the area of the path.

(10). A residential estate is to be developed on a 6-ha piece of land. If 1 500 m^2 is taken up by roads and the rest divided into 40 equal plots, what is the area of each plot?

Calculating the Area of a Triangle

Area of a Triangle

- A triangle is a three sided plane figure
- Area of a triangle is given by the formula;
- $A=\frac{1}{2}\times B\times H$
- A - Area of the triangle
- B - base length of the triangle
- h - height or altitude of the triangle

Example

The base of a rectangle is three times its height. Find its area given that its height is 4 cm

Solution

Base = 3(h)

20. 3 x 4

21. 12cm

4. $=\frac{1}{2}\times B\times H$

$=\frac{1}{2}\times 12\times 4$

=

$24cm^2$

Assignment

(1). A photograph measuring 14 cm by 10 cm is fixed inside a triangular frame of height 24 cm and base 18 cm. What is the background area of the space not covered by the photograph?

(2). A plot in the shape of right angled triangle 300 m by 400 m by 500 m. Find its area hectares.

(3). A triangular mat measuring 0.7 m by 2.4 m 2.5 m covers an area inside a floor measuring 14 m by 12 m. Find the area not covered by the mat.

(4). The height of a triangle is three times its base. If its area is 24 cm. what is the height of the triangle?

(5). The area of a right angled triangle whose sides are x, 2x and x - 5 cm is $24cm^2$. Find the exact value for the height of the triangle.

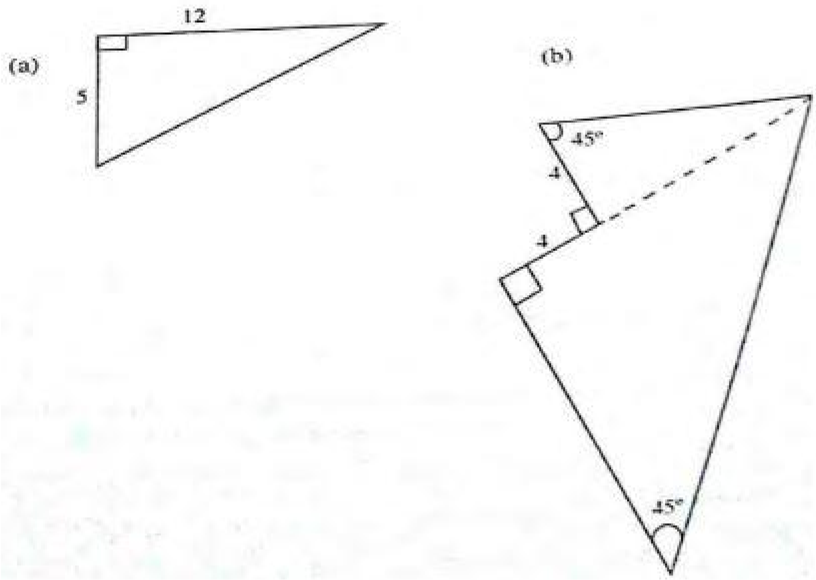
(6). The area of 10 similar triangular plots is 16 000 ares. Find the in metres the longest side of each plot given that one of the shorter sides is three times the other.

(7). The area of a right-angled triangle whose sides are x cm. 5 cm and 13 cm is 30 cm^3. Find the perimeter of the triangle

(8). A triangle has an area of 23 cm^2. Find the base given that its height 5 cm.

(9). The height a triangle is 23 cm and base is 25cm. Find the area of 23 such triangles

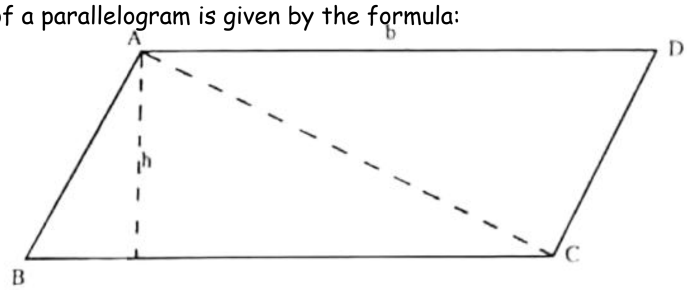
(10). Find the area of each of the following shapes.



Area of a Parallelogram

AREA OF A PARALLELLOGRAM.

The area of a parallelogram is given by the formula:



A = b × h

- Where ***b*** is the base of the parallelogram and ***h*** is the height.

Example

Find the area of a parallelogram with a base 6 cm and a height 4 cm.

Solution

Area of a parallelogram,

A = b × h

10. *6 (cm) × 4 (cm)*

= 24 cm^2

Assignment

(1). Find the area of a parallelogram with a base of 5 cm and a height of 3 cm.

(2). Determine the area of a parallelogram with a base of 12 m and a height of 6 m.

(3). A parallelogram has a height of 10.5 m and an area of 94.5 m^3. What is the length of the base?

(4). Calculate the area of a parallelogram with a base of 8 cm and a height of 2 cm.

(5). Given that the base of a parallelogram is 15 m and a height of 10 m. Find the area of a parallelogram.

(6). If the area of a parallelogram is 48 cm^2 and the base is 8 cm, what is the height?

(7). Determine the area of a parallelogram with a base of 7 cm and a height of 9 cm.

(8). Find the height of a parallelogram with an area of 286 cm^2 and a base of 22 cm

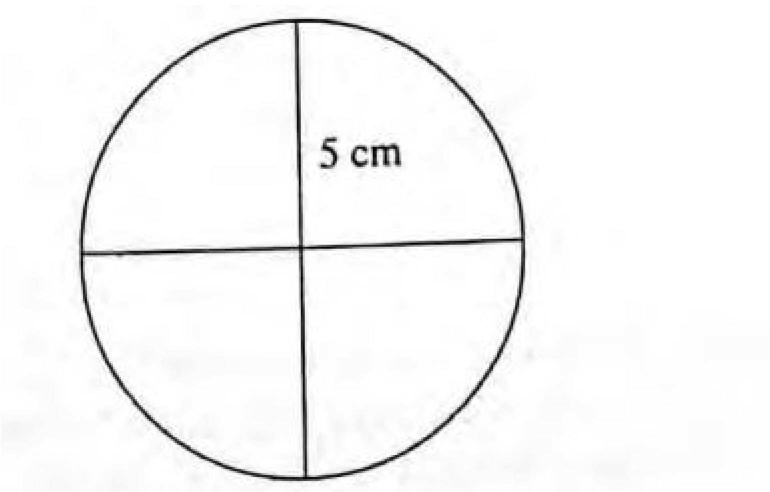
(9). The area of a parallelogram is 144 cm^2 and the base is 12 cm. What is the height of the parallelogram?

(10). If the area of a parallelogram is 350 cm^2 and the base is 14 cm, what is the height?

Area of a Circle

A circle

- A round plane figure whose boundary (the **circumference)** consists of points **equidistant** from a fixed point (the centre).



Area of a circle is given by the formula:

$A = \pi r^2$

Examples

Find the area of circle of radius 5cm

11. =

πr^2

=

$^{22}/$

$_7$

$\times 5^2$

=

78.

57c

m^2

Assignment

(1). A wire of length 44 cm is bent to form a complete circle, find the area of the circle.

(2). Find the area of a circle of diameter 14 cm.

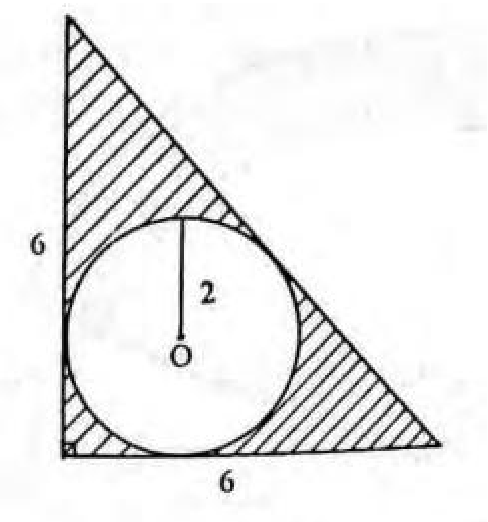
(3). The radius of a circle is quarter of its circumference. Find the area of the circle given that the circumference of the circle is 20 cm.

(4). The area of a circle is 38.5 cm^2. Find the radius of the circle. (Take $\pi = {}^{22}/_7$).

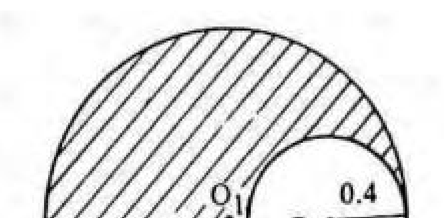
(5). An arc PQ of a circle of radius 15 cm subtends an angle of 160° at the centre of the circle.

(6). Find the area of a circle of radius 10 cm correct to 2 significant figures. (Take = 3.142).

(7). Find the area of the shaded region



(8). Find the area of the shaded region.



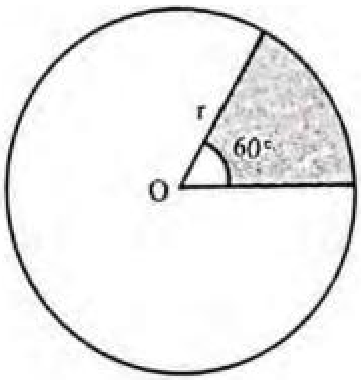
(9). Find the area of grass watered by a sprinkler which is capable of spraying to a maximum distance of 10 m.

(10). A goat is tethered to a post by a rope 6.3 m long. Find its maximum grazing area.

Area of a Sector

A sector :

- The plane figure enclosed by two **radii** of a circle and the arc between them.



Area of a sector is given by the formula:

$$A = \frac{\theta}{360} \pi r^2$$

Examples;

Find the area of the sector of a circle of radius 3 cm if the angle subtended at the centre is 140°. (Take $\pi = {}^{22}/_{7}$)

$$A = \frac{\theta}{360} \pi r^2$$

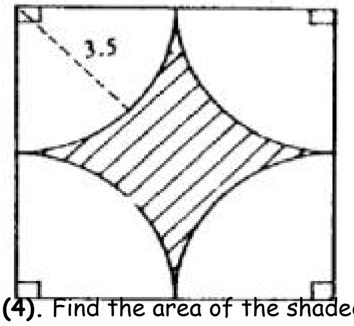
20. ${}^{140}/_{360} \times {}^{22}/_{7} \times 3 \times 3$
21. $11\ cm^2$

Assignment

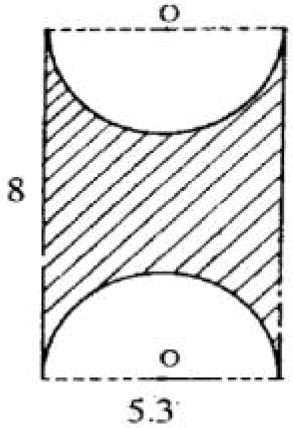
(1). The area of a sector of a circle is 38.5 cm^2. Find the radius of the circle if the angle subtended at the centre is 90° . (Take $\pi = {}^{22}/_{7}$)

(2). The area of a sector of a circle radius 63 cm is 4 158 cm^2. Calculate the angle subtended at the centre of the circle. (Take $\pi = {}^{22}/_{7}$)

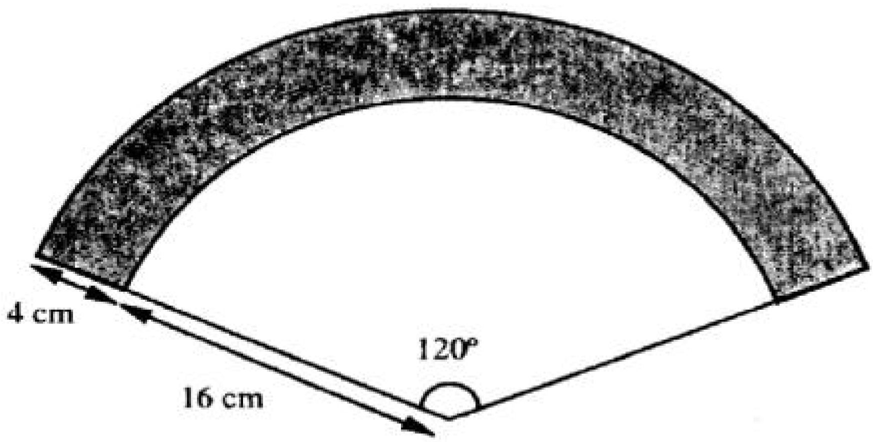
(3). Find the area of the shaded area.



(4). Find the area of the shaded region in the figure below.



(5). The shaded region in the figure below shows the area swept out on a flat windscreen by a wiper. Calculate the area of the region.



(6). Find the difference between the area swept out by the minute hand of a clock which is 3.6 cm long and the hour hand which is 2.9 cm long between a duration of 15 minutes.

(7). The length of a minute hand of a clock is 3.5 cm. Find the angle it turns through if it sweeps an area of 4.8 cm^2. (Take $\pi = {}^{22}/_{7}$)

(8). The perimeter of a quadrant of a circle is 50 cm^2. Find the area of the quadrant (a quadrant is a quarter of a circle). (Take $\pi = {}^{22}/_{7}$)

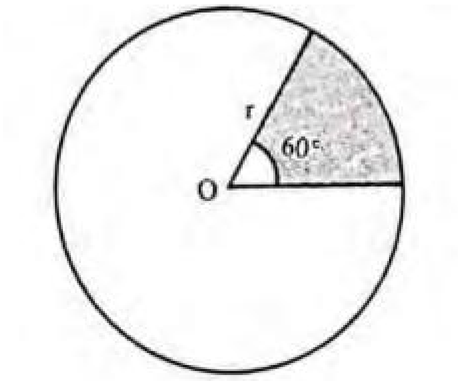
(9). The two arms of a pair of divider are spread so that the angle between them is 45°. Find the area of the sector formed if the length of each arm is 8.4 cm. (Take $\pi = {}^{22}/_{7}$)

(10). An arc PQ of a circle of radius 15 cm subtends an angle of 160° at the centre of the circle. Find the area of the sector formed by the arc PQ correct to 2 significant figures. (Take = 3.142).

Real Life Application of the Area of a Sector

A sector

- The plane figure enclosed by two **radii** of a circle and the arc between them.

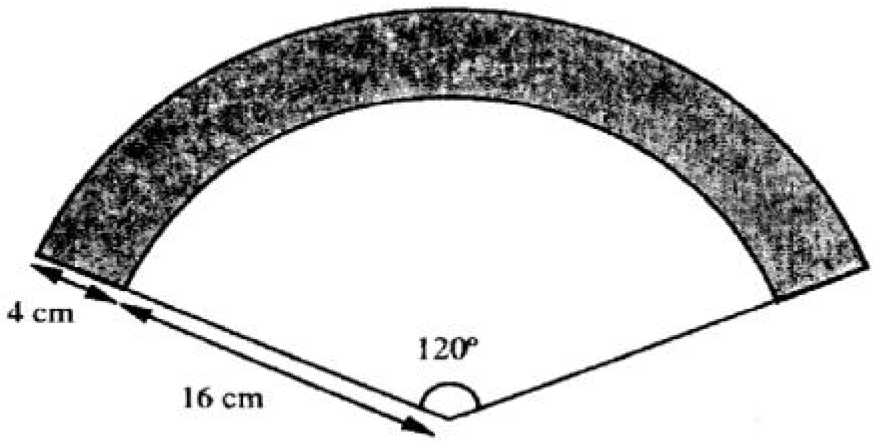


Area of a sector is given by the formula;

$$A = \frac{\theta}{360}\pi r^2$$

Example

Find the area of the shaded region.



A=A1–A2

$$\frac{\theta}{360}\pi R^2 - \frac{\theta}{360}\pi r^2$$

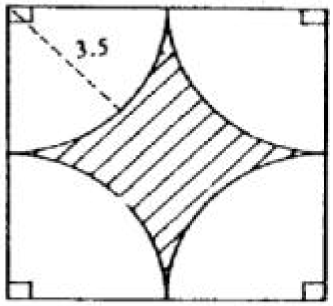
22. $^{120}/_{360} \times {}^{22}/_{7} \times 20 \times 20 - {}^{120}/_{360} \times {}^{22}/_{7} \times 16 \times 16$

23. 419.047619 - 268.190476

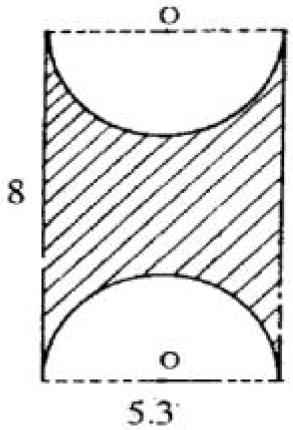
24. $150.8571cm^2$

Assignment

(1). Find the area of the shaded area.



(2). Find the area of the shaded region in the figure below.



(3). Find the difference between the area swept out by the minute hand of a clock which is 3.5 cm long and the hour hand which is 4.9 cm long between a duration of 30minutes.

(4). The length of a minute hand of a clock is 3.5 cm. Find the angle it turns through if it sweeps an area of 44.8 cm^2. (Take $\pi = {}^{22}/_{7}$)

(5). The two arms of a pair of divider are spread so that the angle between them is 75°. Find the area of the sector formed if the length of each arm is 10.5 cm. (Take $\pi = {}^{22}/_{7}$)

(6). The area of the sector of a circle is 138.5 cm^2 . Find the radius of the circle if the angle subtended at the centre is 50°. (Take $\pi = {}^{22}/_{7}$).

(7). The perimeter of a quadrant of a circle is 50 cm. Find the radius of the quadrant (a quadrant is a quarter of a circle). Find the area of quadrant.

(8). The length of a minute hand of a clock is 10.5 cm. Find the angle it turns through if it sweeps an area of 4.8 cm^2. (Take $\pi = 3.142$)

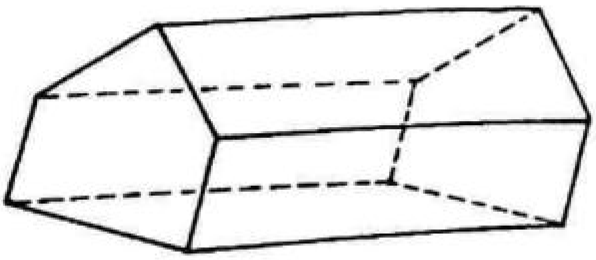
(9). The difference between the area swept out by the minute hand of a clock is 3.6 cm^2. How long is the hour hand given the that the minute hand is 2.9 cm long and they both turn through an angle of 30°.

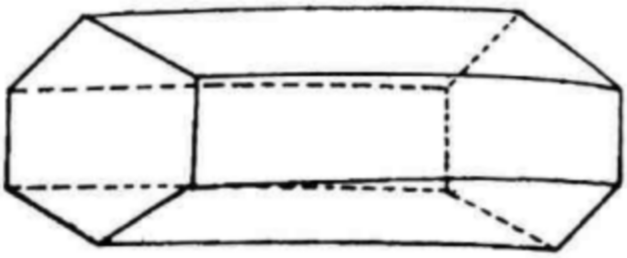
(10). Two equal sheets of metal in the shape of sectors and with radii 0.7 m are cut out from a rectangular sheet measuring 2 m by 3 m. Find the area of the remaining sheet. Given that the angle between their radii is 45°

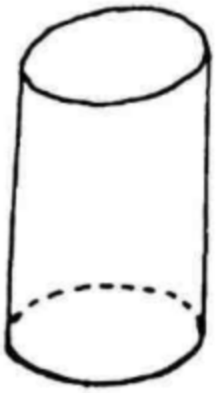
Surface Area of a Prism

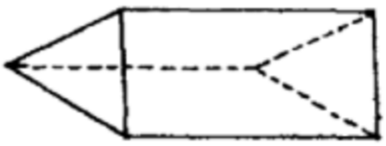
A PRISM

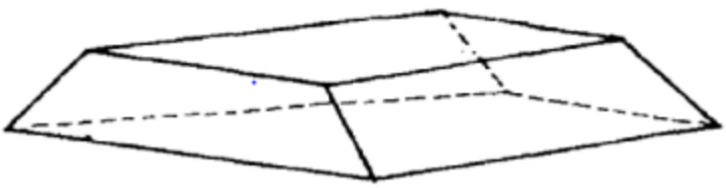
- A solid geometric figure whose two ends are similar, equal, and parallel rectilinear figures, and whose sides are parallelograms.









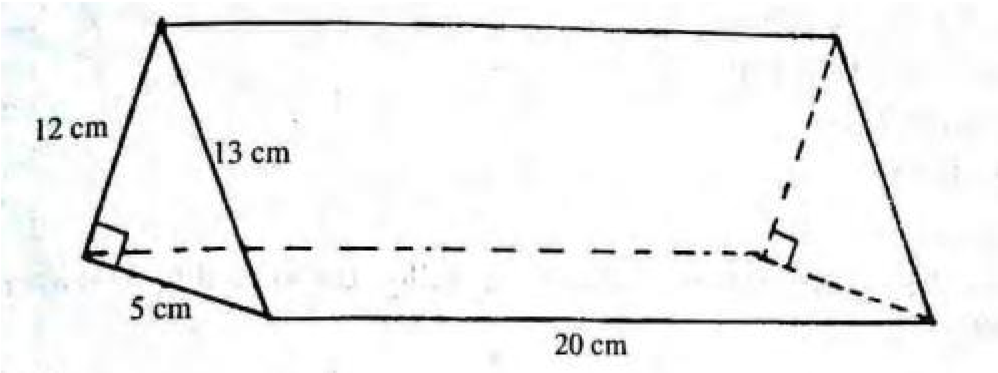


Total surface area of a prism is given by the formula:

A = 2(Cross sectional area) + Perimeter of cross section x Length of the prism.

Examples

Find the total surface of the triangular prism below.



Solution

Cross sectional area = $\frac{1}{2} \times 5 \times 12$

= 30 cm^2

Perimeter of cross section

25. 5+12+13
26. 30 cm

22. $2(30) + 30 \times 20$

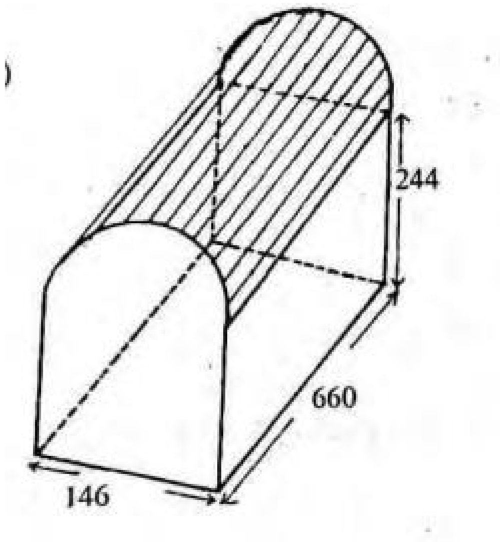
23. $60 + 600$

24. 660 mcm^2

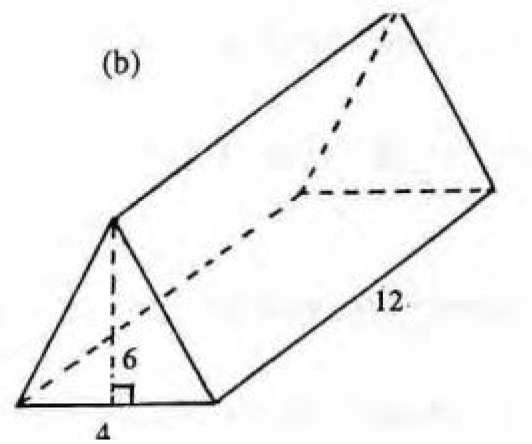
Assignment

(1). A right angled triangular prism has length 3 m, breadth 2 m and height 2.5 m. Find the total surface area of the prism.

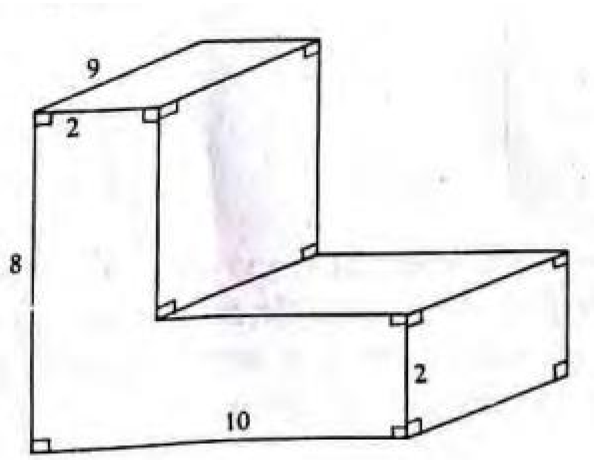
(2). Find the total surface area of the prism below.



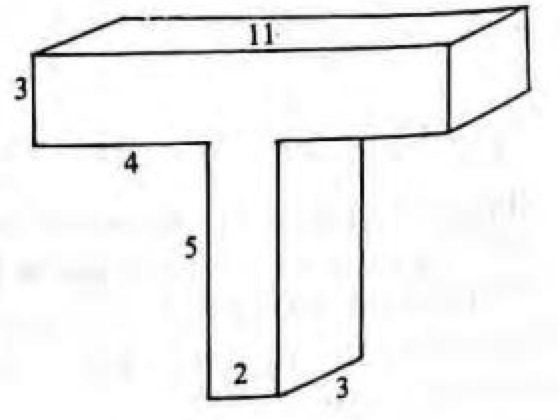
(3). Find the surface area of the prism



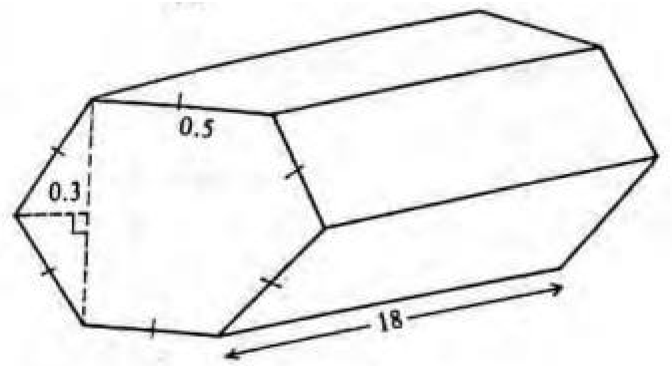
(4). Find the surface area of the prism.



(5). What is the surface area of the solid below.



(6). Determine the surface area of the prism below.



(7). Find the surface area of building block measuring 9 cm by 12 cm by 4 cm.

(8). What is the surface area of a rectangular eraser which measures 2.3 cm by 2 cm by 0.5 cm?

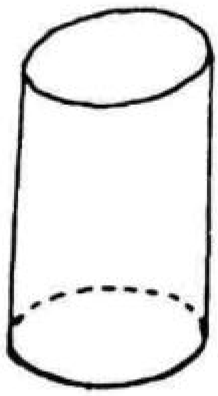
(9). An open chalk box is 15 cm long, 12 cm wide and 10 cm high. Find its external surface area.

(10). Find the surface area of a triangular prism of length 25 cm, height 4.5 cm and base 6 cm.

Surface Area of a Cylinder

A CYLINDER

- Cylinder is one of the basic 3d shapes, in geometry, which has two parallel circular bases at a distance. The two circular bases are joined by a curved surface, at a fixed distance from the center.



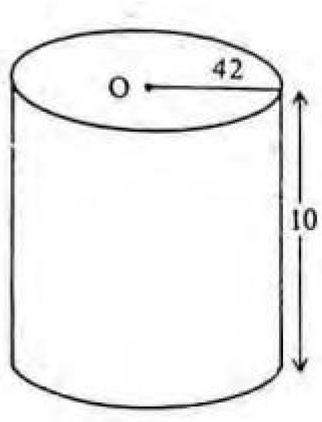
Total surface area of a cylinder is given by the formula:

A = 2(Cross sectional area) + Perimeter of cross section Length of the prism.

$= 2r^2 + 2rh$

Examples

Find the total surface of the closed cylinder below.



Solution

$A = 2r^2 + 2\pi rh$

(vi) $2 \times {}^{22}/_{7} \times 42^2 + 2 \times {}^{22}/_{7} \times 42 \times 10$

(vii) $11088 + 2640$

(viii) $13720cm^2$

<u>**Assignment**</u>

(1). A cylindrical water-tank with no top was constructed at a dining hall corner. If the diameter of the tank was 2.8 m and height 4.8 m, what was its surface area?

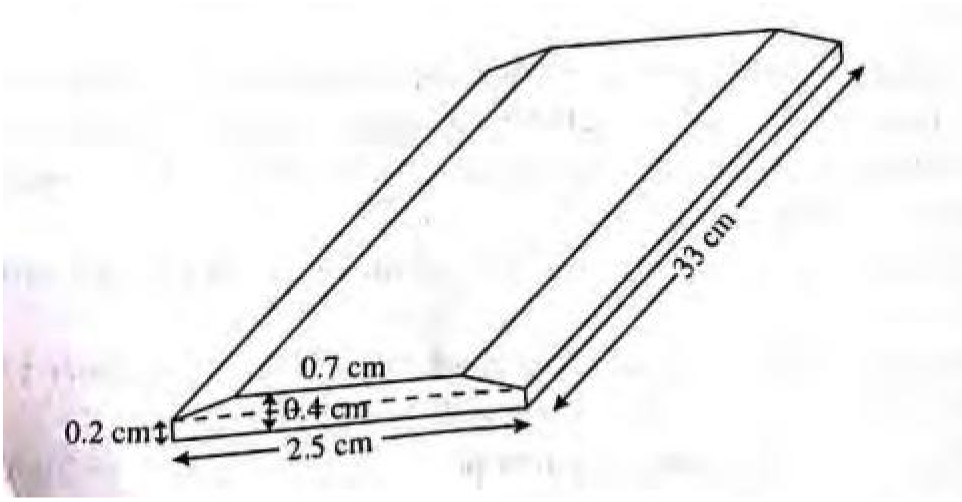
(2). Find the number of revolutions made by a roller of diameter 1.02 m and thickness 1.3 m if it rolls over a surface of $291.72\ m^2$.

(3). Calculate the thickness of a disc of diameter 14 cm and surface area 352 cm^2.

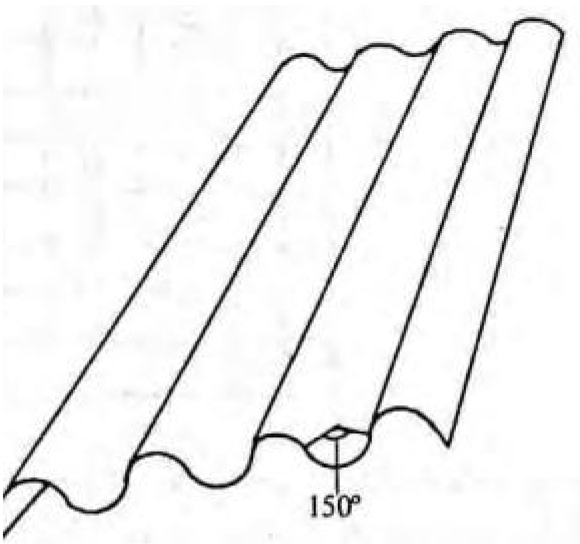
(4). Two metallic pipes, each of length 3 m and external diameter 10 cm, are used as netball posts. Find their total external surface area.

(5). The diameter of a cylindrical unsharpened pencil is 8 mm and its length is 17.5 cm. Calculate its surface area.

(6). The Figure below shows cross-section of a ruler which is a rectangle of 2.5 cm by 0.2 cm on which is surmounted an isosceles trapezium (one in which the non-parallel sides area of equal length). The shorter of the parallel sides of the trapezium is 0.7 cm long. If the greatest height of the ruler is 0.4 cm and it is 33 cm long, calculate its surface area.



(7). Figure 13.24 shows a corrugated iron sheet made of sections, each of which is the minor are of a circle of radius 4.2 cm subtending an angle of 150° at the centre of the circle. If there are 50 sections and the sheet is 2 m long, calculate the area of the curved top surface of the sheet.



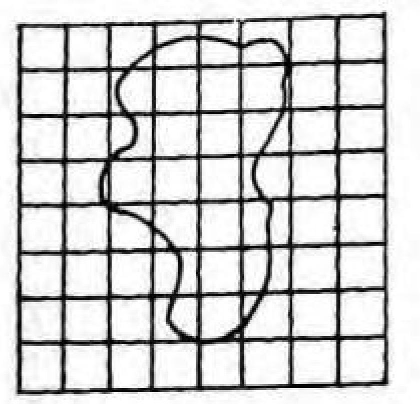
(8). A solid block in the shape of a cylinder has a height of 14 cm and a radius of 10cm, find the total surface area of the cylinder. (Take = $^{22}/_{7}$)

(9). A cylindrical container of diameter 14 cm and depth 20 cm is half full of juice. Calculate the area of the container not in contact with juice .

(10). The diameter of a cylindrical container, closed at both ends is 0.28 m and its height is 14 m. Find its surface area.

Area of Irregular Shapes

Area of Irregular Shapes



Area of irregular figure cannot be found accurately, but it can be the area of an estimated as follows:

- Draw a grid of unit squares on the figure or copy the figure on such a grid. As indicated in the figure below.
- Count all the unit squares fully enclosed within the figure.
- Count all the partially enclosed unit squares and divide the total by two, i.e., treat each one of them as half of a unit square.
- The sum of the numbers in (2) and (3) gives an estimate of the area of the figure.

From the figure,

The number of full squares is 9.

Number of partial squares = 18

Total number of squares = $9 + \frac{1}{2} \times 18$

7. 9 + 9

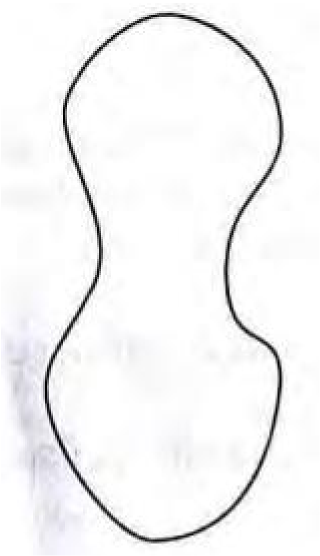
8. 18 Square units

Assignment

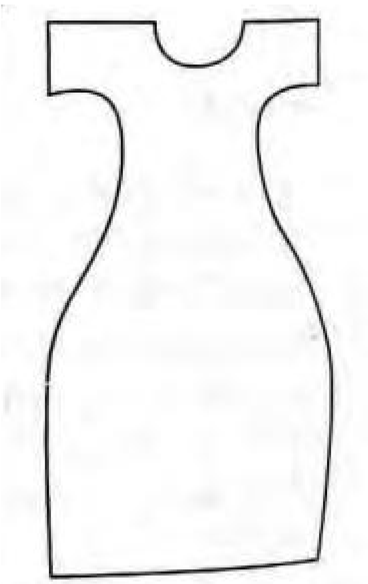
(1). Trace the outline of the palm of your hand on a graph paper and estimate its area and estimate the area of the outline using the counting technique.

(2). Trace the outline for your foot and estimate the area of the outline using the counting technique.

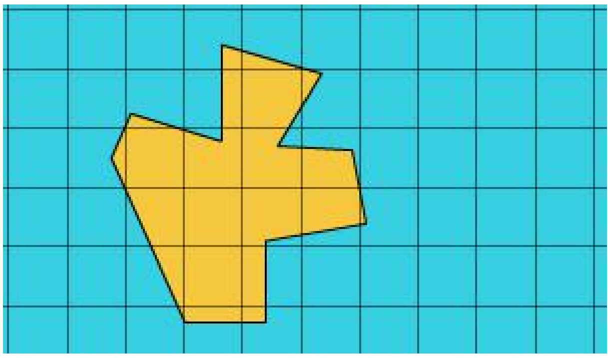
(3). Estimate the area of the shape below.



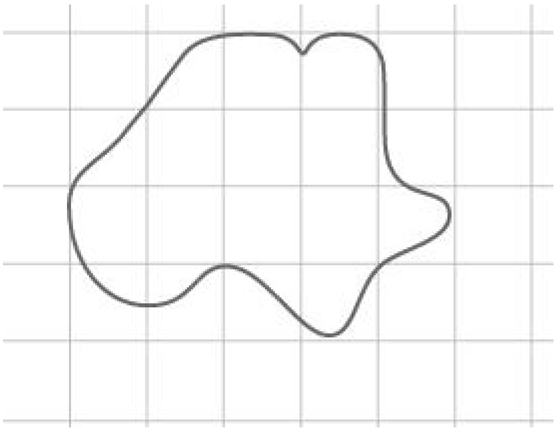
(4). Estimate the area of the figure below by drawing a grid of unit square on the figure below.



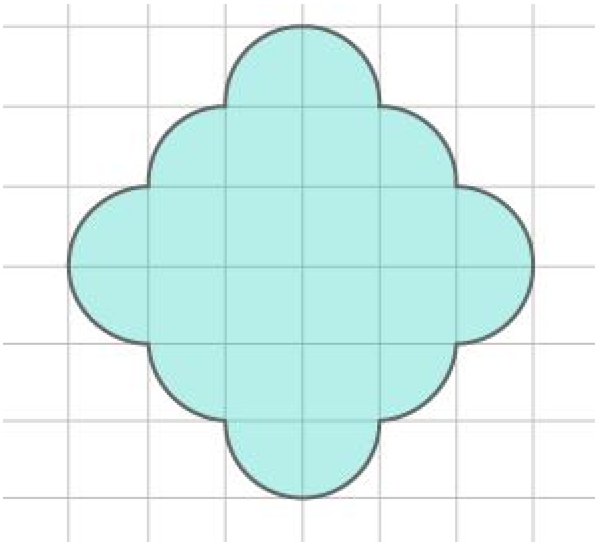
(5). Estimate the area of the outline below.



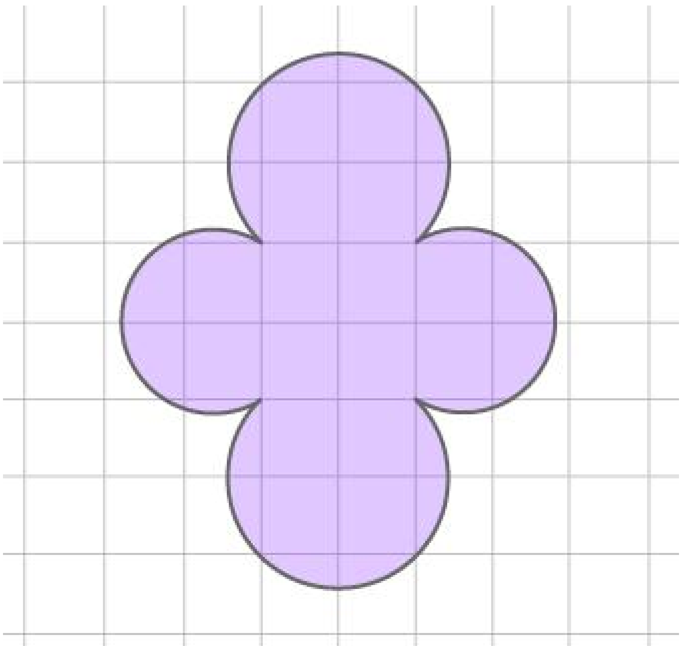
(6). Find the area of the outline in the figure below.



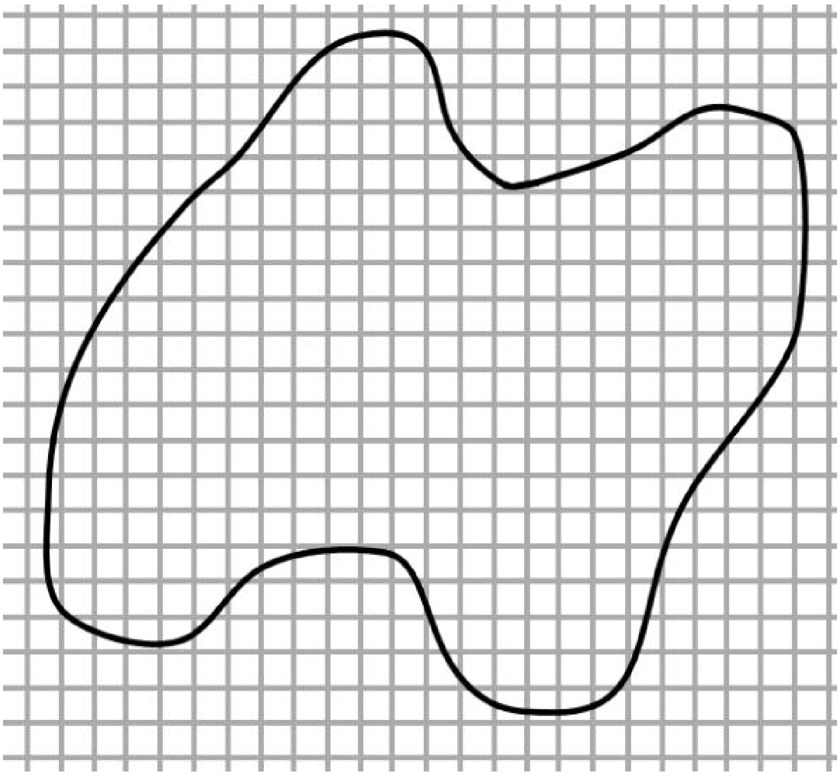
(7). Estimate the area of the figure below.



(8). Estimate the area of the outline.



(9). Find the area of the figure in the outline.



Common Solids

Introduction to Common Solids

Introduction to Common Solids.

- A solid is an object which occupies space and has a definite or fixed shape. Solids are either regular or irregular.

Definition of Terms.

Faces:

- A face is a flat surface of a three-dimensional object.
- It is a polygon that forms one of the sides of the solid shape.

Vertices (Vertex - singular):

- A vertex is a point where edges meet in a three-dimensional object.
- It is the intersection point of three or more edges.
- The plural form is vertices.

Edges:

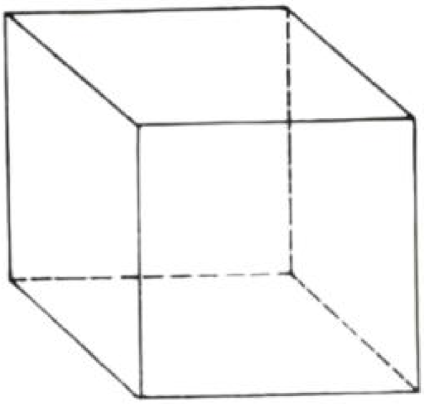
- An edge is a line segment where two faces of a solid shape meet.
- It is the connection between vertices.

All solids have surfaces. Some have faces, edges and vertices. Such solids are called ***polyhedra (singular polyhedron)***.

Types of Solids.

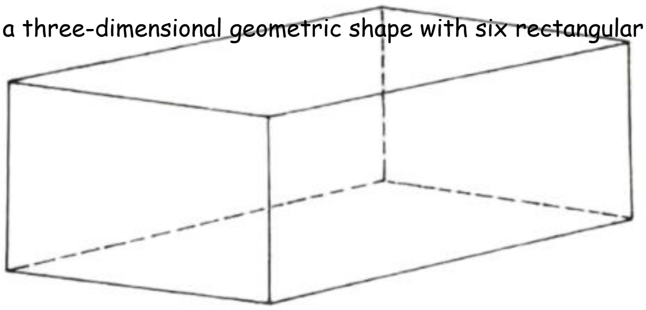
(1). Cube

- A cube is a three-dimensional geometric shape that has six square faces, twelve straight edges, and eight vertices.



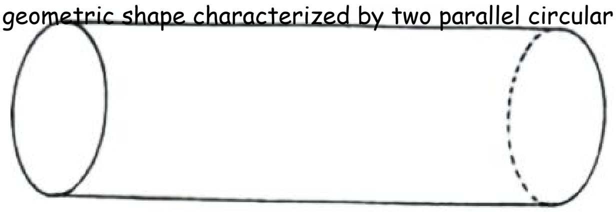
(2). Cuboid.

- A cuboid is a three-dimensional geometric shape with six rectangular faces, twelve straight edges, and eight vertices.



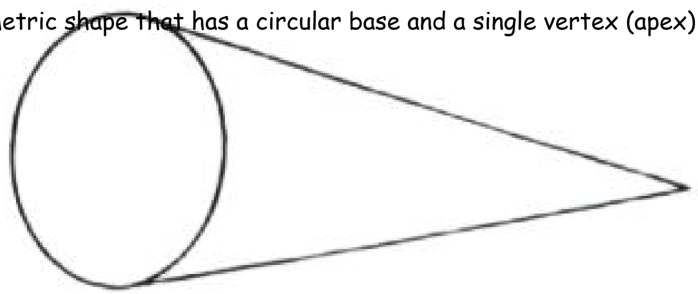
(3). Cylinder.

- A cylinder is a three-dimensional geometric shape characterized by two parallel circular bases of equal size, connected by a curved surface.



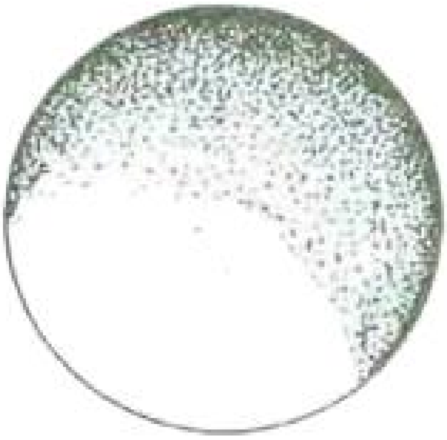
(4). Cone.

- A cone is a three-dimensional geometric shape that has a circular base and a single vertex (apex) located above the center of the base.



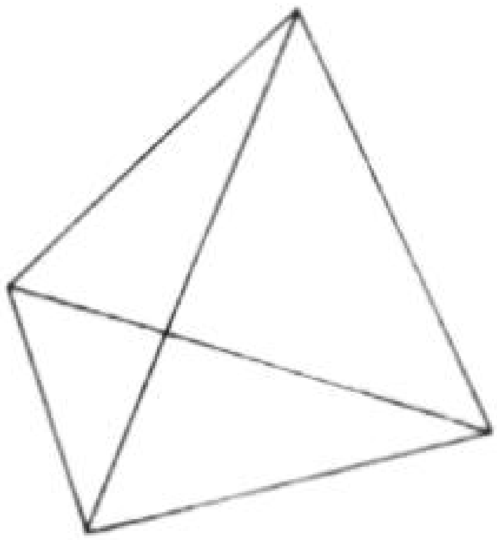
(5). Sphere.

- A sphere is a three-dimensional geometric shape that is perfectly round and symmetrical. It is defined as the set of all points in space that are equidistant from a common center.



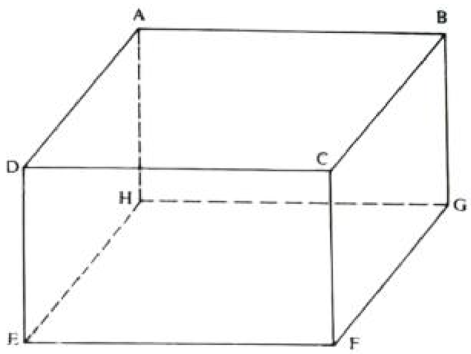
(6). Tetrahedron.

- A tetrahedron is a three-dimensional geometric shape with four triangular faces, four vertices, and six edges. Each triangular face of a tetrahedron is adjacent to every other face, and the faces are equilateral triangles.



Example.

(1). The figure below shows a cuboid ABCDEFGH. How many faces, edges and vertices does it has.



Solution

Faces = 6

Edges = 12

Vertices = 8.

<u>Assignment</u>

How many faces, edges and vertices does:

(1). a cuboid has?

(2). a triangular prism has?

(3). a cone has?

(4). a Tetrahedron has?

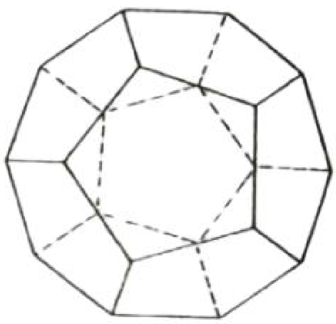
(5). a sphere has?

(6). Name some common solid that have no vertices.

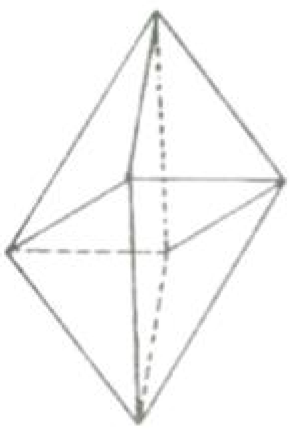
(7). Name a common solid that has neither a vertex nor an edge. How many faces does that solid have?

- *Polyhedra are named according to the number of faces they have. State the number of faces in the figure below.*

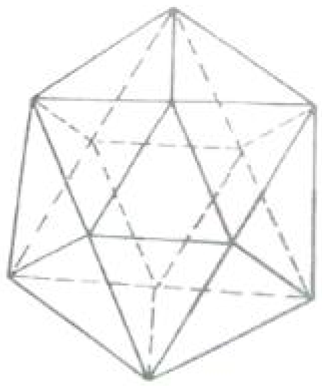
(8).



(9).



(10).



Sketching of Solids

Sketching of Solids.

- To draw a reasonable sketch of a solid on a plain paper, the following ideas are helpful:

(1). Use of Isometric Projection.

- In this method, the following points should be observed:

22. Each edge should be drawn to the correct length.

23. All rectangular faces must be drawn as parallelograms.

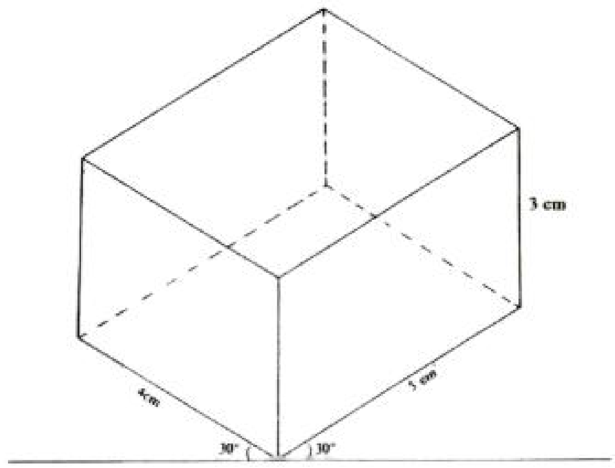
24. Horizontal and vertical edges must be drawn accurately to scale.

25. The base edges are drawn at an angle of 30° with the horizontal lines.

26. Parallel lines are drawn parallel.

Example.

(1). An isometric projection of a cuboid 5 cm long, 4 cm wide and 3 cm high is shown in figure below.



(2). The use of Perspective Projection

- In this method, solids are drawn bearing the following points in mind:

5. Parallel lines are not drawn parallel. Horizontal parallel lines appear to

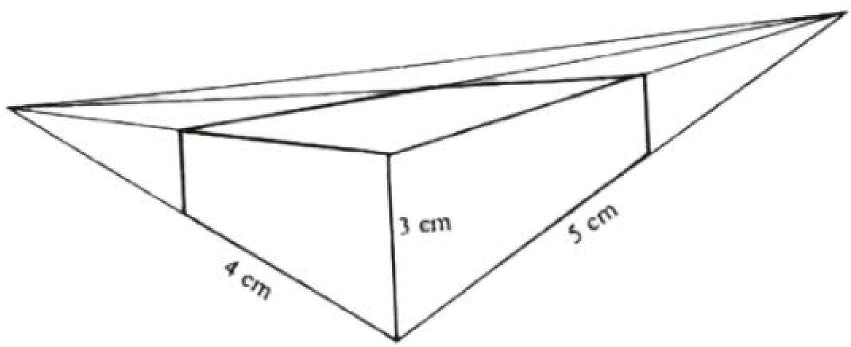
   meet at a vanishing point.

6. Vertical lines are drawn vertical.

(iii) For a front view of a solid, the measurements of the visible face are accurately drawn to scale.

Example

A perspective projection of a cuboid 5 cm and 4 cm by 3 cm is shown in figure below.



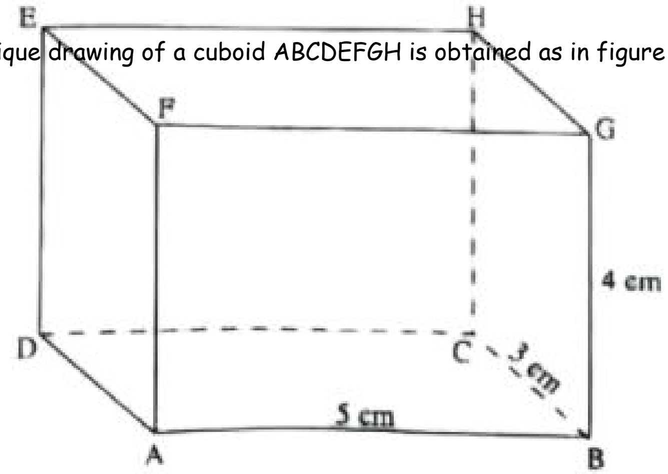
(3). Oblique Projection

12. Draw the horizontal line AB = 5 cm.
13. Draw the vertical line AF = 4 cm.
14. Draw AD and BC reduced to about of their true lengths, so that they make angles of 45° with AB.
15. Draw the vertical lines BG, CH and DE accurately.

16. Join EF, FG, GH and HE.

Example

An oblique drawing of a cuboid ABCDEFGH is obtained as in figure below in which AB = 5 cm, BC = 3 cm and BG =



4 cm.

Assignment

(1). Draw an isometric projection of a pyramid 7 cm high on a square base of side 4 cm.

(2). A water tank is in the shape of a cuboid 3 m long, 2 m long wide and 3 m high. Draw:

(3). An isometric projection of the tank using a scale of 2 cm for 1 m.

(4). An oblique projection of the tank.

(5). A perspective drawing of the tank.

(6). Draw an oblique projection of a cube of edge 4 cm.

(7). Make a perspective drawing of a rail tunnel.

(8). Make a perspective drawing of a classroom door half open, as viewed from outside.

(9). Draw an oblique view of a long line of coffee trees, showing the vanishing points clearly.

(10). A pyramid 8 cm high on a square base of side 3 cm. Draw its isometric projection.

Nets of Solids

Nets of Solids.

- A geometry net is a two-dimensional shape that can be folded to form a three-dimensional shape or a solid. When the surface of a three-dimensional figure is laid out flat showing each face of the figure, the pattern obtained is the net.
- Infinite patterns like nets of models are called tessellations.

- A regular tessellation is a pattern of congruent regular polygons, all of one kind, filling a whole space, e.g., a squared paper. Tessellations are therefore widely used in the construction of models of solids.

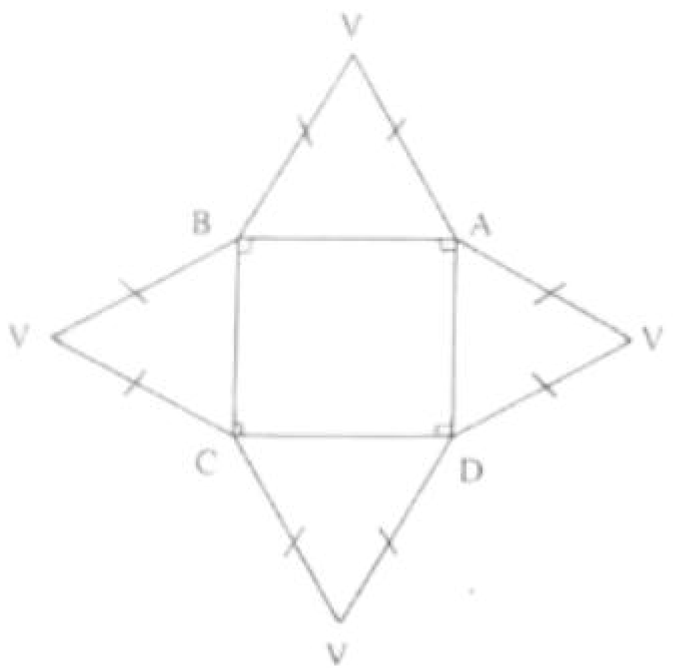
Example.

(1). The figure below shows a sketch of a cardboard model of a right pyramid on a square base. Draw the net of the shape formed.

Solution

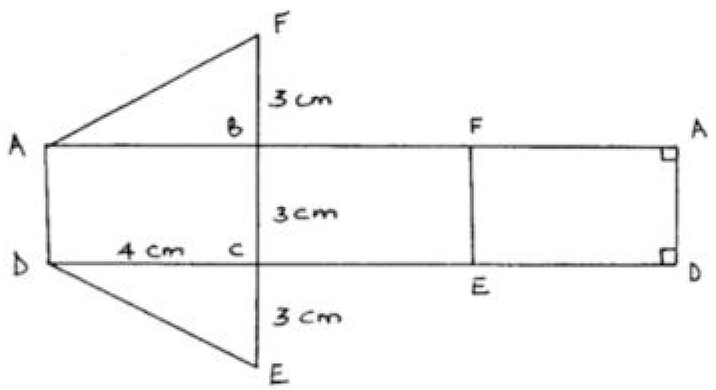
If the pyramid is cut along the edges VA, VB, VC and VD, the faces can be laid out

flat. The flat shape forms the net of the pyramid. This is as shown below.



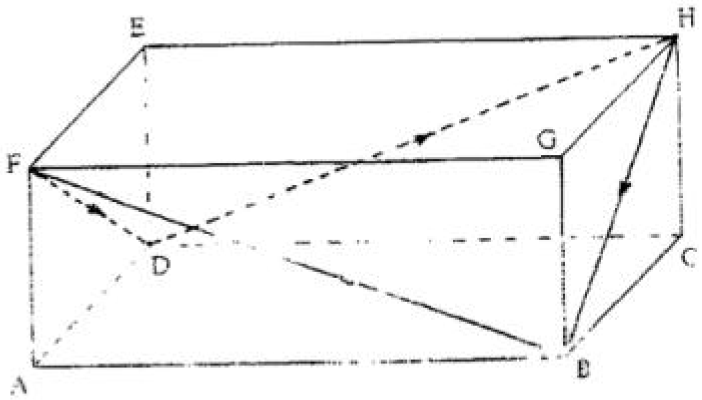
<u>Assignment</u>

(1). The net of a solid is as shown below. Sketch the solid if ABCD is the base.

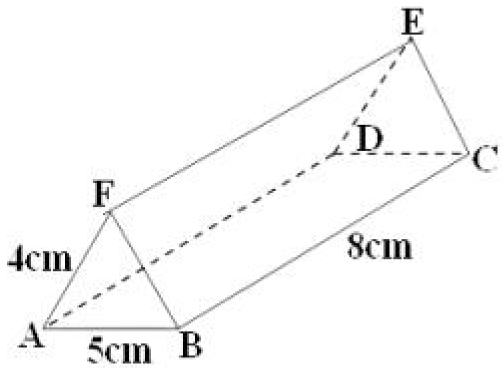


(2). On The Surface of a Cuboid ABCDEFGH A Continuous Path BFDHB Is Drawn as Shown by The Arrows Below.

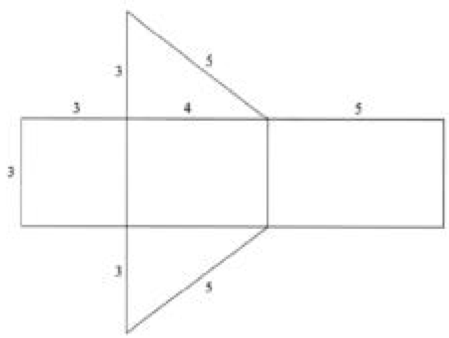
Draw and label a net of cuboid showing the path.



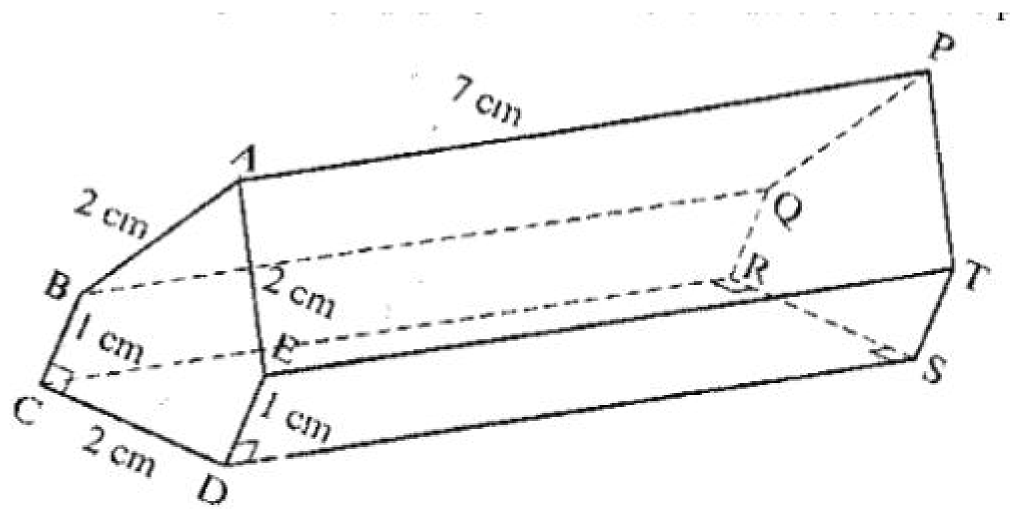
(3). The figure below is a triangular prism of uniform cross section in which AF = 4 cm, AB = 5 cm and BC = 8 cm. Draw and clearly labelled net of the prisms.



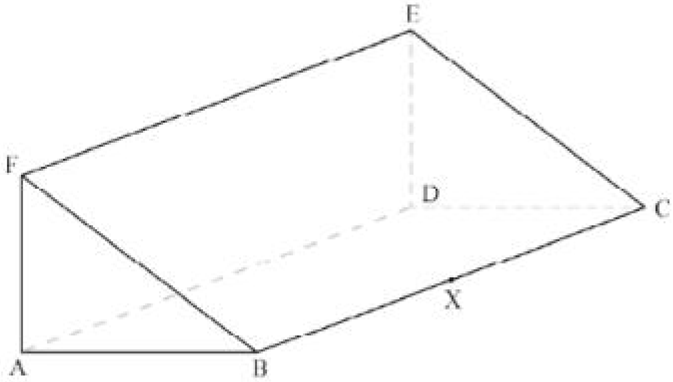
(4). The figure below shows a net of a solid (measurements are in centimeters). Complete the solid showing the hidden parts.



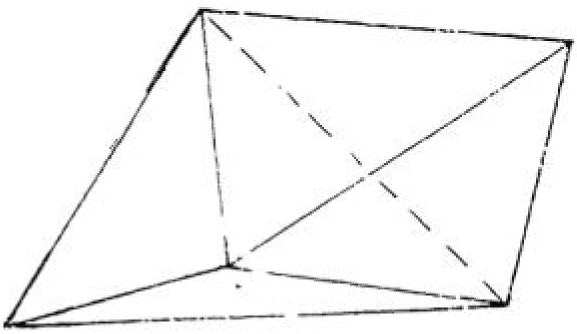
(5). The figure below represents below represents a prism of length 7 cm AB = AE = CD = 2 cm and BC ED = 1 cm. Draw the net of the prism.



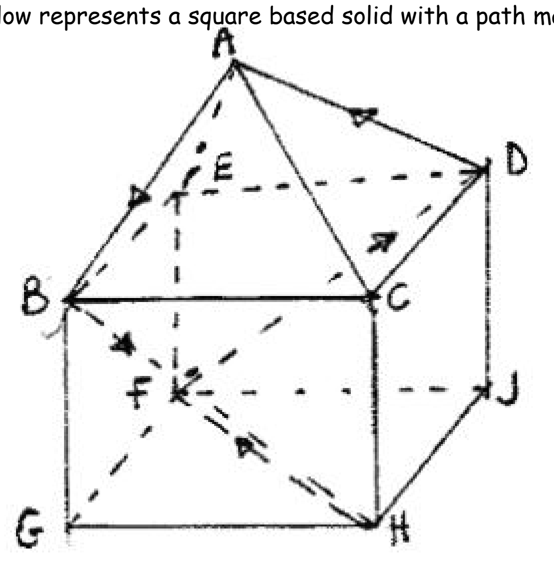
(6). The figure below represents a triangular prism ABCDEF. X is a point on BC. Draw a net of the prism.



(7). The figure below shows a solid made by passing two equals regular tetrahedra. Draw a net solid.

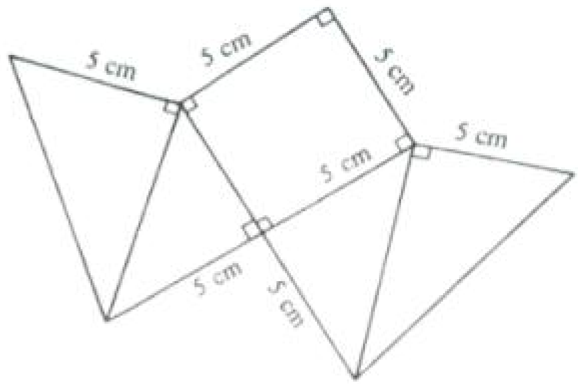


(8). The figure below represents a square based solid with a path marked on it. Sketch and label the net of the solid

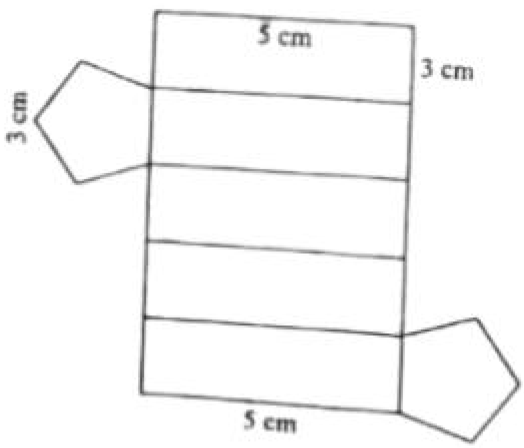


Draw the solids of each of each of the nets below:

(9).



(10).



Models of Common Solids

Models of Common Solids

(a). Cardboard models

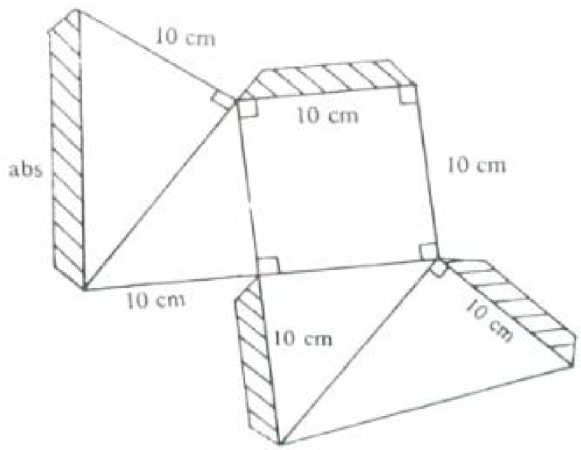
- It involves use of manilla papers.

Procedure.

(1). Draw accurately the net of a pyramid on whose base is a square of side 10 cm and slant edges are each 15 cm.

(2). Cut out the net, fold it to form a pyramid. Secure the edges using a cellotape.

The net of a pyramid can be cut out as shown. with tabs. Construct the net.

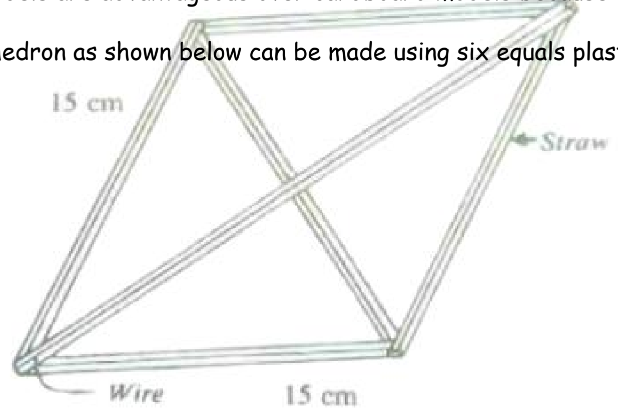


(b). Skeleton models

- To make skeleton models, the following can be used:

17. Wire of suitable thickness.
18. Plastic straws with an appropriate wire or string.
19. Pair of pliers for cutting and bending the wires.

- These models are advantageous over cardboard models because it is easier to see all the angles and edges. • A model of a tetrahedron as shown below can be made using six equals plastic straws, each 15 cm long and

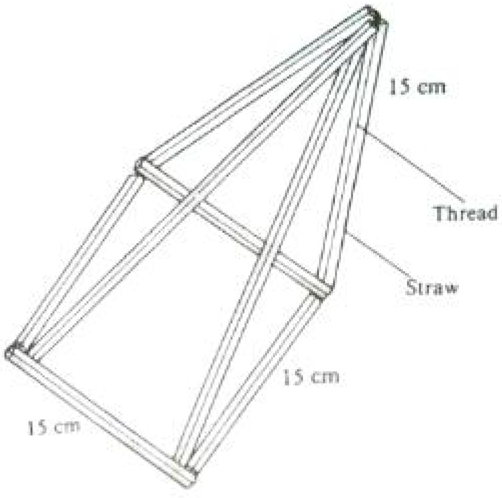


a wire.

Example.

Make a skeleton model of a reasonable pyramid with a square base using a straw of 15cm length.

Solution



Assignment

(1). Make a model of a cube of side 10 cm.

Make a model of the following solids:

(2). Tetrahedron.

(3). Cylinder.

(4). Octahedron.

(5). Triangular prism.

(6). Draw a regular pentagon of side 5 cm. From each side, draw other regular pentagons so that you have six pentagons in total. Similarly obtain another set of six pentagons. Join any side of one set to the other. The net so formed is of a regular dodecahedron. Join all the sides to obtain the model of the solid.

(7). Make a skeleton model of a reasonable of an octahedron

(8). Make a skeleton model of a reasonable of a wedge

(9). Draw accurately the model of a pyramid on whose base is a square of side 5 cm and slant edges are each 15 cm.

(10). Draw a model of tetrahedron of sides 10 cm.

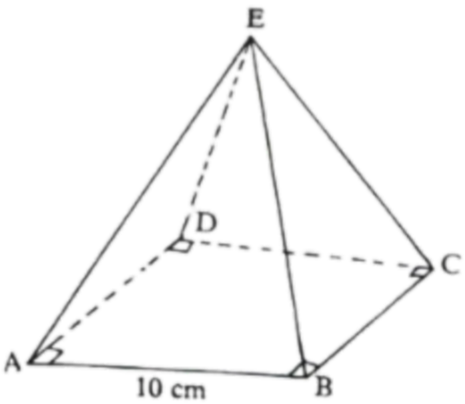
Surface Area of Common Solids From Nets

<u>**Surface Area of Solids from Nets.**</u>

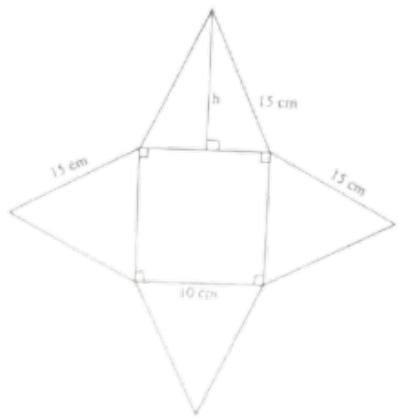
- The surface area of a solid may be calculated by finding the area of its net.
- Find the area of all the faces of the solid.
- Add up the areas of all the faces to get the total surface area of the solid.

Example.

(1). The figure below shows a right pyramid whose base is a square of side 10 cm and its slant side 15 cm long. Calculate its surface area.



Solution



The net of the pyramid is shown in figure below.

Area of square = 10 x 10

= 100 cm²

Height of each triangle = $\sqrt{(15^2 - 5^2)}$

= 14.1 (to 1 d p.)

Area of each triangle = $^1/_2 \times 10 \times 14.1$

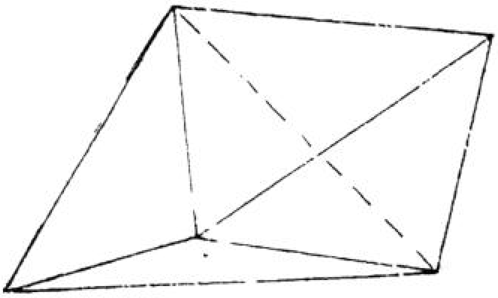
= 70.5 cm²

Thus, Surface area of the pyramid = 100 + 70.5 x 4

= 382 cm²

Assignment

(1). The figure below shows a solid made by passing two equal regular tetrahedral. Draw a net of the solid and hence find the surface area of the solid, if each face is an equilateral triangle of side 5cm.



For each of the following solids, draw the net and hence use the net to calculate the surface area of the solid.

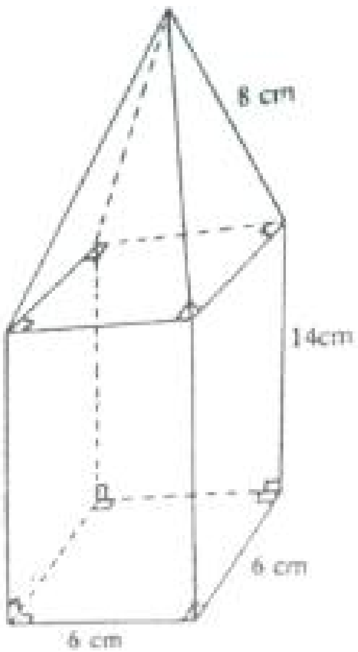
(2). A cube of side 8 cm.

(3). A cuboid measuring 12 cm by 6 cm by 8 cm.

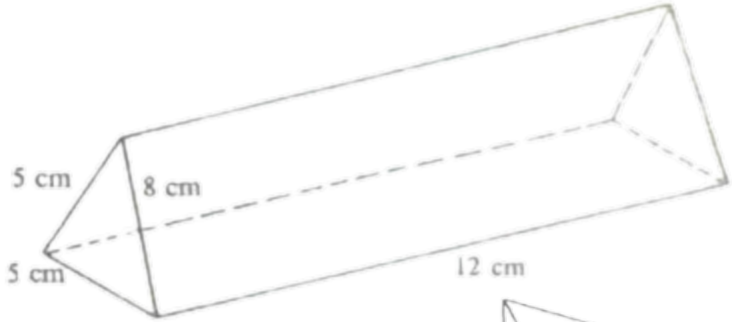
(4). A tetrahedron whose faces are equilateral triangles of side 10 cm.

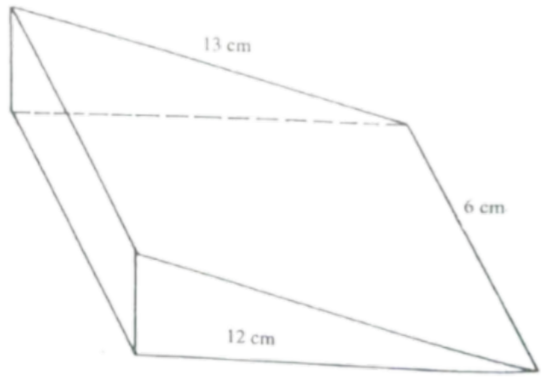
(5). A cylinder whose radius and height are 7 cm and 20 cm respectively.

(6). A polyhedron made up of a pyramid with isosceles triangles and a cuboid as shown in figure below.



(7). A triangular prism. as shown in figure below.

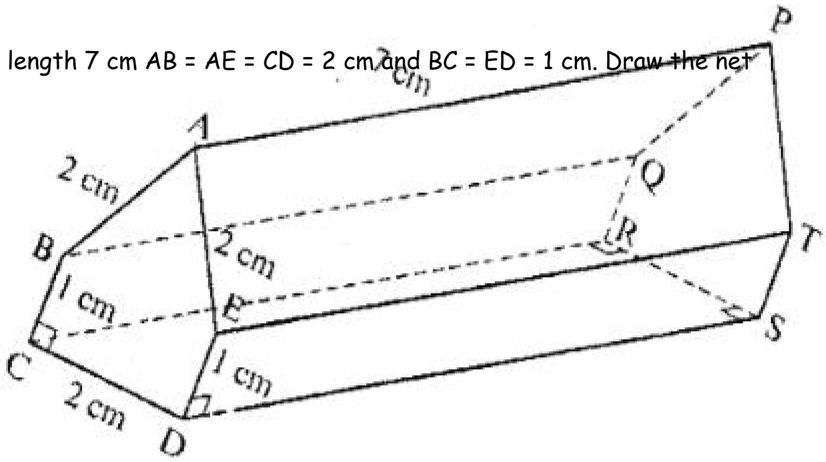




(8). The wedge shown in figure below.

(9). A cone of radius 7 cm and height 10 cm.

(10). The figure below represents a prism of length 7 cm AB = AE = CD = 2 cm and BC = ED = 1 cm. Draw the net



of the prism hence find its Surface Area.

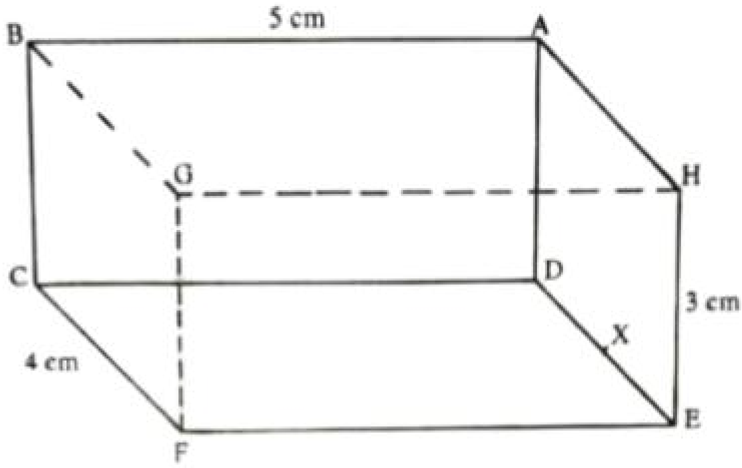
Distance Between Two Points on the Surface of a Solid

Distance between Two Points on the Surface of a Solid.

- To find the distance between two points on the surface of a solid, first open up the solid into its net.

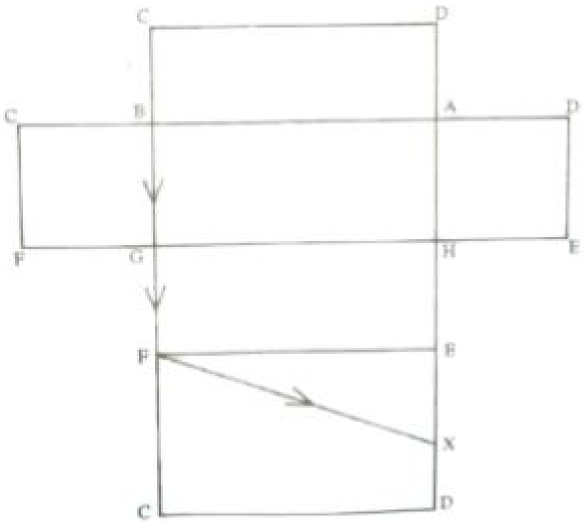
Example.

Find the distance between B and X through G and F in figure below, if BA = 5 cm, AD = 3 cm and DE = 4 cm.



Solution

Open the cuboid into a net:



DA BG+GF+ FX

27. 4+3+√ (52+22)

28. 7+ √29

29. 7+5.385

30. 12.385

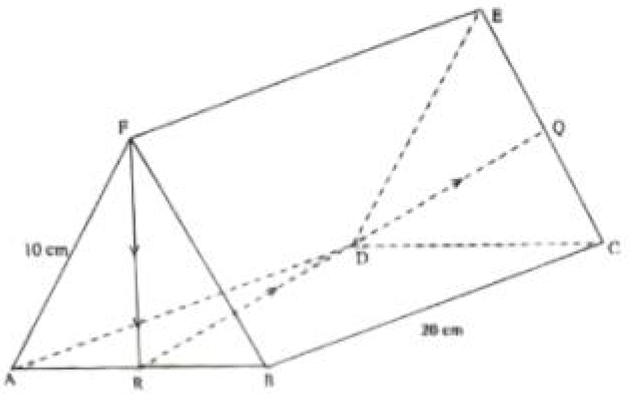
Thus, BX= 12.4 cm (to 1 d.p.)

Assignment

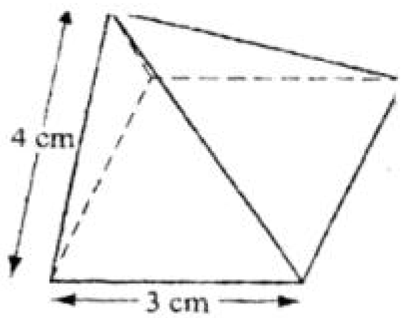
(1). The figure below shows a triangular prism ABCDEF. Its cross section is an equilateral triangle of side 10 cm and its length is 20 cm. A string runs from F to Q through R and D. Along what edges should the cube be opened so that

F, Q, R, and S lie on a straight line? What is the length of the straight

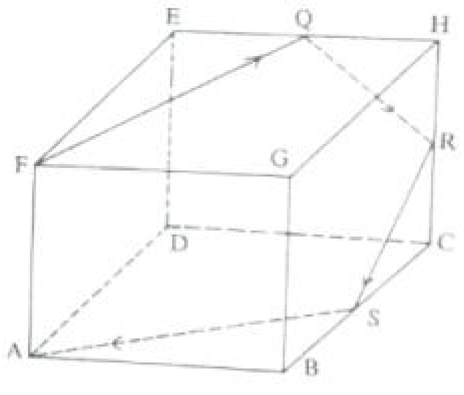
line?



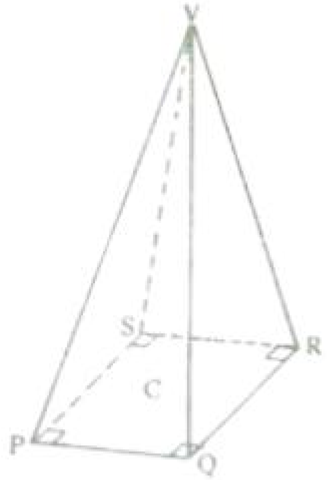
(2). The diagram below represents a right pyramid on a square base of side 3 cm. The slant of the pyramid is 4 cm. Draw a net of the pyramid and on the net drawn, measure the height of a triangular face from the top of the Pyramid.



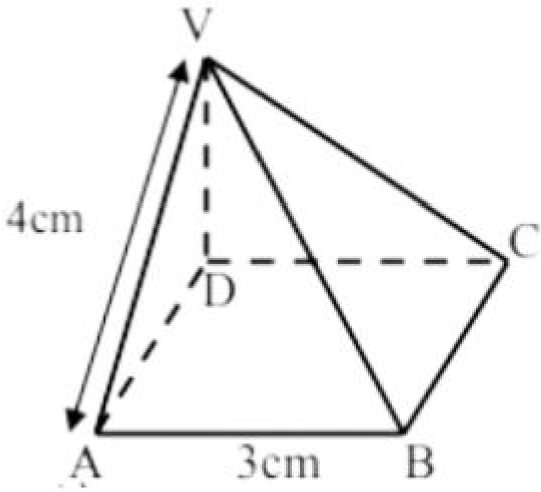
(3). The figure shows a cube of side 8 cm. The points Q, R and S are midpoints of EH, HC and BC respectively. A string runs from F to Q on face EFGH, Q to R on face CDEH, R to S on face BCHG and S to A on face ABCD. Along what edges should the cube be opened so that the points F, Q, R, S and A lie on a straight line? What is the length of the line?



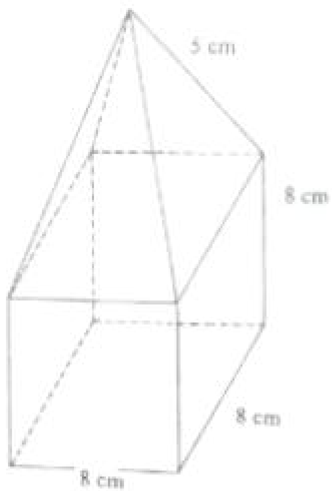
(4). The figure below shows a pyramid on a square base PQRS. Given that PV = QV = RV = SV = 5 cm, draw accurately the net of the pyramid. Use the net to calculate the distance PQ = QR = RS = SP of the pyramid.



(5). The diagram below represents a right pyramid on a square base of side 3cm. The slant edge of the pyramid is 4cm. Draw a labelled net of the pyramid and on the net drawn, measure the height of a triangular face from the top of the pyramid.



(6). A model of a tent consists of a cube and a pyramid on a square base, see the figure below. Draw accurately the net of the model. Use the net to calculate the total height of the model.



Volume of Prisms

- Volume is the amount of space occupied by an object. It�s measured in cubic units.
- Generally volume of objects is base area x height

Volume of a Prism

- A prism is a solid with uniform cross section.
- The volume V of a prism with cross section area A and length l is given by V = AL

Example;

- A rectangular box has a length of 5cm, a width of 3cm, and a height of 2cm. Find its volume.

Solution:

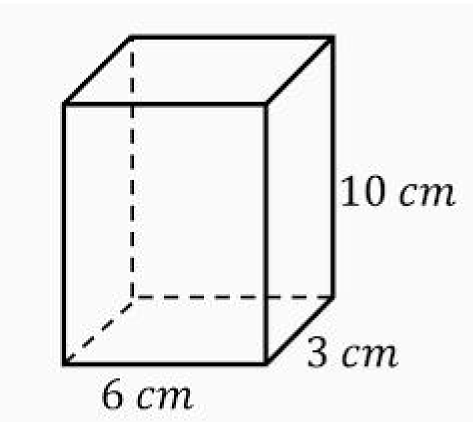
- Find the area of the base: Base area = length × width = 5cm × 3cm = 15 sq cm
- Calculate the volume: V = B × h = 15 sq cm × 2 cm = 30 cubic cm (cm^3, read as cubic centimeters)

QUESTIONS

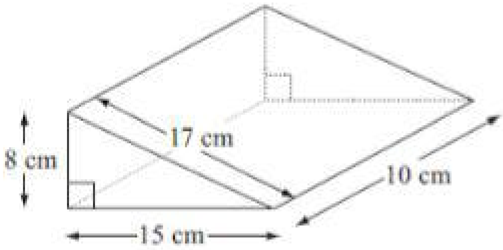
(1). A triangular prism has a base area of 24 sq cm and a height of 8cm. Find its volume.

(2). A regular hexagonal prism has a side length of 4cm and a height of 10cm. The base area of a regular hexagon can be calculated using a specific formula, but for this example, we can assume the base area (B) is provided as 24 sq cm.

(3).



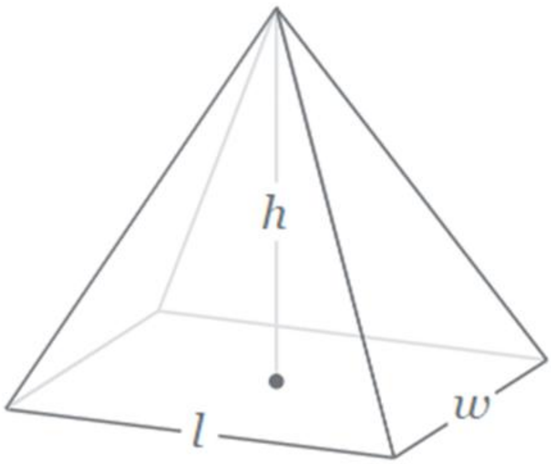
(4).



Volume of a Pyramid

Volume of a Pyramid

- Volume of a pyramid = $^1/_3$ Ah
- Where A = area of the base and h = vertical height



- A square-based pyramid has a base side length of 6cm and a height of 8cm. Find its volume.

Solution:

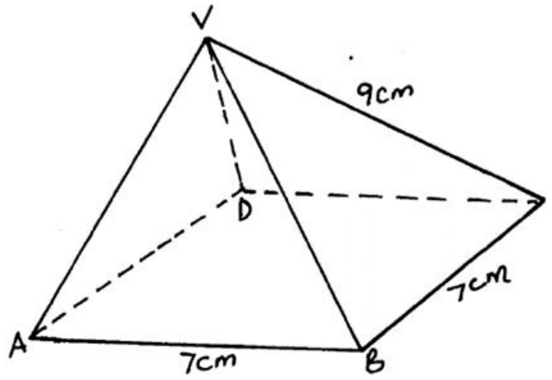
- Find the area of the base: Base area = side^2 = 6^2 cm^2 = 36 sq cm
- Calculate the volume: V = ($^1/_3$) × Base Area × Height = ($^1/_3$) × 36 sq cm × 8 cm = 96 cubic cm

QUESTIONS

(1). A triangular pyramid has a base with an area of 15 sq cm and a height of 10cm. Find its volume

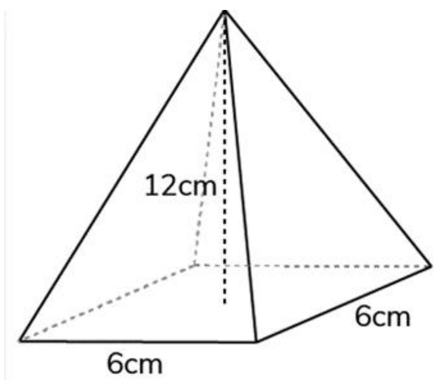
(2). An irregular pyramid has a trapezoidal base with parallel bases of 4cm and 7cm and a height of 5cm. Find its volume.

(3). The figure below is a square based pyramid, ABCDV, such that AB= 7 cm,and VA=VB= VC = VD= 9cm.



a. Find the height of the vertex V above the centre of the base.

(3). Find the volume of;

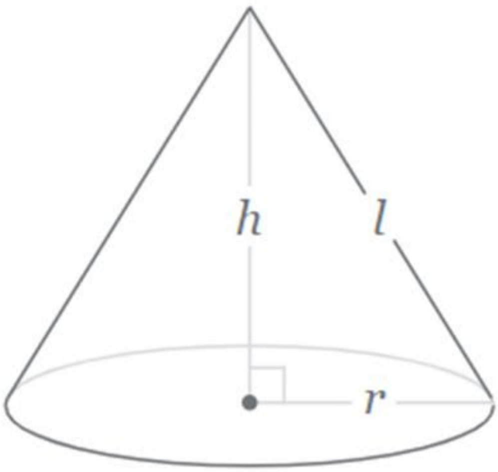


Volume of a Cone

Volume of a Cone

- Volume = $^1/_3$ area of base × height

  = $^1/_3\,\pi r^2 h$



EXAMPLES

- An ice cream cone has a base diameter of 5cm and a height of 8cm. Find its volume (excluding the space occupied by the ice cream).

Solution:

- Find the radius: Radius (r) = diameter (d) / 2 = 5cm / 2 = 2.5cm
- Calculate the volume: $V = (1/3)\pi r^2 h = (^1/_3)\pi \times 2.5^2\ cm^2 \times 8\ cm \approx 33.51$ cubic cm (rounded to two decimal places)

<u>QUESTIONS</u>

(1). A party hat is shaped like a cone with a base radius of 3cm and a height of 12cm. Find the volume of the hat.

(2). A cone has a base radius of 4cm and a volume of 47.12 cubic cm. If the base area is 50.24 sq cm (which can be calculated using πr^2), find the height.

(3). Calculate the volume of a cone whose height is 12cm and length of the slant height is 13cm

Volume of a Frustum of a Cone

<u>Volume of a Frustum of a cone</u>

- Volume = volume of large cone – volume of smaller cone

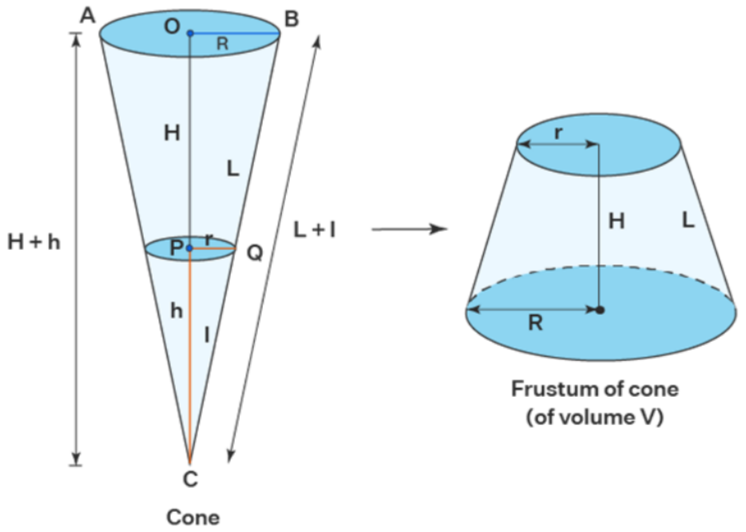


Fig. 7.35

Now, Volume = $^1/_3\pi\ [R^2 + Rr + r^2]h$ cu.units

27. $\quad ^1/_3 \times\ ^{22}/_7 \times [28^2 + (28 \times 7^2) \times 45$

28. $\quad ^1/_3 \times\ ^{22}/_7 \times 1029 \times 45 = 4810$

QUESTIONS

(1). A watering can has the shape of a frustum with a base diameter of 12cm (R = 6cm) and a top diameter of 8cm (r = 4cm). The slant height is 10cm. Find the volume of the can.

(2). A traffic cone has the shape of a frustum with a base diameter of 50cm (R = 25cm), a top diameter of 20cm (r = 10cm), and a height of 70 cm. Find the volume of the cone.

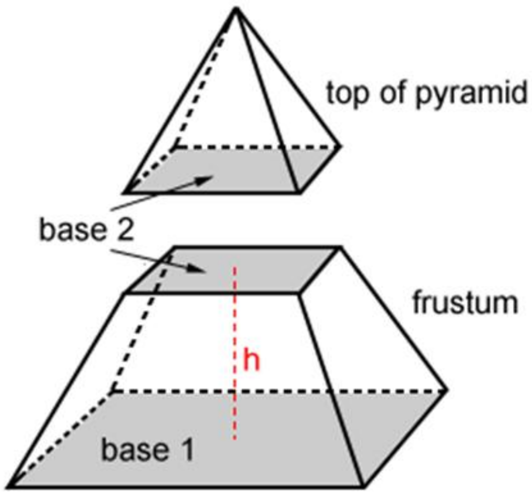
(3). A frustum of a cone has a base radius of 10cm, a top radius of 5cm, and a height of 12cm. The volume is 261.8 cubic cm. Find the larger base radius (R).

Volume of a frustum of a pyramid

Volume of a Frustum of a pyramid

- Volume = volume of large pyramid − volume of smaller pyramid

Volume of a pyramid = $^{1}/_{3}$ Ah



It is given that

- The height of the frustum = H =15cm
- The Radius of the larger base = R = 5cm
- The radius of the smaller base = r = 2.5cm
- The volume of the pyramid is V = $\frac{\pi H}{3}$ $(R^2 + Rr + r^2)$

V = $\frac{\pi \times 15}{3}$ $(5^2 + 5 \times 2.5 + 2.5^2)$

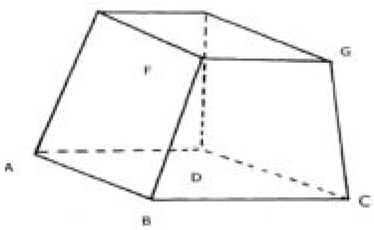
V = 687.22cm^3

QUESTIONS

(1). A square-based frustum has a base side length of 8 cm, a height of 10 cm, and the top is cut off such that the remaining top side length is 4 cm. Find the volume.

(2). A triangular-based frustum has a base with side lengths of 6 cm, 8 cm, and 10 cm. The height of the frustum is 12 cm. The top is cut off parallel to the base, removing the top 4 cm from each side length. Find the volume.

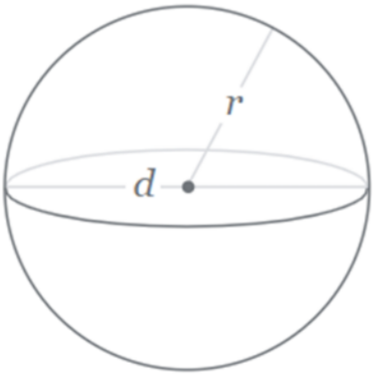
(3). The figure below represents a frustum of a right pyramid on a square base. The vertical height of the frustum is 3cm. Given that EF = FG = 6 cm and that AB = BC + 9cm

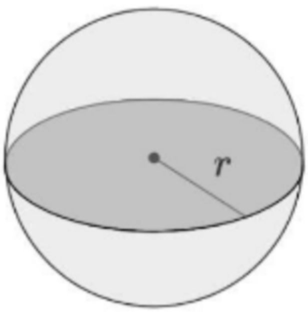


Volume of a Sphere

<u>Volume of a Sphere</u>

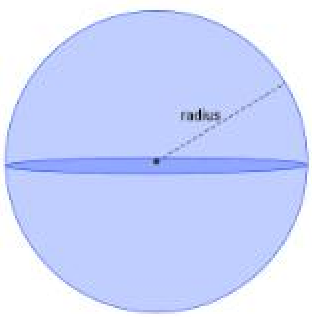
- $V = {}^4/_3 \pi r^3$





$$V = \frac{2}{3}\,\tau r^3$$

Example;



r = 7.7

Volume of a sphere = ${}^4/_3\ \pi r^3$ units^3

7. ${}^4/_3\ \pi 7.7^3$
8. 1912.32 units^3

Volume of Hemisphere =

$\frac{\frac{4}{3}\pi r^3}{2}$ units³

= $\frac{\frac{4}{3}\pi 7.7^3}{2}$

= 956.16 units³

QUESTIONS

(1). A basketball has a diameter of 24 cm. Find its volume.

(2). A bowl has the shape of a hemisphere with a radius of 8 cm. Find the volume of water it can hold.

(3). A spherical container which is 30 cm in diameter is $^3/_4$ full of water. The water is emptied into a cylindrical container of diameter 12 cm. What is the depth of the water in the cylindrical container?

Mass, Weight and Density

MASS

Mass as a Unit of Measurement

- The mass of an object is the quantity of matter in it. Mass is constant quantity, wherever the object is, and matter is anything that occupies space. The three states of matter are solid, liquid and gas.
- The SI unit of mass is the kilogram. Other common units are tonne, gram and milligram.

The following table shows units of mass and their equivalent in kilograms.

Unit	*Abbreviation*	*Equivalence in kg*
1 hectogram	Hg	0.1 kg
1 decagram	Dg	0.01 kg
1 gram	g	0.001 kg
1 decigram	dg	0.000i kg
1 centigram	cg	0.00001 kg
1 milligram	mg	0.000001 kg
1 microgram	µg	0.000000001 kg
1 megagram	Mg	1 000 kg
1 tonne (1 megagram)	t	1 000 kg

Exercise

(1). A car has a mass of 1500 kilograms. If a passenger with a mass of 70 kilograms gets into the car, what is the total mass of the car and the passenger?

(2). A bakery uses 4.5 kilograms of flour to make one batch of bread. If the bakery wants to make 12 batches of bread, how much flour in total will they need?

(3). A package contains three items with masses of 0.75 kg, 1.2 kg, and 0.5 kg. What is the total mass of the items in the package?

(4). A swimming pool is filled with water, which has a mass of 1000 kilograms per cubic meter. If the pool has dimensions of 10 meters by 5 meters by 2 meters, what is the total mass of the water in the pool?

(5). Sarah has a bag of apples that weighs 3.6 kilograms. If each apple weighs 0.2 kilograms, how many apples are in the bag?

(6). Mary bought 2 kg of meat. Half of the meat was cooked for supper and a quarter of the remainder used to make burgers for the following days breakfast. How much meat in grams was left?

(7). John requires 2 100 kg of sand to construct his house. How many lorries of sand will he buy if 1 lorry carries 7 tonnes of sand?

(8). Express each of the following masses in kilograms; 20 Hg,

(9). A textbook has 268 leaves. Each leaf has a mass of 50 g and the cover 20g find the mass of the book in kilograms.

(10). Express each of the following in grams: 4538 μg

WEIGHT

Weight as a Unit of Measurement - Video Lesson and Notes PDF

Density

Density as a Unit of Measurement

DENSITY

- The density of a substance is the mass of a unit cube of the substance. A body of mass (m) kg and volume (V) m^3 has

(i).

$$\text{Density } (d) = \frac{\text{mass } (m)}{\text{volume } (v)}$$

(ii) Mass (m) = density (d) x volume (v)

(iii)

$$\text{Volume } (v) = \frac{\text{mass}}{\text{density}}$$

Units of Density

- The SI unit of density is kg/m^3. The other common unit is g/cm^3 •

1 g/cm^3 = 1000 kg/m^3

Example;

Find the mass of an ice cube of side 6 cm, if the density of ice is 0.92 g/cm^3

Solution:

Volume of cube = 6 x 6 x 6 = 216 cm^3

Mass density x volume

29. 216 × 0.92

30. 198.72g

EXCERCISE

(1). What is the mass of water that can fill a cylindrical tank whose diameter and height are 2.8 m and 3 m respectively? (Take density of water as 1 kg/l)

(2). A cylindrical milk churn contains 15 litres of milk. Find the density of milk in g/cm^3 if the total mass of milk in the churn is 14 kg.

(3). The reading of liquid in a measuring cylinder is 45 cm^3. A solid of mass 150 g is put into the container. If the density of the solid i 8.6 g/cm^3, find the new reading.

(4). A right-angled triangular prism has length 3 m, breadth 2 m and height 2.5 m. If the mass of the prism is 3.4 kg, find its density.

(5). The density of a certain type of wood is 0.48 g/cm^3. Find the mass of a log of this wood with diameter 49 cm and length 3 m.

(6). A wooden block measuring 20 cm by 30 cm cm by 50 cm has a mass of 22.5 kg. Find the density of this wood in g/cm^3.

(7). Find the density in kg/m^3 of petrol if the mass of 1.5 litres of petrol is 1.2 kg.

(8). Calculate the mass in grams of 205 cm^3 of steel if it has a density of 97 800 kg/m^3.

(9). The density of gold is 19.3 g/cm^3, Calculate the volume, in m^3, of a golden ring mass of 57.9 g.

(10). 2 000 cm^3 of a mixture consists of 2.5 kg of substance A and 7.5 kg of substance B. Find the density of the mixture.

Volume and Capacity

What is Unit Conversion for Volume?

Unit Conversion for Volume

- Volume is the amount of space occupied by a solid object. The unit of volume is cubic units. •

Conversion of Units of Volume

A cube of edge 1 cm has a volume of 1 cm x 1 cm x 1 cm = 1 cm^3 A cube

of side m has a volume of 1 m^3 But 1m = 100cm

Therefore 1m × 1m × 1m = 100cm × 100cm × 100cm

Thus; $1m^3 = 1000000cm^3$

Example;

Convert 20 m^3 to cm^3

Sol

$20m^3 = 20 \times 1000000$

$= 20000000$ cm^3

Exercise

(1). Conver 19.7 cm^3 in to m^3

(2). Convert 750 mm^3 into cm^3

(3). Convert 105 cm^3 into cm^3

(4). Convert 750 mm^3 into cm^3

(5). Convert 0.2 Hm^3 into cm^3

(6). Convert 0.01 km^3 into cm^3

(7). Convert 7.5 dm^3 into cm^3

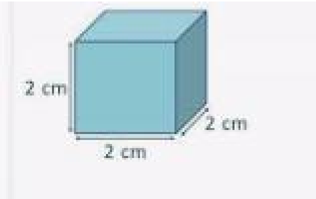
(8). A rectangular tin measures 20 cm by 20 cm by 30 cm. What is its volume in m^3

(9). A school water tank has a radius of 2.1 m and a height of 450 cm. How many cm^3 of water does it carry when full?

(10). Cylindrical solid of radius 7 cm has a conical top of the same radius. The height of the cylindrical part of the solid is 17 cm. The conical top has a vertical height of 9 cm. Calculate the volume of the solid in m^3

Volume of a Cube

- A cube is a solid having six plane square faces in which the angle between two adjacent faces is a right-angle.



Volume of a cube = area of base x height

31. $l^2 \times l$

32. l^3

Example;

Find the volume of a cube of side 6cm

Sol

V=66×6

= $216cm^3$

Exercise;

(1). A cubic tin measures 20 cm by 20 cm by 20 cm. What is its volume?

(2). Find the volume of water in a full cubic tank 4-m long, 4 m wide and 4 m deep?

(3). A school water tank has a square base of side of 450m and a height of 450 cm. Determine the maximum quantity of water it can carry

(4). A cubic container can hold 120 cm^3 of liquid. Find its length.

(5). 150 cm^3 of milk is poured into a cubic container of length 10 cm. Calculate the depth of the milk.

(6). A cubic room contains 1200 cm^3 of air. Find the length of the room.

(7). Find the volume of a cube of length 2.5 cm.

(8). Find in term of x the volume of a cube of side (x-4) cm.

(9). A school water tank is in the shape of a cube. Given that the volume of water in the tank when full is 369 cm^3 Calculate the surface area of the tank when closed, correct to 2 decimal places.

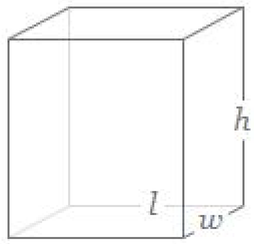
(10). The base of a cube are of length 80 cm and width 80 cm. Calculate the volume of the cube.

Volume of a Cuboid

- A cuboid is a solid wit six faces which are not necessarily square.

Volume of a cuboid = length x width x height

= A sq.units × h



= Ah units cubic

Example;

Find the volume of a cuboid of sides 6cm by 8cm by 10cm

Sol

V=LWH

9. 6810

   a. $480cm^3$

Exercise

(1). A cuboid tin measures 20 cm by 25 cm by 30 cm. What is its volume?

(2). Find the volume of water in a full cuboid tank 4-m long, 8m wide and 7m deep?

(3). A school water tank has a square base of side of 450m and a height of 4050 cm. Determine the maximum quantity of water it can carry

(4). A cuboid container can hold $120cm^3$ of liquid. Find its height given that it has a square base of length 10cm.

(5). $150cm^3$ of milk is poured into a cuboid container of length 10cm and width 3cm. Calculate the depth of the milk.

(6). A cuboid room contains $1200cm^3$ of air. Find the height of the room given that the room measures 30cm by 20cm.

(7). Find the volume of a cube of length 2.5cm .

(8). Find in term of x the volume of a cube of side (x-4) cm.

(9). A school water tank is in the shape of a cuboid. Given that the volume of water in the tank when full is 369m3 Calculate the surface area of the tank when closed , correct to 2 decimal places given that the tank has a square base of side 5m .

(10). A cuboid measures 80 cm by width 100 cm by 300cm. Calculate the volume of the cuboid.

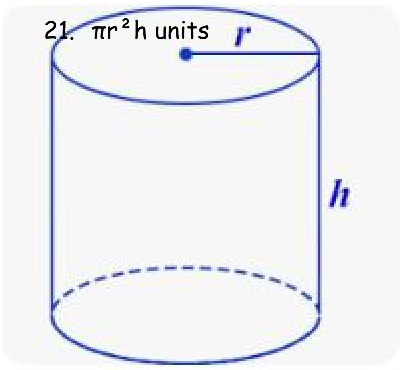
Volume of a Cylinder

- A cylinder is a three-dimensional solid that holds two parallel bases joined by a curved surface, at a fixed distance. These bases are normally circular in shape (like a circle) and the center of the two bases are joined by a line segment, which is called the axis.

Volume of a cylinder = area of base x height

20. $\pi r^2 h$

21. $\pi r^2 h$ units



Example;

Find the volume of a cylinder of of radius 7cm and height 10cm.

Volume of a cylinder = $\pi r^2 h$

22. $^{22}/_{7} \times 7 \times 7 \times 10$

23. 1540 cm^3

EXCERISE

(1). A cylindrical tin has a height of 20, a radius of 7 cm contains a liquid of volume 1540 cm^3. What is the height of the tin that is not in contact with the liquid ?

(2). Find the volume of water in a full cylindrical tank of radius 14 m long, and 17 m deep?

(3). A school water tank has a radius 50 m and a height of 4050 cm. Determine the maximum quantity of water it can carry

(4). A cylindrical container can hold 120 cm^3 of liquid. Find its height given that it has a square base of length 10cm.

(5). 150 cm^3 of milk is poured into a cuboid container of length 10 cm and width 3 cm. Calculate the depth of the milk.

(6). A cylindrical container contains 1200 cm^3 of air. Find the height of the room given that the room has a radius 20cm.

(7). Find the volume of a cylinder of radius 2.5 cm and height 2.5 m .

(8). Find in term of x the volume of a cylinder of diameter (x-4) cm and height 10 m.

(9). A school water tank is in the shape of a cylinder. Given that the volume of water in the tank when full is 369 m^3

Calculate the surface area of the tank when closed, correct to 2 decimal places given that the tank has a radius of 5 m.

(10). A cylinder has a radius of 80 cm and height of 30 m. Calculate the volume of the cylinder.

Volume of a Prism

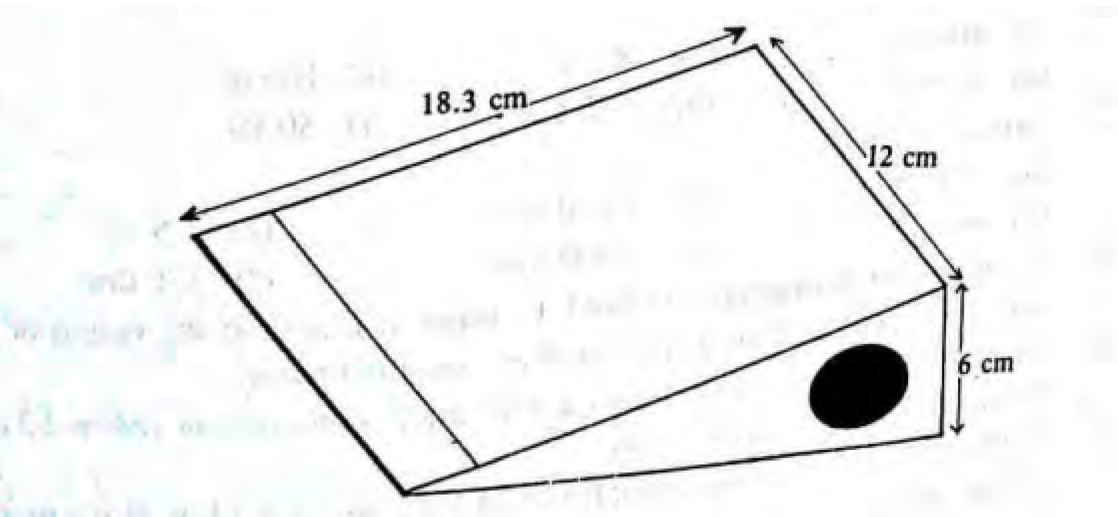
Meaning of a prism

- A solid geometric figure whose two ends are similar, equal, and parallel **rectilinear** figures, and whose sides are **parallelograms**.

Volume of a prims = Cross sectional area × Length

Example;

Find the volume of the prism below.



Cross sectional area $=\sqrt{s(s-a)(s-b)(s-c)}$ s = ½(18.3+18.3+6)

31. ½ × 42.6

32. 21.3

Cross sectional area$=\sqrt{s(s-a)(s-b)(s-c)}$

33. 21.3(21.3-18.321.3-18.321.3-6)

$= \sqrt{21.3 \times 3 \times 3 \times 15.3}$

$= \sqrt{2933.01}$

a. 54.16 cm^2

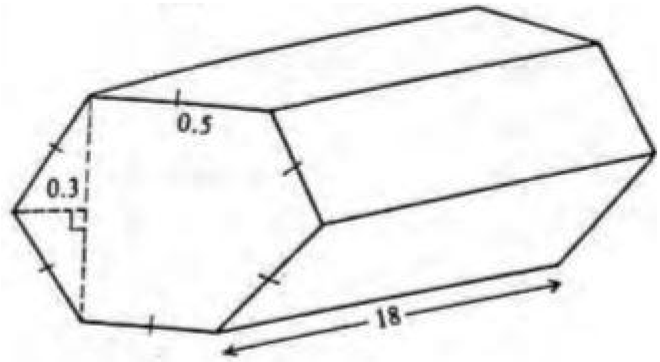
Volume of the prism = Cross sectional area × length

34. **54.16 × 12**

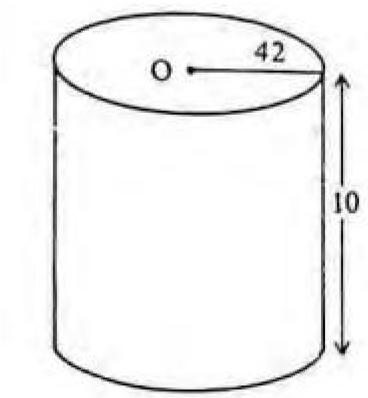
35. **649.92cm^3**

EXERCISE

(1). Find the volume of the prism below



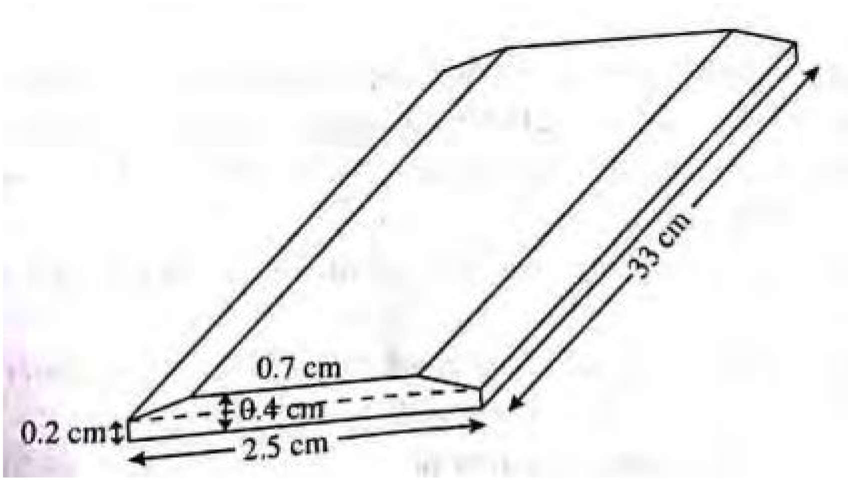
(2). Determine the volume of the prism below



(3). A rectangular tin measures 20 cm by 20 cm by 30 cm. What is its volume in cubic meters?

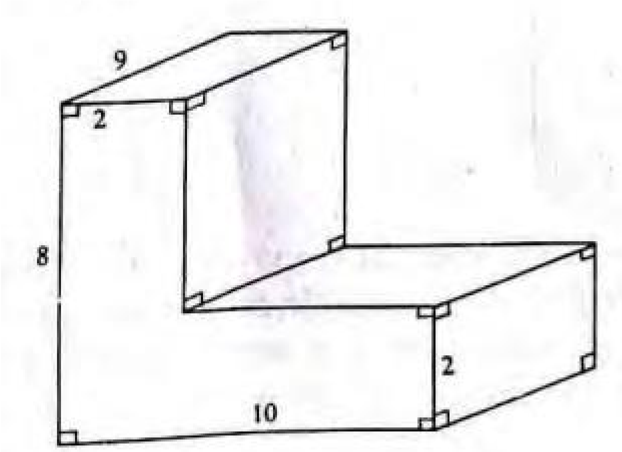
(4). How many cubic centimetre of water are there in a full rectangular tank 4m long, 4 m wide and 2 m deep?

(5). Figure 13.23 shows cross-section of a ruler which is a rectangle of 2.5 cm by 0.2 cm on which is surmounted an isosceles trapezium (one in which the non-parallel sides area of equal length). The shorter of the parallel sides of the trapezium is 0.7 cm long. If the greatest height of the ruler is 0.4 cm and it is 33 cm long, calculate its volume.

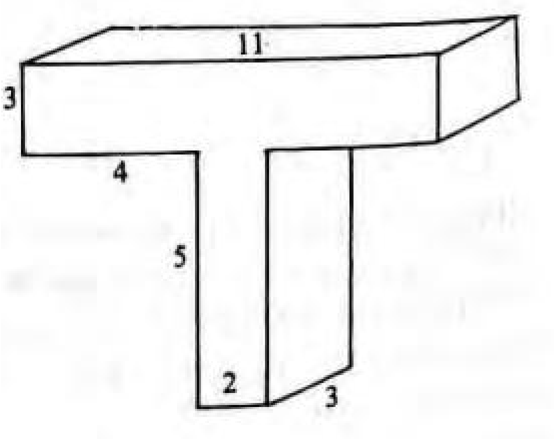


(6). A rectangular slab of glass measures 8 cm by 2 cm by 14 cm. Calculate its volume.

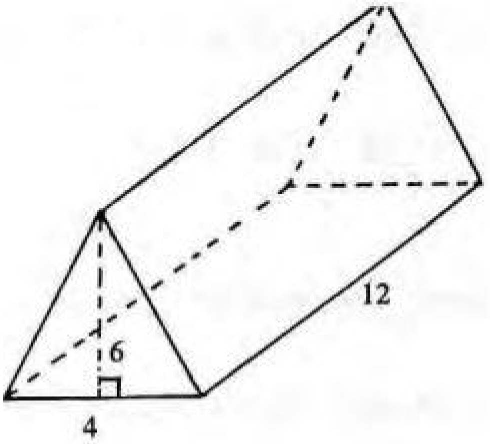
(7). Find the volume of the prism below. The measurements are in metres.



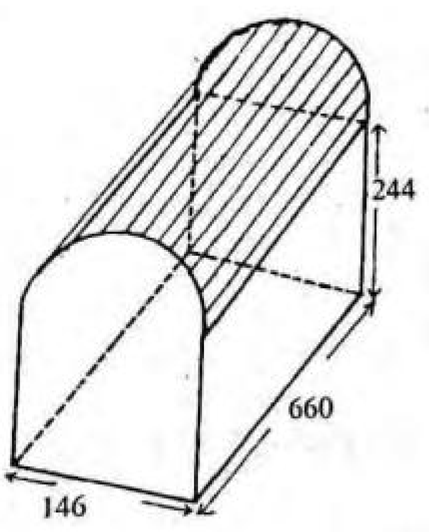
(8). Determine the volume of the prism below. The measurements are in milimetres.



(9). Find the volume of the prism below given that the measurements are in metres.



(10). What the volume of the prism below given that the measurement are in cubic centimetres.



Units Conversion of Capacity

Meaning of capacity

- Volume indicates the total amount of space covered by an object in three-dimensional space. Capacity refers to the ability of something (like a solid substance, gas or liquid) to hold, absorb or receive by an object. Both solid and hollow objects have volume. Only hollow objects have the capacity.
- Units used for capacity include: Litres, milliliteres etc.

Capacity

- Capacity is the ability of a container to hold fluids. The SI unit of capacity is the litre (1).

Conversion of Units of Capacity;

- 1 centilitre (c/) = 10 millilitres (ml)
- 1 decilitre (d/) = 10 centilitres (c/)
- 1 litre (1) = 10 decilitres (d/)
- 1 Decalitre (D/) = 10 litres (7)
- 1 hectolitre (HI) = 10 Decalitres (DI)
- 1 kilolitre (kl) = 10 Hectolitres (HI)
- 1 kilolitre (kl) = 1000 litres (1)
- 1 litre (1) = 1000 millilitres (ml)

Relationship between Volume and Capacity

- A cube of edge 10 cm holds 1 litre of liquid. 1 litre = 10 cm x 10 cm x 10 cm = 1000 cm^3 1 m^2 = 10 cm^3 1 m^2 = 10 litres

EXERCISE

Express in litres:

(1). 400 ml

(2). 536 ml

(3). 375 Hl

(4). 100 dl

(5). A cylindrical container can hold 12 litres of liquid. If the height of the container is 0.4 m, find its radius to one decimal place.

(6). One litre of milk is poured into a cylindrical container of radius 10 cm. Calculate the depth of the milk

(7). A rectangular tin measures 20 cm by 20 cm by 30 cm. What is its capacity in litres?

(8). How many kilolitres of water are there in a full rectangular tank 4-m long, 4 m wide and 2 m deep?

(9). A school water tank has a radius of 2.1 m and a height of 450 cm. How many litres of water does it carry when full?

(10). A school uses 5, 000 litres of water a day, approximately how many days will a full cubic tank of side 5 m last?

Applications of volume and capacity

EXCERCISE

(1). A cylindrical container can hold 12 litres of liquid. If the height of the container is 0.4 m, find its radius to one decimal place.

(2). The British government hired two planes to airlift football fans to South Africa for the World cup tournament. Each plane took 10 $\frac{1}{2}$ hours to reach the destination. Boeng 747 has carrying capacity of 300 people and consumes fuel at 120 litres per minute. It makes 5 trips at full capacity. Boeng 740 has carrying capacity of 140 people and consumes fuel at 200 litres per minute. It makes 8 trips at full capacity. If the government sponsored the fans one way at the cost of 800 dollars per fan, calculate the total cost of fuel used if one litre costs 0.3 dollars.

(3). One litre of milk is poured into a cylindrical container of radius 10 cm. Calculate the depth of the milk

(4). A village water tank is in the form of a frustrum of a cone of height 3.2 m. The top and bottom radii are 18 m and 24 m respectively (a) Calculate The capacity of the water tank

(5). 15 families each having 15 members use the water tank in question 4 above and each person uses 65 litres of water daily. How long will it take for the full tank to be emptied

(6). A rectangular water tank measures 2.6 m by 4.8 m at the base and has water to a height of 3.2 m. Find the volume of water in litres that is in the tank

(7). A rectangular tin measures 20 cm by 20 cm by 30 cm. What is its capacity in litres?

(8). How many kilolitres of water are there in a full rectangular tank 4-m long, 4 m wide and 2 m deep?

(9). A school water tank has a radius of 2.1 m and a height of 450 cm. How many litres of water does it carry when full?

(10). A rectangular tank whose internal dimensions are 2.2 m by 1.4 m by 1.7 m is three fifth full of milk.

a. Calculate the volume of milk in litres

b. The milk is packed in small packets in the shape of a right pyramid with an equilateral base triangle of sides 10 cm. The vertical height of each packet is 13.6 cm. Full packets obtained are sold at Shs. 30 per packet. Calculate:

25. The volume in cm^3 of each packet to the nearest whole number

26. The number of full packets of milk

27. The amount of money realized from the sale of milk 12. An 890 kg culvert is made of a hollow cylindrical material with

Time,distance and speed

Parameters of Motion

Introduction

- Distance between the two points is the length of the path joining them while displacement is the distance in a specified direction Speed
- Average speed = distance covered time taken

Example

A man walks for 40 minutes at 60 km/hour, then travels for two hours in a minibus at 80 km/hour.

Finally, he travels by bus for one hour at 60 km/h. Find his speed for the whole journey.

Solution

Average speed = distance covered

time taken

Total distance = (40/60 × 60)km + (2 × 80)km + (1 × 60)km = 260 km

Total time = 4 /6 + 2 + 1 = 32 /3 hrs

Average speed = 260

32 /3

33. 260 x 3 = 70.9 km/h

11

QUESTIONS

(1). Anne takes two hours to walk from home to her place of work, a distance of 8km. On a certain day, after walking for 30 minutes, she stopped for ten minutes to talk to a friend. At what average speed should she walk to reach on time?

(2). A motorist drove for 1 hour at 100 km/hr. She then travelled for $1\ ^1/_2$ hours at a different speed. If the average speed for the whole journey was 88 km/hr, what was the average speed for the latter part of the journey?

(3). A commuter train moves from station A to station D via stations B and C in that order. The distance from A to C via B is 70 km and that from B to D via C is 88 km. Between the stations A and B, the train travels at an average speed of 48 km/h and takes 15 minutes. Between the stations C and D, the average speed of the train is 45 km/h. Find:

a. This distance from B to C

b. The time taken between C and D

Velocity and Acceleration

Velocity and Acceleration

- For motion under constant acceleration;
- Average velocity = initial velocity + final velocity /2

Example

A car moving in a given direction under constant acceleration. If its velocity at a certain time is 75 km/h

and 1 0 seconds later its 90 km/hr.

Solution

Acceleration = change in velocity /time taken

=(90 – 75)km/h 10s

21. (90 – 75) x 1000 m/s^2

10 x 60 x 60

22. 5 /12 m/s^2

QUESTIONS

(1). The initial velocity of a car is 10 m/s. The velocity of the car after 4 seconds is 30 m/s. Find its acceleration.

(2). A bus accelerates from a velocity of 12 m/s to a velocity of 25 m/s. Find the average velocity during this interval.

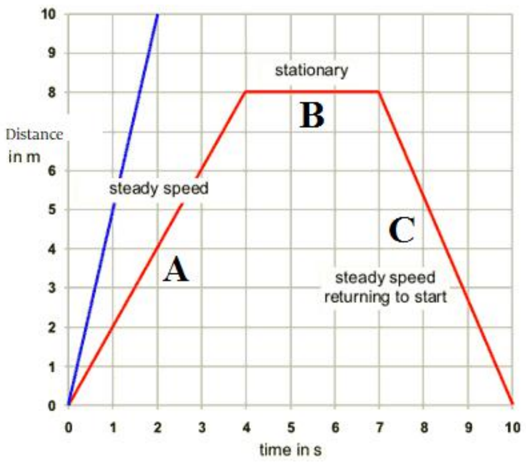
(3). A car moves with constant acceleration of $8m/s^2$ for 5 seconds. if the final velocity is 40 m/s, find the initial velocity.

(4). A train driver is moving 40 km/h applies brakes so that there is a constant retardation of 0.5 m/s^2 . Find the time taken before the train stops.

Distance Time Graphs

Distance Time Graph

- When distance is plotted against time, a distance time graph is obtained.



- When describing the motion of an object try to be as detailed as possible. For instance...
- During 'Part A' of the journey the object travels +8 m in 4s. It is travelling at a constant velocity of $+2ms^{-1}$
- During 'Part B' of the journey the object travels 0m in 3s. It is stationary for 3 seconds
- During 'Part C' of the journey the object travels -8m in 3s. It is travelling at a 'constant velocity' of '$-2.7ms^{-1}$' back to its starting point, our reference point 0

QUESTIONS

Example

(1). Table 17.1 shows the distance covered by a motorist from Limuru to Kisumu:

Time	9.00 a.m.	10.00 a.m.	11.00 a.m.	11.30 a.m.	12.00 noon	1.00 p.m.
Distance (km)	0	80	160	160	210	310

a. Draw the distance-time graph

b. Use the graph to answer the following questions:

i. How far was the motorist from limuru at 10.30 am?

ii. What was the average speed during the first part of the journey?

46. What was the average speed for the whole journey?

(2). A man leaves home at 9.00 am, and walks to a bus stop 6 km away at an average speed of 4 km/h. He then waits at the bus stop for 25 minutes before boarding a bus to a town 105 km away. The bus travels at an average speed of 60 km/h. Draw a distance time-graph for the journey and use it to answer the following questions

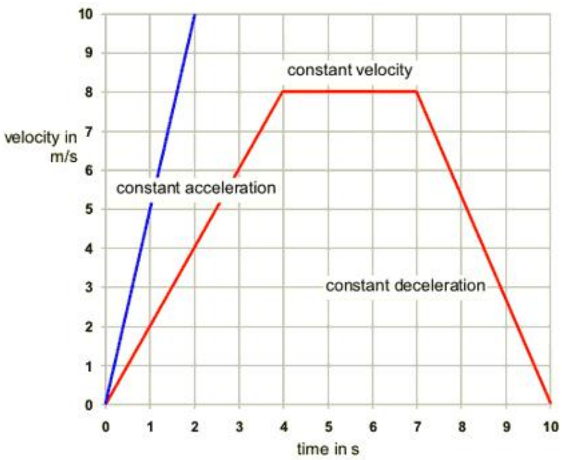
29. At what time was the man 100 km from home?

30. How far away from home was he at 10.15 am?

Velocity Time Graph

Velocity-time Graph

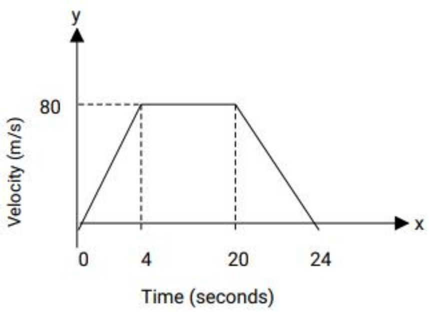
- When velocity is plotted against time, a velocity time graph is obtained.



- The distance travelled is the area under the graph
- The acceleration and deceleration can be found by finding the gradient of the lines.

QUESTIONS

(1). The figure below is a velocity time graph for a car.



a. Find the total distance traveled by the car. (2 marks)

b. Calculate the deceleration of the car. (2 marks)

(2). A car is travelling at 40 m/s. its brakes are applied and it then decelerates at 8 m/s^2. Use a velocity-time graph to find the distance it travels before stopping

(3). A particle is projected vertically upwards with a velocity of 30 m/s. If the retardation to motion is 10 m/s^2, use a graphical method to find the maximum height reached by the particle.

Approaching Bodies

Relative Speed

- Consider two bodies moving in the different direction at different speeds. Their relative speed is the sum of the individual speeds.

Example

A truck left Nyeri at 7.00 am for Nairobi at an average speed of 60 km/h. At 8.00 am a bus left Nairobi for Nyeri at speed of 120 km/h .How far from Nyeri did the vehicles meet if Nyeri is 160 km from Nairobi?

Solution

Distance covered by the lorry in 1 hour = 1 x 60 = 60 km

Distance between the two vehicle at 8.00 am = 160 – 1 00

= 100km

Relative speed = 60 km/h + 120 km/h

Time taken for the vehicle to meet = $^{100}/_{180}$

= $^{5}/_{9}$ hours

Distance from Nyeri = 60 x $^{5}/_{9}$ x 60

22. 60 + 33.3

23. 93.3 km

__QUESTIONS__

(1). A matatus left town A at 7 a.m. and travelled towards a town B at an average speed of 60 km/h.

A second matatus left town B at 8 a.m. and travelled towards town A at 60 km/h. If the distance between the two towns is 400 km, find;

i. The time at which the two matatus met

ii. The distance of the meeting point from town A

(2). Two towns P and Q are 400 km apart. A bus left P for Q. It stopped at Q for one hour and then started the return journey to P. One hour after the departure of the bus from P, a trailer also heading for Q left P. The trailer met the returning bus $\frac{3}{4}$ of the way from P to Q. They met hours after the departure of the bus from P.

a. Express the average speed of the trailer in terms of t

b. Find the ratio of the speed of the bus to that of the trailer.

Overtaking Bodies

Relative Speed

- Consider two bodies moving in the same direction at different speeds. Their relative speed is the difference between the individual speeds.

Example

A van left Nairobi for kakamega at an average speed of 80 km/h. After half an hour, a car left Nairobi for Kakamega at a speed of 100 km/h.

a. Find the relative speed of the two vehicles.

b. How far from Nairobi did the car over take the van

Solution

Relative speed = difference between the speeds

10. 100 80

11. 20 km/h

Distance covered by the van in 30 minutes

Distance = $^{30}/_{60}$ x 80 = 40 km

Time taken for car to overtake matatu = $^{40}/_{20}$ = 2 hours

Distance from Nairobi = 2 x 100 = 200 km

QUESTIONS

(1). Two Lorries A and B ferry goods between two towns which are 3120 km apart. Lorry A traveled at km/h faster than lorry B and B takes 4 hours more than lorry A to cover the distance. Calculate the speed of lorry B

(2). Nairobi and Eldoret are each 250 km from Nakuru. At 8.1 5 am a lorry leaves Nakuru for Nairobi. At 9.30 am a car leaves Eldoret for Nairobi along the same route at 100 km/h. Both vehicles arrive at Nairobi at the same time.

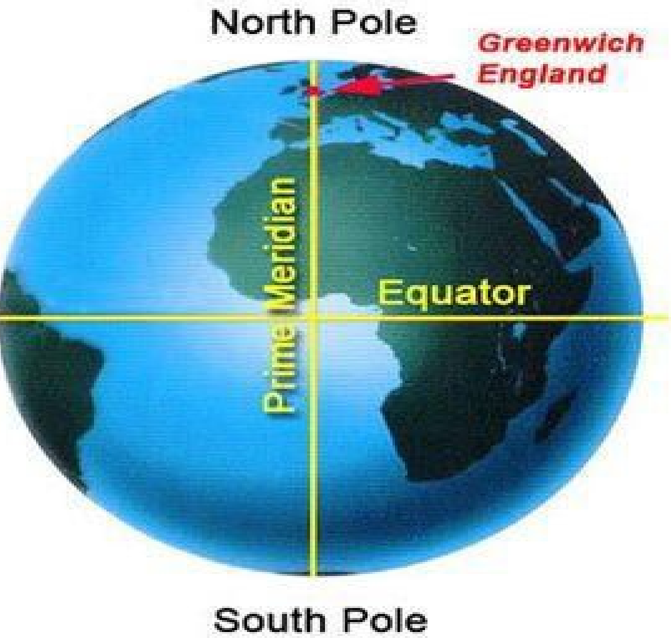
a. Calculate their time of arrival in Nairobi

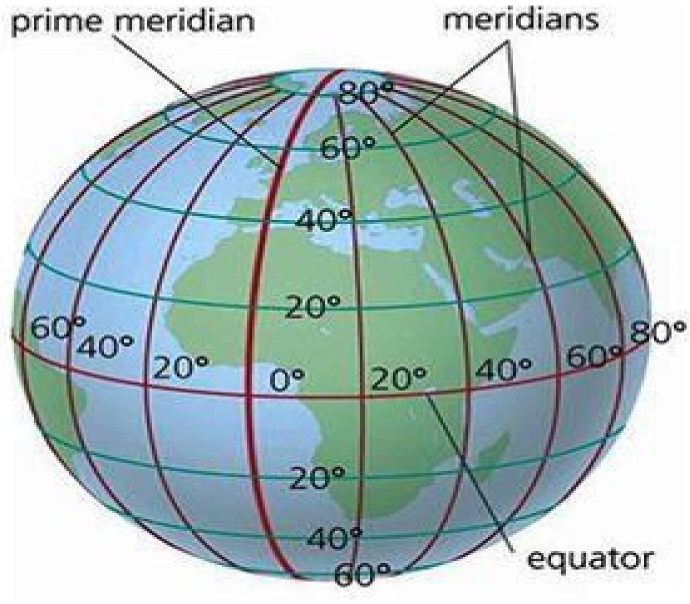
b. Find the cars speed relative to that of the lorry.

c. How far apart are the vehicles at 1 2.45 pm.

Determining time

LONGITUDES





Calculating the time of different places in the world using longitudes

- Rotation of the earth- This is movement of earth on its own axis
- Distance between longitudes is measured in degrees
- There are 360 meridians or longitudes. That is, 180^0 to the East and 180^0 to the west.
- One complete rotation is 360°
- The direction of the rotation is from west to east i.e. anticlockwise direction.
- One complete rotation takes 24 hours
- All places found in the east of the Greenwich meridian will see sunrise first and therefore they are one hour ahead of those to the west

❖ If it takes 24 hours for the earth to rotate, it means in 1 hour, the earth covers 15^0 and 4 minutes to cover 1^0.

❖ when calculating time to the east of Greenwich Meridian, we add the time difference to the local time.

❖ When calculating time to the West of Greenwich Meridian, we subtract the time difference to the local time.

24hrs = 360°
1hr = ?
360×1 ÷24 =15
Therefore 1hr =15° or ***(24×60)minutes=1440min***
360 °=
°= 1440÷360 ×1=4min
I Hr the earth covers 15° and 1° it covers 4 minutes

<u>Calculating time of places found to the east of Greenwich Meridian</u>

<u>Example 1</u>

The time in Accra 0° is 7.00am. Calculate time in bermbera 45 °E
1hr =15 °
? = 45 ° = 45÷15×1 =3hrs
So 3hrs is equivalent to 45 ° then add 3hrs to 7.00am to get 10.00am

Example 2
Suppose the time at GWM is 12 noon what is the local time at Watamu 40° E?
Time gained=40×4=160min=2 hours 40min
Local time at Watamu is 12.00+2.40=14.40-1200=2.40pm.

Example 3

At Dar-es-Salaam 40◦E time is 12pm what is the time at Ecuador 40◦E?

40◦+20◦=60◦

60×4=240min=4hours

Ecuador is behind in time =12.00-4=8 am.

NB

When calculating time to the east of Greenwich meridian, we add the time difference to the local time.

<u>Calculating time of places found to the west of Greenwich Meridian</u>

- When calculating time to the west of Greenwich meridian we subtract the time difference to the local time.

Example 1

If the time in Accra is 12.00 noon, what is the time in Dakar 17^0 W?

Find the difference in degree.

17^0-0^0

$=17^0$.

Calculate the difference in time between the two cities.

If 360 =24 hours

17^0 =?

17x24 ÷ 360

= 1 hr 08 minutes.

If the time in Accra 0^0 is 12.00 noon, then subtract the time difference to get the local time in Dakar.

12.00 noon

- 1. 08 mins

10. 52 am Dakar local time.

<u>Calculating time of places found to the East and west of Greenwich Meridian</u>

<u>Example 1</u>

A plane took off in Freetown 15^0 W at 7am local time.

What is the local time in Cape Town, 18^0 E

Calculate the number of degrees between Freetown and Cape town.

$15^0 + 18^0 = 33^0$.

Calculate the time difference between the two cities.

If 360^0 = 24 hrs

Then 33^0 =?

33x24÷ 360

=2 hrs 12 minutes.

Since cape town is to the east of the Greenwich meridian, it means the city is a head of Freetown.

Therefore, to get time in Cape town, add the time difference to Freetown local time.
am + 2hrs 12 minutes= 9.12 am (Cape Town local time

MONEY

Calculating Profit

Profit

- The difference between the **cost price** and the **selling price** is either **profit or loss**. If the selling price is **greater than** the cost price, the difference is a **profit**.

Note:

- Selling price cost price = profit

  Percentage profit = profit cost price x 100

Example:

Profit

Tirop bought a cow at Sh. 18 000 and sold it at sh. 21 000. What percentage profit did he make?

Solution

Selling price = Sh. 21 000

Cost price = Sh. 18 000

Profit = Sh. (21 000 18 000)

= sh. 3 000

Percentage profit = 3000/18000 x 10

= 1623%

EXCERCISE

(1). Abdi bought a pair of trousers for Sh. 650 and later sold it at Sh. 720. What profit did he make?

(2). A trader bought a 50 kg bag of sugar at Sh. 2 100. She sold the sugar at Sh. 50 per kilogram. What was the percentage profit?

(3). Parpai bought a textbook at Sh. 450 and later sold it at Sh. 500. What was the percentage profit?

(4). A businessman bought a bag containing 50 mangoes for Sh. 250. He sold the mangoes at sh. 10 each. If 5 mangoes were bad, what was his percentage profit?

(5). A lady Sold blouses at Sh. 1 500 each. She made a profit of sh. 150 for each blouse. How much did she pay for each?

(6). Tina bought a bag containing 80 tomatoes for Sh. 270. She sold the tomatoes in piles of four, making a profit of 50%. For how much did she sell each pile?

(7). A trader sold an article at Sh. 4800 after allowing his customer a 12% discount on the marked price of the article. In so doing he made a profit of 45%.

a. Calculate the price at which the trader had bought the

b. If the trader had sold the same article without giving a discount. Calculate the percentage profit he would have made.

(8). A man imported a vehicle at Shs. 600,000 and sold it at Sh. 1,080,0000. Find his percentage profit if he spent Sh. 60,000 for clearing the vehicle from the port and a further Sh. 40,000 for shipping.

(9). A farmer made a profit of 28% by selling a goat for Sh.1440. What percentage profit would he have made if he had sold the goat for Sh. 2100?

(10). Mr. Sitienei sold a house to Mr. Lagat at a profit of 10%. Mr. Lagat then sold it to Mr. Rotich at a profit of 5%. Mr. Rotich paid Ksh 110,000 more than Mr. Lagat for the house. Find how much Mr. Rotich paid for the house.

Calculating Loss

Loss

- This is the difference between the **cost price** and the **selling price** when the cost price is **greater than** the selling price.

Formula

Loss = Cost price - selling price

Percentage Loss =

$$\frac{Loss}{Cost\ price} \times 100\ \%$$

Example;

(i). Abdi bought a pair of trousers for Sh 720 and sold it at Sh 650. What percentage loss did he make?

Solution

Cost price = sh. 720

Selling price = sh. 650

Loss = sh. 720 – sh. 650

= sh. 70

34. Loss =

$\frac{sh.\ 70}{sh.\ 720} \times 100\%$ = 9.722%

(ii). Calculate the loss incurred if Juma bought a 60 kg bag of sugar at Sh. 2 100. She sold the sugar at sh. 30 per kilogram.

Solution

Loss = Cost price – selling price

Selling Price for 60 kg = Selling Price per kg × Weight (kg)

Selling Price for 60 kg = Sh 30 × 60 kg = Sh 1800

Selling Price for 60 kg = Sh 30 × 60kg = Sh 1800

Now, we can calculate the loss:

Loss = Cost Price – Selling Price

Loss = Cost Price – Selling Price

Loss = Sh 2100 – 1800

Loss = Sh 2100 – 1800

Loss = Sh 300

EXERCISE;

(1). An entrepreneur purchased a bag that held 50 apples for Sh. 250. He charged sh. 8 for each apple. What loss did he make if 15 apples were bad?

(2). Jane sold a dress she had purchased for Sh. 2 800 after paying Sh. 3 500 for it. What was her percentage loss?

(3). A retailer bought a batch of 50 shirts for Sh 1,000. Due to some defects, 5 shirts were unsellable. If the retailer sold the remaining shirts at Sh 25 each, find the overall loss.

(4). There is a 25 % loss when an article is sold at Sh. 200. At what price should it be sold to reduce the loss to 5 %?

(5). Parpai bought a textbook at Sh. 450 and later sold it at Sh. 400. What was the percentage loss?

(6). A shopkeeper made a loss of 30% by selling an electric iron at Sh 700. What loss would he have made had he sold it at Sh 500?

(7). A man bought 10 mangoes at Sh. 10.00 each. He ate four of the mangoes and sold the remainder, making an overall loss of Sh. 16.00 Calculate his selling price per mango, hence the percentage loss on each mango.

(8). A book seller sold Distinction Mathematics text book for Sh. 720 making a 10% loss. How much would he have sold the book to reduce the loss to 2% ?

(9). Kombo bought a bull for Sh. 28 000 and later sold it for Sh. 26 600. What percentage loss did he make?

(10). A trader bought 500 oranges for Sh 4 000. During the transportation 20 of them got spoilt. She sold the remaining in piles of 5 at Sh 20. What percentage loss did she make?

What is Discount?

Discount

- A shopkeeper may decide to sell an article at a reduced price. The difference between the marked price and the reduced price is referred to as the discount.
- Discount is usually expressed as a percentage of the actual marked price.

Example:

The price of an article is marked at Sh. 120.00. A discount is allowed and the article sold at Sh. 96.00 Calculate the percentage discount.

Solution

Actual price = Sh. 120.00

Reduced price = Sh. 96.00

Discount

10. Sh. (120.00 96.00)
11. Sh. 24

Percentage discount

$$= \frac{24 \times 100}{120}$$

= 20%

EXERCISE

(1). The marked price of a shirt was Sh. 500.00. The shopkeeper offered a discount and sold it at Sh. 480. Calculate the percentage discount.

(2). Mama Mwanyumba bought the following goods from a supermarket: 3 kg of sugar @Sh. 46.00

2 loaves of bread @Sh. 22.50

4 packets of milk @Sh. 25.50

a. How much did she pay, for the goods?

b. How much would she have paid for the goods had she been allowed a 10% discount?

(3). Jane paid Sh. 12 000 for a T.V. set after she was allowed a discount of 16%. What was the marked price of the T.V?

(4). A school bought textbooks worth Sh. 27 027 from a bookseller. If the bookseller allowed a discount of 10%, what was the cost of the books without the discount?

(5). A farmer was allowed a cash discount of Sh. 175 on farm implements worth Sh. 3 500. What was the percentage discount?

(6). Calculate the marked price on a bag of cement selling at Sh. 570 after a discount of 5% is offered.

(7). An umbrella and a pen are sold at a discount of 8% and 3% respectively. Calculate the overall discount offered on the two commodities, if the cost of the umbrella is four times that of the pen.

(8). One day Mr. Makori bought some oranges worth Ksh 45, on another day of the same week his wife Mrs. Makori spent the same amount of Money but bought the oranges at a discount of 75 cents per orange

a. If Mr. Makori bought an orange at Ksh. x, write down and simplify an expression for the total number of oranges bought by the two in the week.

b. If Mrs. Makori bought 2 oranges more than her husband, find how much each spent on an orange.

(9). The marked price of a shirt is Sh. 800. A customer buys the shirt after being given a discount of 13%. The seller then realizes that he made a profit of 20% on this sale. Find how much the seller had bought the shirt.

(10). A trader sold an article at Sh. 4800 after allowing his customer a 12% discount on the marked price of the article. In so doing he made a profit of 45%. Calculate the marked price of the article.

What is a Commission?

Commission

- A commission is an agreed rate of payment, usually expressed as a percentage, to an agent for his services. Some employers offer a commission as an additional reward on top of a fixed salary, whereas others provide a commission-based salary only.
- Commission can be an excellent tool for motivating employees to meet performance objectives in terms of sales and profit growth. It can be especially beneficial to small businesses, as the wages they pay out are proportional to the performance outcomes of their workforce.

Formula

Commission Rate = $\frac{Commission}{sales} \times 100$

Example:

Mr. Nyongesa, a salesman in a soap industry, sold 250 pieces of toilet soap at Sh. 45.00 and 215 packets of detergent at Sh. 75.00 per packet. If he got a 5% commission on the sales, how much money did he get as commission?

Solution

Sales for the toilet soap was 250 x 45 = Sh. 11,250

Sales for the detergent was 215 x 75 = Sh. 16,125

Commission = $^5/_{100}$ (11,250+ 16,125)

24. $^5/_{100}$ x 27,375
25. Sh. 1368.75

EXERCISE

(1). Miss Onyango sold goods worth Sh. 12 000 at a commission of 5%. How much commission did she get?

(2). Chris works as a salesman. He is paid a salary of Sh. 24 000 per month plus a commission of 2% of his sales. In one month, he sold goods worth Sh. 100 000. How much did he earn altogether during that month?

(3). A salesman is paid a salary of Sh. 12 000 per month. He is also paid a commission of 2% on sales up to Sh. 15 000 and 21% on sales above that amount. In one month, he sold goods worth Sh. 2 500. How much was he paid that month?

(4). Simon earned Sh. 400 as a commission for a sale of goods worth Sh. 16 000. What would be his commission for a total sale of Sh. 7 000?

(5). A saleswoman was paid a monthly salary of Sh. 20 000 plus commission on goods sold. In one month, she sold goods worth Sh. 40 000. At the end of that month, her total earnings were Sh. 21 200. What percentage commission was she given?

(6). A saleslady was paid a monthly salary plus a commission of 8% on goods sold. In one month, she sold goods worth Sh. 64 000 and her total earnings were Sh. 23 120. What was her basic salary without commission?

(7). A salesman earns 25% commission. His sales amounted to Sh. 2 450 after giving buyers a 2% discount. Calculate his commission. Suppose all the goods were sold at the marked price, what would be his earnings?

(8). A company saleslady sold goods worth Kshs 240,000 from this sale she earned a commission of Kshs 4 000. Calculate the rate of commission;

a. If she sold good whose total marked price was Kshs 360 000 and allowed a discount of 2% calculate the amount of commission she received.

(9). A car dealer charges 5% commission for selling a car. He received a commission of Kshs17 500 for selling car. How much money did the owner receive from the sale of his car?

(10). A salesman gets a commission of 2.4 % on sales up to Kshs 100 000. He gets an additional commission of 1.5% on sales above this. Calculate the commission he gets on sales worth Kshs 280 000.

Simple Interest

- Interest is the money charged for the use of borrowed money for a specific period of time.
- If money is borrowed or deposited it earns interest, Principle is the sum of money borrowed or deposited P, Rate is the ratio of interest earned in a given period of time to the principle.
- The rate is expressed as a percentage of the principal per annum (P.A).
- When interest is calculated using only the initial principal at a given rate and time, it is called **simple interest** (I).

Simple Interest Formulae

Simple interest = principle x $\frac{\text{rate}}{100}$ x time

Example

Franny invests ksh 16,000 in a savings account. She earns a simple interest rate of 14%, paid annually on her investment. She intends to hold the investment for $1\frac{1}{2}$years. Determine the future value of the investment at maturity.

Solution

35. = $\frac{PRT}{100}$

12. sh. 16000 x $\frac{14}{100}$ x $\frac{3}{2}$
13. sh 3360

26. sh.16000 + sh 3360
27. sh.19360

Example

Calculate the rate of interest if sh 4500 earns sh 500 after $1\frac{1}{2}$years.

Solution

From the simple interest formulae

28. =

P

R

T

1

0

0

R=$\frac{100 \times I}{P \times T}$

P = sh 4500

I = sh 500

T = $1\frac{1}{2}$years

Therefore R = $\frac{100 \times 500}{4500 \times {}^{3}/_{2}}$

R=7.4%

Example

Esha invested a certain amount of money in a bank which paid 1 2% p.a. simple interest. After 5 years, his total savings were sh 5600.Determine the amount of money he invested initially.

Solution

Let the amount invested be sh P

T = 5 years

R = 12 % p.a.

A =sh 5600

But A = P + I

Therefore 5600 = P + P X $\frac{12}{100}$ X 5

36. P+0.60P
37. 1.6 P

Therefore P = $\frac{5600}{1.6}$

= sh 3500

Compound Interest

- Suppose you deposit money into a financial institution, it earns interest in a specified period of time.
- Instead of the interest being paid to the owner it may be added to (compounded with) the principle and therefore also earns interest.
- The interest earned is called compound interest. The period after which its compounded to the principle is called interest period.
- The compound interest maybe calculated annually, semi-annually, quarterly, monthly etc.
- If the rate of compound interest is R% p.a and the interest is calculated n times per year, then the rate of interest per period is $({}^{R}/_{n})$%

Example

Moyo lent ksh.2000 at interest of 5% per annum for 2 years. First we know that simple interest for 1 st year and 2nd year will be same

i.e. = 2000 x 5 x $^{1}/_{100}$ = Ksh. 100

Total simple interest for 2 years will be = 100 + 100 = ksh. 200

In Compound Interest (CI) the first year Interest will be same as of Simple Interest (SI) i.e. Ksh.100.

But year II interest is calculated on P + SI of 1 st year i.e. on ksh. 2000 + ksh. 100 = ksh. 2100. So,

year II interest in Compound Interest becomes

$= 2100 \times 5 \times {}^{1}/_{100}$ = Ksh. 105

So it is Ksh. 5 more than the simple interest. This increase is due to the fact that SI is added to the principal and this ksh.

105 is also added in the principal if we have to find the compound interest after 3 years.

Direct formula in case of compound interest is

$$A = P\left(\frac{1 + r}{100}\right)^t$$

Where A = Amount

P = Principal

r = Rate % per annum

t = Time

A=P+CI

$$P\left(1 + \frac{r}{100}\right)^t = P + CI$$

Types of Question

- Type I: To find CI and Amount
- Type II: To find rate, principal or time
- Type III: When difference between CI and SI is given.
- Type IV: When interest is calculated half yearly or quarterly etc.
- Type V: When both rate and principal have to be found.

Type 1

Example

Find the amount of ksh. 1000 in 2 years at 10% per annum compound interest.

Solution.

$A = P (1 + {}^{r}/_{100})^{t}$
$=1000 (1 + {}^{10}/_{100})^{2}$
$= 1000 \times {}^{121}/_{100}$
=ksh. 1210

Example

Find the amount of ksh. 6250 in 2 years at 4% per annum compound interest.

Solution

$A = P (1 + {}^{r}/_{100})^{t}$

38. $6250 (1 + {}^{4}/_{100})^{2}$
    $=6250 \times {}^{676}/_{625}$
39. ksh. 6760

Example

What will be the compound interest on ksh 31 250 at a rate of 4% per annum for 2 years?

Solution.

$CI = P(1 + {}^{r}/_{100})^{t} - 1$

$=31250 \{ (1 + {}^{4}/_{100})^{2} - 1 \}$

$=31250 ({}^{676}/_{625} - 1)$

$=31250 \times {}^{51}/_{625}$ = ksh. 2550

Example

A sum amounts to ksh. 24200 in 2 years at 10% per annum compound interest. Find the sum ?

Solution.

24200 = P (1 + 10/100)2

40. P $(^{11}/_{10})^2$
41. 24200 x $^{100}/_{121}$
42. ksh. 20000

Type II

Example.

The time in which ksh. 15625 will amount to ksh. 17576 at 4% compound interest is?

Solution

$A = P (1 + {}^{r}/_{100})^{t}$

$17576 = 15625 (1 + {}^{4}/_{100})^{t}$

$^{17576}/_{15625} = (^{26}/_{25})^{t}$

$(^{26}/_{25})^{t} = (^{26}/_{25})^{3}$

t = 3 years

Example

The rate percent if compound interest of ksh. 15625 for 3 years is Ksh. 1951.

Solution.

A=P+CI

= 15625 + 1951 = ksh. 1 7576

$A = P(1 + {}^{r}/_{100})^{t}$

$17576 = 15625 (1 + {}^{r}/_{100})^{3}$

$17576/1\,5625 = (1 + {}^{r}/_{100})^{3}$

$(^{26}/_{25})^{3} = (1 + {}^{r}/_{100})^{3}$

$^{26}/_{25} = 1 + {}^{r}/_{100}$

$^{26}/_{25} - 1 = {}^{r}/_{100}$

$^{1}/_{25} = {}^{r}/_{100}$

r = 4%

Type IV

1. Remember
    - When interest is compounded half yearly then Amount = P (1 + R/2)^{2t} 100

      I.e. in half yearly compound interest rate is halved and time is doubled.

28. When interest is compounded quarterly then rate is made ¼ and time is made 4 times. Then A = P [(1 +R/4)/100]4t

29. When rate of interest is $R_1\%$, $R_2\%$, and $R_3\%$ for 1st, 2nd and 3rd year respectively; then $A = P(1 + R_1/100)(1 + R_2/100)(1 + R_3/100)$

Example

Find the compound interest on ksh.5000 at 20% per annum for 1.5 year compound half yearly.

Solution.

Amount = 5000 [(1 + 20/2)/100]3/2 × 2

(ix) 5000 (1 + $^{10}/_{100})^3$

=5000 × $^{1331}/_{1000}$

(x) ksh 6655

CI = 6655 – 5000 = ksh. 1655

Example

Find compound interest ksh. 47145 at 12% per annum for 6 months, compounded quarterly.

Solution.

A = 471 45 [(1 + 12/4)/100]½ x 4

9. 47145 (1 + $^{3}/_{100})^2$

10. 47145 x $^{103}/_{100}$ x $^{103}/_{100}$

11. ksh. 50016.13

CI = 50016.13 – 471 45

= ksh. 2871.13

Example

Find the compound interest on ksh. 1 8750 for 2 years when the rate of interest for 1st year is 4% and for 2nd year 8%.

Solution.

A = P (1 + $^{R1}/_{100}$) (1 + $^{R1}/_{100}$)

= 18750 × $^{104}/_{100}$ × $^{108}/_{100}$

=ksh. 21060

CI = 21060 – 18750

= ksh. 2310

Type V

Example

The compound interest on a certain sum for two years is ksh. 52 and simple interest for the same period at same rate is ksh. 50, find the sum and the rate.

Solution.

We will do this question by basic concept. Simple interest is same every year and there is no difference between SI and CI for 1 st year.

The difference arises in the 2nd year because interest of 1 st year is added in principal and interest is now charged on principal + simple interest of 1 st year.

So in this question

2 year SI = ksh. 50

1 year SI = ksh. 25

Now CI for 1 st year = 52 - 25 = Ksh. 27

This additional interest 27 -25 = ksh. 2 is due to the fact that 1 st year SI i.e. ksh. 25 is added in principal.

It means that additional ksh. 2 interest is charged on ksh. 25. Rate % = $^{2}/_{25} \times 100 = 8\%$

Shortcut:

Rate % = [(CI - SI)/(SI/2)] x 1 00

6. $[(^{2}/_{50})/2] \times$

$100\ ^{2}/_{25} \times 100$
$=8\%$

$P = SI \times\ ^{100}/_{R} \times T = 50 \times\ ^{100}/_{8} \times 2$
= ksh. 312.50

Example

A sum of money lent CI amounts in 2 year to ksh. 8820 and in 3 years to ksh. 9261 . Find the sum and rate.

Solution.

Amount after 3 years = ksh. 9261

Amount after 2 years = ksh. 8820
By subtracting last year's interest ksh. 441
It is clear that this ksh. 441 is SI on ksh. 8820 from 2nd to 3rd year i.e. for 1 year.

Rate % = $441 \times\ ^{100}/_{8820} \times 1$
$=5\ \%$

Also $A = P(1 +\ ^{r}/_{100})^{t}$
$8820 = P(1 +\ ^{5}/_{100})^{2}$
$= P(^{21}/_{20})^{2}$
$P = 8820 \times 400/_{441}$
= ksh. 8000

Appreciation and Depreciation

Appreciation is the gain of value of an asset while depreciation is the loss of value of an asset.

Example

An iron box cost ksh 500 and every year it depreciates by 1 0% of its value at the beginning of that that year. What will its value be after value 4 years?

Solution

Value after the first year = sh $(500 - \frac{10}{100} \times 500)$
= sh 450
Value after the second year = sh $(450 - \frac{10}{100} \times 450)$
= sh 405
Value after the third year = sh $(405 - \frac{10}{100} \times 405)$
= sh 364.50
Value after the fourth year = sh $(364.50 - \frac{10}{100} \times 364.50)$

= sh 328.05

In general if P is the initial value of an asset, A the value after depreciation for n periods and r the rate of depreciation per period.

$A=P(1 - {}^{r}/_{100})^n$

Example

A minibus cost sh 400000.Due to wear and tear, it depreciates in value by 2 % every month. Find its value after one year,

Solution

$A=P(1 - {}^{r}/_{100})^{n}$

Substituting P= 400,000 , r = 2 , and n =12 in the formula ;

A =sh.400000 $(1 - 0.02)^{12}$

=sh.400,000$(0.98)^{12}$

= sh.313700

Example

The initial cost of a ranch is sh.5000, 000.At the end of each year, the land value increases by 2%.What will be the value of the ranch at the end of 3 years?

Solution

The value of the ranch after 3 years =sh 5,000, 000$(1 + {}^{2}/_{100})^{3}$

68. sh. 5,000,000$(1.02)^{3}$
69. sh 5,306,040

Hire Purchase

- Method of buying goods and services by instalments. The interest charged for buying goods or services on credit is called carrying charge.
  Hire purchase = Deposit + (instalments x time)

Example

Achieng wants to buy a sewing machine on hire purchase. It has a cash price of ksh 7500. She can pay a cash price or make a down payment of sh 2250 and 15 monthly instalments of sh.550 each. How much interest does she pay under the instalment plan?

Solution

Total amount of instalments = sh 550 x 15 = sh 8250

Down payment (deposit) = sh 2250

Total payment = sh (8250 + 2250) = sh 10500

Amount of interest charged = sh (10500-7500)

= sh3000

Note;

- Always use the above formula to find other variables.

Income Tax

- Taxes on personal income is income tax. Gross income is the total amount of money due to the individual at the end of the month or the year.
  Gross income = salary + allowances / benefits

- Taxable income is the amount on which tax is levied. This is the gross income less any special benefits on which taxes are not levied. Such benefits include refunds for expenses incurred while one is on official duty.
- In order to calculate the income tax that one has to pay, we convert the taxable income into Kenya pounds K£ per annum or per month as dictated by the by the table of rates given.

Relief

- Every employee in kenya is entitled to an automatic personal tax relief of sh.12672 p.a (sh.1 056 per month) • An employee with a life insurance policy on his life, that of his wife or child, may make a tax claim on the premiums paid towards the policy at sh.3 per pound subject to a maximum claim of sh .3000 per month.

Example

Mr. John earns a total of K£ 12300 p.a.Calculate how much tax he should pay per annum.Using the tax table below.

Income tax K£ per annum	Rate (sh per pound)
- 1 5808	2
5809 - 1 1 280	3
1 1 289 - 1 6752	4
6753 1 - 22224	5
Excess over 22224	6

Solution

His salary lies between £ 1 and £1 2300.The highest tax band is therefore the third band.

For the first £5808, tax due is sh 5808 x 2 = sh 11616
For the next £5472, tax due is sh 5472 x 2 = sh 16416
Remaining £1020, tax due sh. 1020 x 4 = sh 4080 +
Total tax due sh 32112
Less personal relief of sh.1056 x 12 = sh.12672 –
Sh 19440
Therefore payable p.a is sh.19400.

Example

Mr. Ogembo earns a basic salary of sh 15000 per month.in addition he gets a medical allowance of sh 2400 and a house allowance of sh 12000.Use the tax table above to calculate the tax he pays per year.

Solution

Taxable income per month = sh (15000 + 2400 + 12000) = sh.29400

Converting to K£ p.a = K£ $\frac{29400}{20}$ x 12

= K£ 1 7640

Tax due
First £ 5808 = sh.5808 x 2 = sh. 11616
Next £ 5472 = sh.5472 x 3 = sh. 16416
Next £ 5472 = sh.5472 x 4 = sh. 21888
Remaining £ 888 = sh.888 x 5 = sh 4440+
Total tax due sh 54360
Less personal relief sh 12672 –
Therefore, tax payable p.a sh 41688

PAYE

- In Kenya, every employer is required by the law to deduct income tax from the monthly earnings of his employees every month and to remit the money to the income tax department.
- This system is called Pay As You Earn (PAYE).

Housing

- If an employee is provided with a house by the employer (either freely or for a nominal rent) then 15% of his salary is added to his salary (less rent paid) for purpose of tax calculation.
- If the tax payer is a director and is provided with a free house, then 1 5% of his salary is added to his salary before taxation.

Example

Mr. Omondi who is a civil servant lives in government house who pays a rent of sh 500 per month. If his salary is £9000 p.a, calculate how much PAYE he remits monthly.

Solution

Basic salary £ 9000

Housing £ $\frac{15}{100} \times 9000$ = £1350

Less rent paid = £1350 – £ 300 = £ 1050

Taxable income

Tax charged;

First £ 5808, the tax due is sh.5808 x 2 = sh 11616

Remaining £ 4242, the tax due is sh 4242 x 3 = sh 12726 +

sh 24342

Less personal relief sh 12672 –

sh 11670

PAYE = sh $\frac{11670}{12}$

= sh 972.50

Example

Mr. Odhiambo is a senior teacher on a monthly basic salary of Ksh. 1 6000.On top of his salary he gets a house allowance of sh 1 2000, a medical allowance of Ksh.3060 and a hardship allowance of Ksh 3060 and a hardship allowance of Ksh.4635.He has a life insurance policy for which he pays Ksh.800 per month and claims insurance relief.

10. Use the tax table below to calculate his PAYE.

Income in £ per month	Rate %
1-484	10
485 - 940	15
941 - 1 396	20
1397- 1 852	25
Excess over 1 852	30

11. In addition to PAYEE the following deductions are made on his pay every month

WCPS at 2% of basic salary

HHIF ksh.400

Co operative shares and loan recovery Ksh 4800.

Solution

113. Taxable income = Ksh (16000 + 12000 + 3060 + 4635) =

ksh 35695

Converting to K£ = K£ $\frac{35695}{20}$

= K£ 1784.75

Tax charged is:

First £ 484 = £484 x $\frac{10}{100}$ = £ 48.40

Next £ 456 = £456 x $\frac{15}{100}$ = £ 68.40

Next £ 456 = £456 x $\frac{10}{100}$ = £ 91 .20

Remaining £ 388 = £388 x $\frac{25}{100}$ = £ 97.00.

Total tax due = £305.00

= sh 61 00

Insurance relief = sh $\frac{800}{20}$ x 3 = sh 120

Personal relief = sh 1056

Total relief 120 + 1056 = sh 1176

Tax payable per month is sh 6100

less total relief sh 1176 −

sh 4924

Therefore, PAYE is sh 4924.

Note;

- For the calculation of PAYE, taxable income is rounded down or truncated to the nearest whole number.
- If an employee's due tax is less than the relief allocated, then that employee is exempted from PAYEE

15. Total deductions are

Sh (2 x 16000 + 400 + 4800 + 800 + 4924) = sh 11244 100

Net pay = sh (35695 − 11244) =

sh 24451

Questions

16. A business woman opened an account by depositing Kshs. 12,000 in a bank on 1st July 1995. Each subsequent year, she deposited the same amount on 1st July. The bank offered her 9% per annum compound interest. Calculate the total amount in her account on 30

    30th June 1 996

    30th June 1 997

17. A construction company requires to transport 1 44 tonnes of stones to sites A and B. The company pays Kshs 24,000 to transport 48 tonnes of stone for every 28 km. Kimani transported 96 tonnes to a site A, 49 km away.

    Find how much he paid

    Kimani spends Kshs 3,000 to transport every 8 tonnes of stones to site.

    Calculate his total profit.

    Achieng transported the remaining stones to sites B, 84 km away. If she made 44% profit, find her transport cost.

18. The table shows income tax rates

Monthly taxable pay	Rate of tax Kshs in 1 K£

1–435	2
436 – 870	3
168. -1305	4
1306 – 1740	5
Excess Over 1740	6

A company employee earn a monthly basic salary of Kshs 30,000 and is also given taxable allowances amounting to Kshs 1 0, 480.

Calculate the total income tax

The employee is entitled to a personal tax relief of Kshs 800 per month.

Determine the net tax.

If the employee received a 50% increase in his total income, calculate the corresponding percentage increase on the income tax.

11. A house is to be sold either on cash basis or through a loan. The cash price is Kshs.750, 000. The loan conditions area as follows: there is to be down payment of 10% of the cash price and the rest of the money is to be paid through a loan at 1 0% per annum compound interest. A customer decided to buy the house through a loan.

    i. Calculate the amount of money loaned to the customer.
    ii. The customer paid the loan in 3 year's. Calculate the total amount paid for the house.

    Find how long the customer would have taken to fully pay for the house if she paid a total of Kshs 891,750.

12. A businessman obtained a loan of Kshs. 450,000 from a bank to buy a matatu valued at the same amount. The bank charges interest at 24% per annum compound quarterly

    Calculate the total amount of money the businessman paid to clear the loan in 1 $\frac{1}{2}$ years.

    The average income realized from the matatu per day was Kshs. 1 500. The matatu worked for 3 years at an average of 280 days year. Calculate the total income from the matatu.

    During the three years, the value of the matatu depreciated at the rate of 1 6% per annum. If the businessman sold the matatu at its new value, calculate the total profit he realized by the end of three years.

13. A bank either pays simple interest as 5% p.a or compound interest 5% p.a on deposits. Nekesa deposited Kshs P in the bank for two years on simple interest terms. If she had deposited the same amount for two years on compound interest terms, she would have earned Kshs 210 more. Calculate without using Mathematics Tables, the values of P

7.
    a. A certain sum of money is deposited in a bank that pays simple interest at a certain rate. After 5 years the total amount of money in an account is Kshs 358400. The interest earned each year is 12 800

       Calculate

        i. The amount of money which was deposited
        ii. The annual rate of interest that the bank paid
    b. A computer whose marked price is Kshs 40,000 is sold at Kshs 56,000 on hire purchase terms
        i. Kioko bought the computer on hire purchase terms. He paid a deposit of 25% of the hire purchase price and cleared the balance by equal monthly installments of Kshs 2625. Calculate the number of installments (3mks)
        ii. Had Kioko bought the computer on cash terms he would have been allowed a discount of 1 2$\frac{1}{2}$ % on marked price. Calculate the difference between the cash price and the hire purchase price and express as a percentage of the cash price
        iii. Calculate the difference between the cash price and hire purchase price and express it as a percentage of the cash price.

8. The table below is a part of tax table for monthly income for the year 2004

	Monthly taxable income In (Kshs) Under Kshs 9681

Tax rate percentage	(%) in each shillings
	10%
From Kshs 9681 but under 1 8801	15%
From Kshs 1 8801 but 27921	20%

In the tax year 2004, the tax of Kerubo's monthly income was Kshs 1916. Calculate Kerubo's monthly income

9. The cash price of a T.V set is Kshs 1 3, 800. A customer opts to buy the set on hire purchase terms by paying a deposit of Kshs 2280. If simple interest of 20 p. a is charged on the balance and the customer is required to repay by 24 equal monthly installments. Calculate the amount of each installment.

10. A plot of land valued at Ksh. 50,000 at the start of 1 994. Thereafter, every year, it appreciated by 1 0% of its previous years value find:
    a. The value of the land at the start of 1 995
    b. The value of the land at the end of 1 997
11. The table below shows Kenya tax rates in a certain year.

Income K £ per annum	Tax rates Kshs per K £
1 -4512	2
451 3 - 9024	3
9025 - 1 3536	4
1 3537 - 1 8048	5
1 8049 - 22560	6
Over 22560	6.5

    In that year Muhando earned a salary of Ksh. 1 651 0 per month. He was entitled to a monthly tax relief of Ksh. 960,
    Calculate
    a. Muhando annual salary in K £
    b. The monthly tax paid by Muhando in Ksh
12. A tailor intends to buy a sewing machine which costs Ksh 48,000. He borrows the money from a bank. The loan has to be repaid at the end of the second year. The bank charges an interest at the rate of 24% per annum compounded half yearly. Calculate the total amount payable to the bank.
13. The average rate of depreciation in value of a water pump is 9% per annum. After three complete years its value was Ksh 1 50,700. Find its value at the start of the three year period.
14. A water pump costs Ksh 21 600 when new, at the end of the first year its value depreciates by 25%. The depreciation at the end of the second year is 20% and thereafter the rate of depreciation is 1 5% yearly. Calculate the exact value of the water pump at the end of the fourth year

Approximation and Errors

Approximation

Approximation involves rounding off and truncating numbers to give an estimation

Rounding Off

In rounding off the place value to which a number is to be rounded off must be stated.

The digit occupying the next lower place value is considered.

The number is rounded up if the digit is greater or equal to 5 and rounded down if it�s less than 5.

Example

Round off 395.184 to:

36. The nearest hundreds
37. Four significant figures
38. The nearest whole number
39. Two decimal places

Solution

14. 400
15. 395.2
16. 395
17. 395.18

Truncating

Truncating means cutting off numbers to the given decimal places or significant figures, ignoring the rest.

Example

Truncate 3.2465 to

29. 3 decimal places
30. 3 significant figures

Solution

31.3.246

32. 3.24

Estimation

Estimation involves rounding off numbers in order to carry out a calculation faster to get an approximate answer .This acts as a useful check on the actual answer.

Example

Estimate the answer to $\frac{152 \times 269}{32}$

Solution

The answer should be close to $\frac{150 \times 270}{30} = 1350$

The exact answer is 1 277.75. 1 277.75 writen to 2 significant figures is 1 300 which is close to the estimated answer.

Accuracy and Error

Absolute Error

The absolute error of a stated measurement is half of the least unit of measurement used.

When a measurement is stated as 3.6 cm to the nearest millimeter, it lies between 3.55 cm and 3.65 cm.

The least unit of measurement is milliliter, or 0.1 cm.The greatest possible error is 3.55 - 3.6 = -0.05 or 3.65 - 3.6 = + 0.05.

To get the absolute error we ignore the sign. So the absolute error is 0.05 thus,|-0.05| =| +0.05|= 0.05.

When a measurement is stated as 2.348 cm to the nearest thousandths of a centimeters (0.001) then the absolute error is $^1/_2 \times 0.001 = 0.0005$.

Relative Error

Relative error = $\frac{\text{absolute}}{\text{actual measurements}}$

Example

An error of 0.5 kg was found when measuring the mass of a bull.if the actual mass of the bull was found to be 200kg. Find th relative error

Solution

elative error = $\frac{\text{absolute}}{}$ = $\frac{0.5 \text{ kg}}{}$ = 0.0025 actual measurements 200

Percentage Error

Percentage error = relative error x 1 00%

$= \frac{\text{absolute error}}{\text{actual measurment}} \times 100\%$

Example

The thickness of a coin is 0.20 cm.

43. The percentage error
44. What would be the percentage error if the thickness was stated as 0.2 cm ?

Solution

The smallest unit of measurement is 0.01

Absolute error = $^1/_2 \times 0.01 = 0.005$

Percentage error = $\frac{0.005}{0.20} \times 100\% = 2.5\ \%$

The smallest unit of measurement is 0.1

Absolute error = $^1/_2 \times 0.1 = 0.05$ cm

Percentage error = $^{0.05}/_{0.2} \times 100\% = 25\ \%$

Rounding Off Error

An error found when a number is rounded off to the desired number of decimal places or significant figures, for example when a recurring decimal 1.6 is rounded to the 2 significant figures, it becames 1.7 the rounded off error is;

$1.7 - 1.6 = {^{17}/_{10}} - {^5/_3} = {^1/_{30}}$

Note;

1.6 which is a recurring decimal converted to a fraction is $^5/_3$

Truncating Error

The error introduced due to truncating is called a truncation error.in the case of 1.6 truncated to 2 S.F., the truncated error is; $|1.6 - 1.6| = |1{^6/_{10}} - 1{^2/_3}| = {^1/_{15}}$

Propagation of Errors

Addition and subtraction

What is the error in the sum of 4.5 cm and 6.1 cm, if each represent a measure measurement.

Solution

The limits within which the measurements lie are 4.45, i.e. ., 4.55 or 4.5 ± 0.005 and 6.05 to 6.1 5, i.e. 6.1 ±0.05.

The maximum possible sum is 4.55 + 6.15 =10.7cm

The minimum possible sum is 4.45 + 6.05 =10.5 cm

The working sum is 4.5 + 6.1 = 10.6

The absolute error = maximum sum working sum

$=|10.7 - 10.6|$

$=0.10$

Example

What is the error in the difference between the measurements 0.72 g and 0.31 g?

Solution

The measurement lie within 0.72 ± 0.005 and 0.31 ± 0.005 respectively

The maximum possible difference will be obtained if we substract the minimum value of the second measurement from the maximum value of the first, i.e ;

0.725 − 0.305 cm

The minimum possible difference is 0.71 5 − 0.31 5 = 0.400.the working difference is 0.72 − 0.31 =0.41, which has an absolute error of |0.420 -0.41 | or |0.400 − 0.41 | = 0.1 0.

Since our working difference is 0.41, we give the absolute error as 0.01 (to 2 s.f)

Note:

In both addition and subtraction, the absolute error in the answer is equal to the sum of the absolute errors in the original measurements.

Multiplication

Example

A rectangular card measures 5.3 cm by 2.5 cm. find

45. The absolute error in the rea of the card
46. The relative error in the area of the cord

Solution

47. The length lies within the limits 5.3 ± 0.05 cm
48. The length lies within the limits 2.5 ± 0.05 cm

The maximum possible area is 2.55 x 5.35 =13.6425 cm^2
The minimum possible area is 2.45 x 5.25 =12.8625 cm^2
The working area is 5.3 x 2.5 = 13.25 cm^2
Maximum area − working area = 13.6425 − 13.25 = 0.3925.
Working area − minimum area = 13.25 − 12.8625 = 0.3875
We take the absolute error as the average of the two.

Thus, absolute error = $\frac{0.3925 + 0.3875}{2}$ = 0.3900

The same can also be found by taking half the interval between the maximum area and the minimum area $^1/_2$(13.6425-12.8625) = 0.39
The relative error in the area is :

$\frac{0.39}{13.25}$ = 0.039 (to 2 S.F)

Division

Given 8.6 cm ÷ 3.4 cm.Find:

30. The absolute error in the quotient
31. The relative error in the quotient

Solution

(xi) 8.6 cm has limits 8.55 cm and 8.65 cm. 3.4 has limits 3.35 cm and 3.45 cm.

The maximum possible quotient will be given by the maximum possible value of the numerator and the smallest possible value of the denominator, i.e.,

$\frac{8.65}{3.35} = 2.58$ (to 3 s.f)

The minimum possible quotient will be given by the minimum possible value of the numerator and the biggest possible value of the denominator, i.e.

$\frac{8.65}{3.45} = 2.48$ (to 3 s.f)

The working quotient is;

$\frac{8.6}{3.4} = 2.53$ (to 3 .f.)

The absolute error in the quotient is;

$\frac{2.53 \times 2.48}{2} = {}^1/_2 \times 0.10$

0.050 (to 2 s.f)

(xii) Relative error in the working quotient ;

$\frac{0.05}{2.53} = \frac{5}{253}$

12. 0.01 97

13. 0.020 (to 2 s.f)

Alternatively

Relative error in the numerator is $\frac{0.05}{8.6} = 0.00581$

Relative error in the denominator is $\frac{0.05}{3.4} = 0.0147$

Sum of the relative errors in the numerator and denominator is

0.00581 + 0.01 47 = 0.02051 s

=0.021 to 2 S.F

Questions

1.

a. Work out the exact value of R = $\frac{1}{0.003146 - 0.003130}$

An approximate value of R may be obtained by first correcting each of the decimal in the denominator to 5 decimal places

The approximate value

The error introduced by the approximation

70. The radius of circle is given as 2.8 cm to 2 significant figures

If C is the circumference of the circle, determine the limits between which C/π lies

By taking π to be 3.142, find, to 4 significant figures the line between which the circumference lies.

71. The length and breadth of a rectangular floor were measured and found to be 4.1 m and 2.2 m respectively. If possible error of 0.01 m was made in each of the measurements, find the:

Maximum and minimum possible area of the floor

Maximum possible wastage in carpet ordered to cover the whole floor

72.In this question Mathematical Tables should not be used

The base and perpendicular height of a triangle measured to the nearest centimeter are 6 cm and 4 cm respectively.

Find

The absolute error in calculating the area of the triangle

The percentage error in the area, giving the answer to 1 decimal place

12. By correcting each number to one significant figure, approximate the value of 788 x 0.006. Hence calculate the percentage error arising from this approximation.
13. A rectangular block has a square base whose side is exactly 8 cm. Its height measured to the nearest millimeter is 3.1 cm

    Find in cubic centimeters, the greatest possible error in calculating its volume.
14. Find the limits within the area of a parallegram whose base is 8cm and height is 5 cm lies. Hence find the relative error in the area
15. Find the minimum possible perimeter of a regular pentagon whose side is 1 5.0cm.
16. Given the number 0.237

    Round off to two significant figures and find the round off error

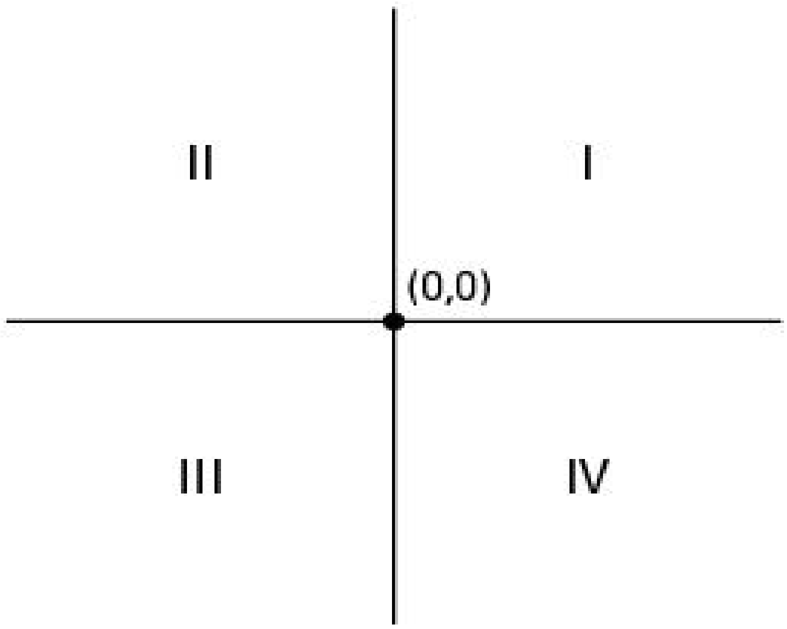
    Truncate to two significant figures and find the truncation error

16. The measurements a = 6.3, b= 1 5.8, c= 1 4.2 and d= 0.001 73 have maximum possible errors of 1%, 2%, 3% and 4% respectively. Find the maximum possible percentage error in $^{ad}/_{bc}$ correct to 1 sf.

Commonly Used Terms in Coordinates and Graphs

Terms used in coordinates and graphs

- The position of a point in a plan is located using an ordered pair of numbers called **coordinates** and is written in the form **(x, y)**.
- The first number represents distance along the x-axis and is called **x coordinates**.
- The second number represents distance along the y-axis and is called the **y coordinates**.
- The x and y axes intersect at the point (0, 0), called **the origin**.
- The coordinate graph is divided into four quarters called **quadrants**. These quadrants are labeled in the **Figure below;**



Notice the following:

- In quadrant I, x is always positive and y is always positive.
- In quadrant II, x is always negative and y is always positive.
- In quadrant III, x and y are both always negative.
- In quadrant IV, x is always positive and y is always negative

EXCERCISE

(1). What is meant by the term coordinates?

(2). What is the meaning of the following terms?

a. y coordinates

b. x coordinates

40. y-axes

41. x-axes

(3). Explain why in the I quadrant, **x** and **y** are always positive

(4). In the iv quadrant, why is **x** positive and **y** negative?

(5). What is meant by the term **the origin?**

(6). Explain why in the iii quadrant, both **x** and **y** are negative

(7). What is the meaning of intersection?

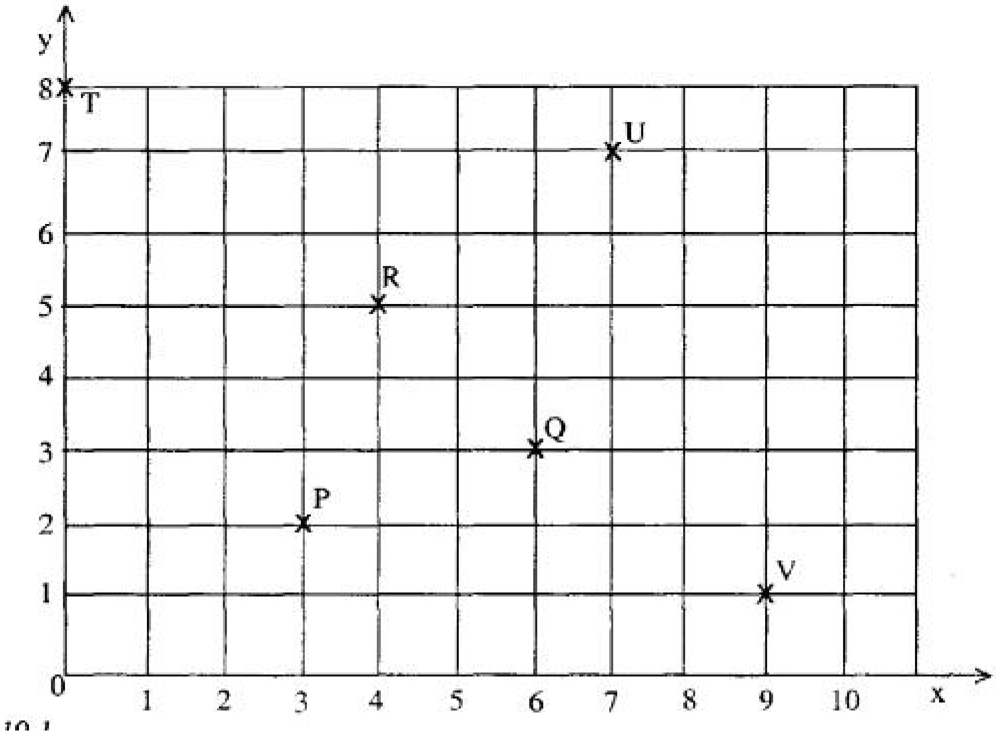
How to Find Coordinates of Points on a Cartesian Plane

Coordinates of points on a cartesian plane

- The position of a point in a plane is located using an ordered pair of numbers called **coordinates** and is written in the form **(x, y).**
- Each point in the plane is identified by its x-coordinate, or horizontal displacement from the origin, and its y-coordinate, or vertical displacement from the origin. Together we write them as an ordered pair indicating the combined distance from the origin in the form (x, y). An ordered pair is also known as a coordinate pair because it consists of x and y-coordinates.

Example;

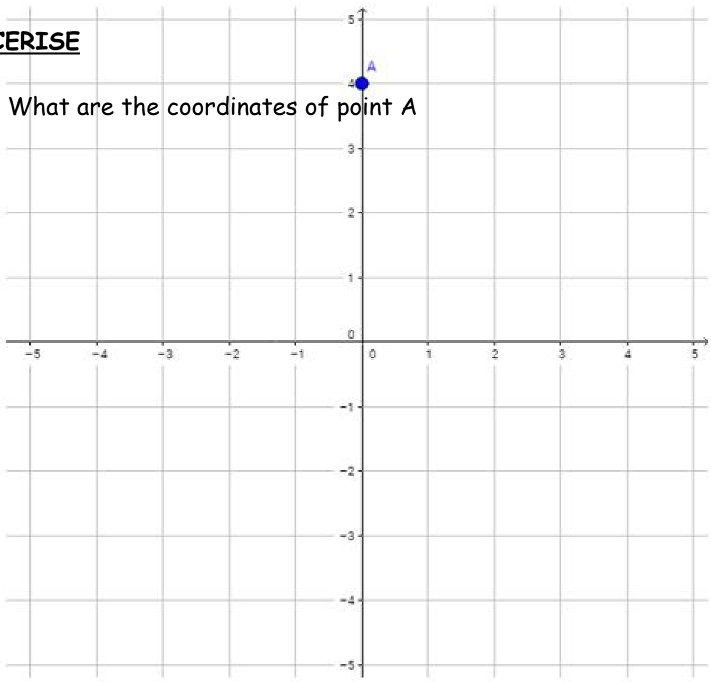
The position of the point P is (3, 2). the position of the points Q, R, S, U and V.



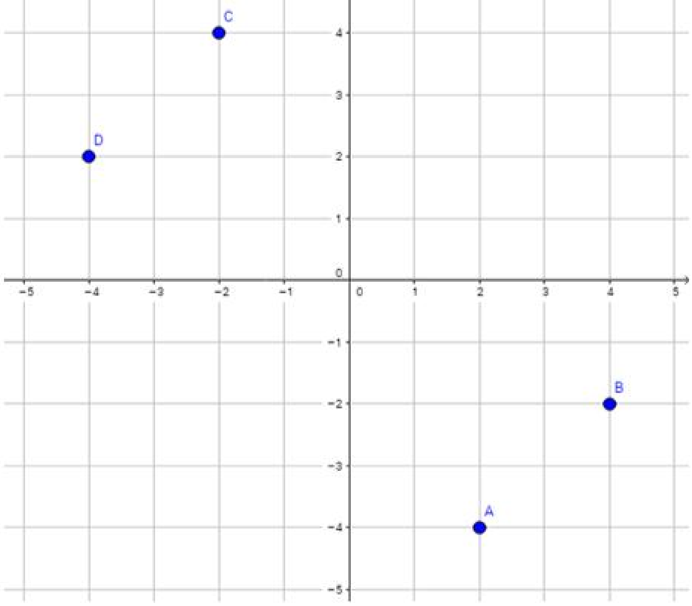
Q (6, 3) R (4, 5) U(7, 7) and V(9, 1) T(0, 8)

EXCERISE

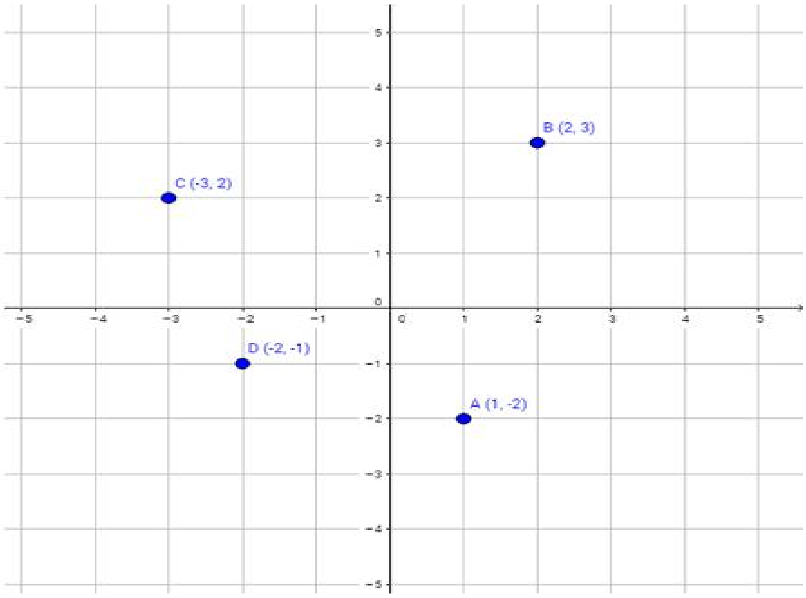
(1). What are the coordinates of point A



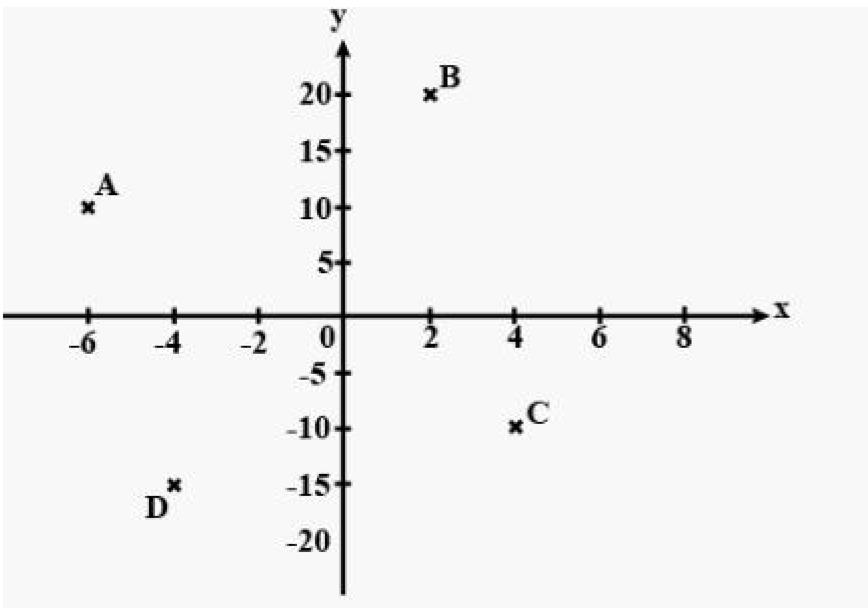
(2). Which of the points has coordinates (-2, 4)?



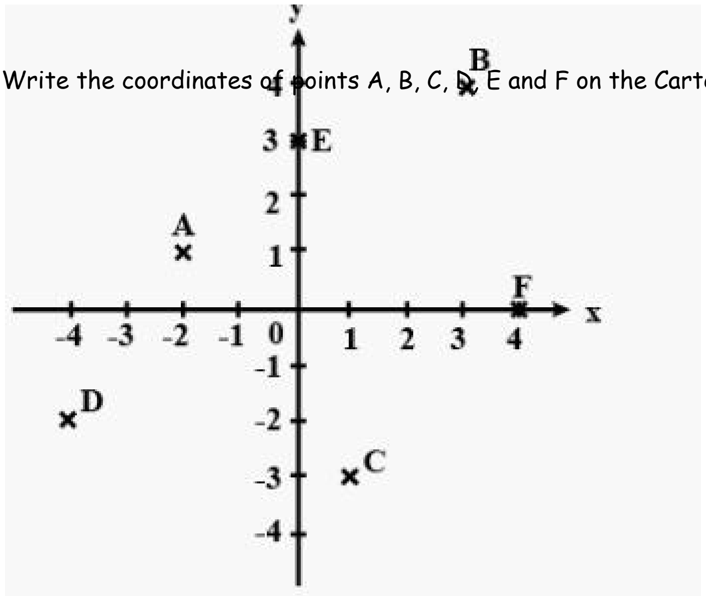
(3). The coordinates of which points are entered correctly?



(4). State the coordinates of points A, B, C and D on the Cartesian plane below.

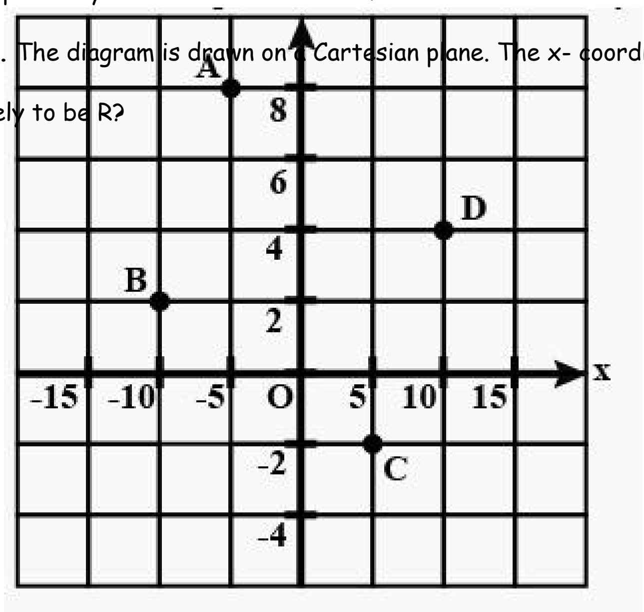


(5). Write the coordinates of points A, B, C, D, E and F on the Cartesian plane.

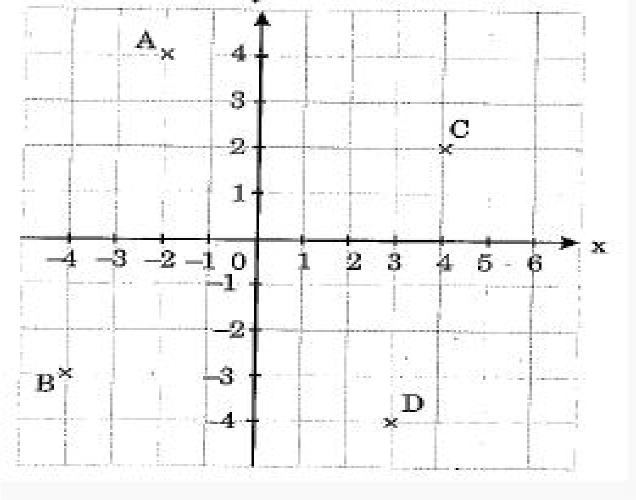


(6). ABCD is a square on the cartesian plane with A, B, and C having coordinates (2, 2), (3, 2), and (3, 1) respectively. Find the coordinates of D.

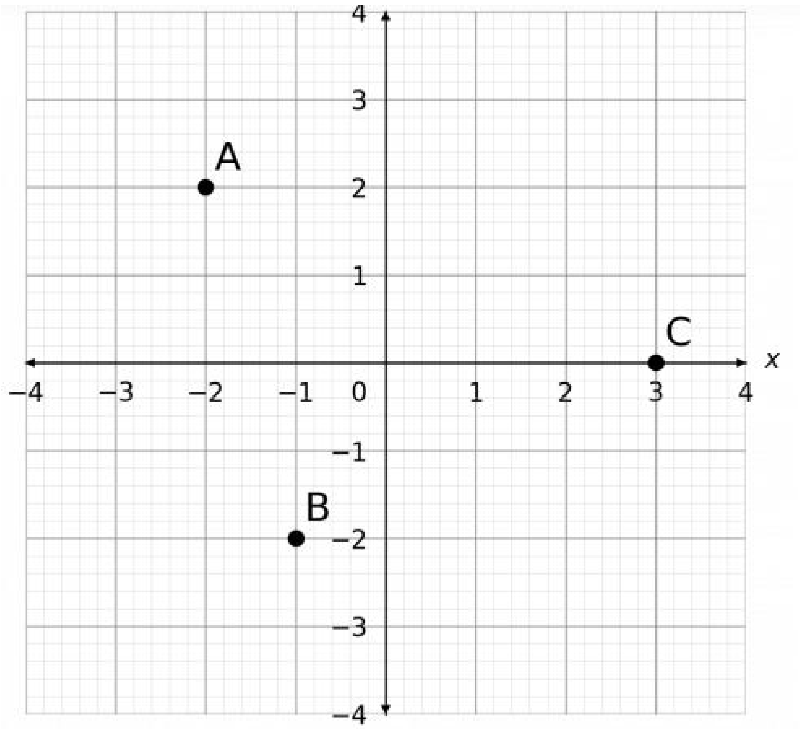
(7). The diagram is drawn on a Cartesian plane. The x- coordinate of point R is -10. Which of the points A, B, C and D is likely to be R?



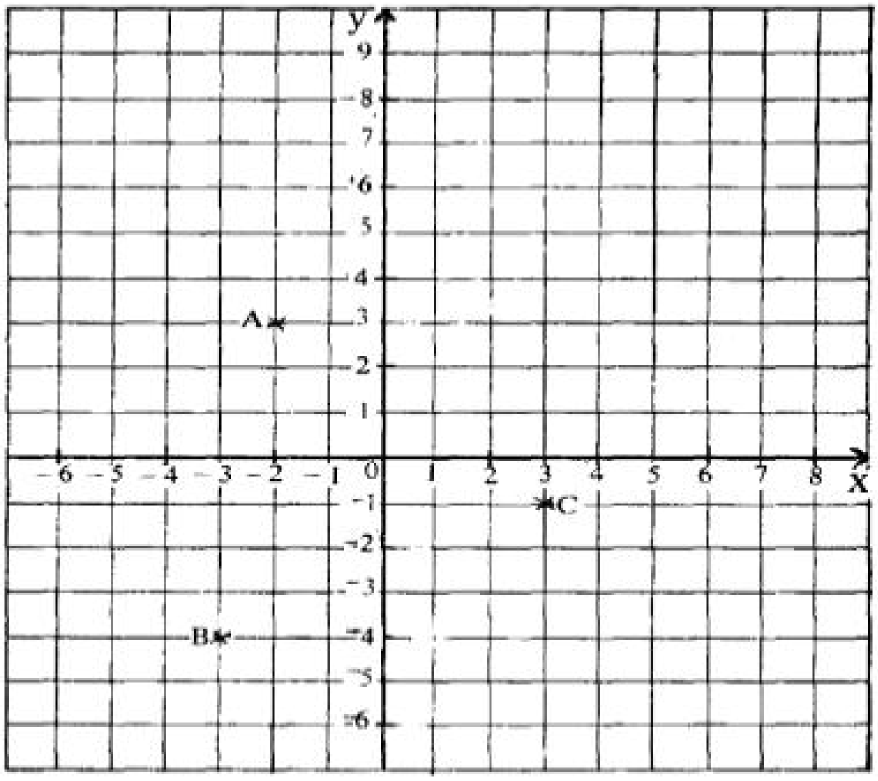
(8). The diagram shows a Cartesian plane. Then y-coordinate of point D is?



(9). Write down the coordinates of points, *A*, *B*, and *C* seen below.



(10). What are the coordinates of points A, B and C.



How to Plot Points on a Cartesian Plane Given Coordinates

<u>Plotting points on a cartesian plane given the coordinates</u>

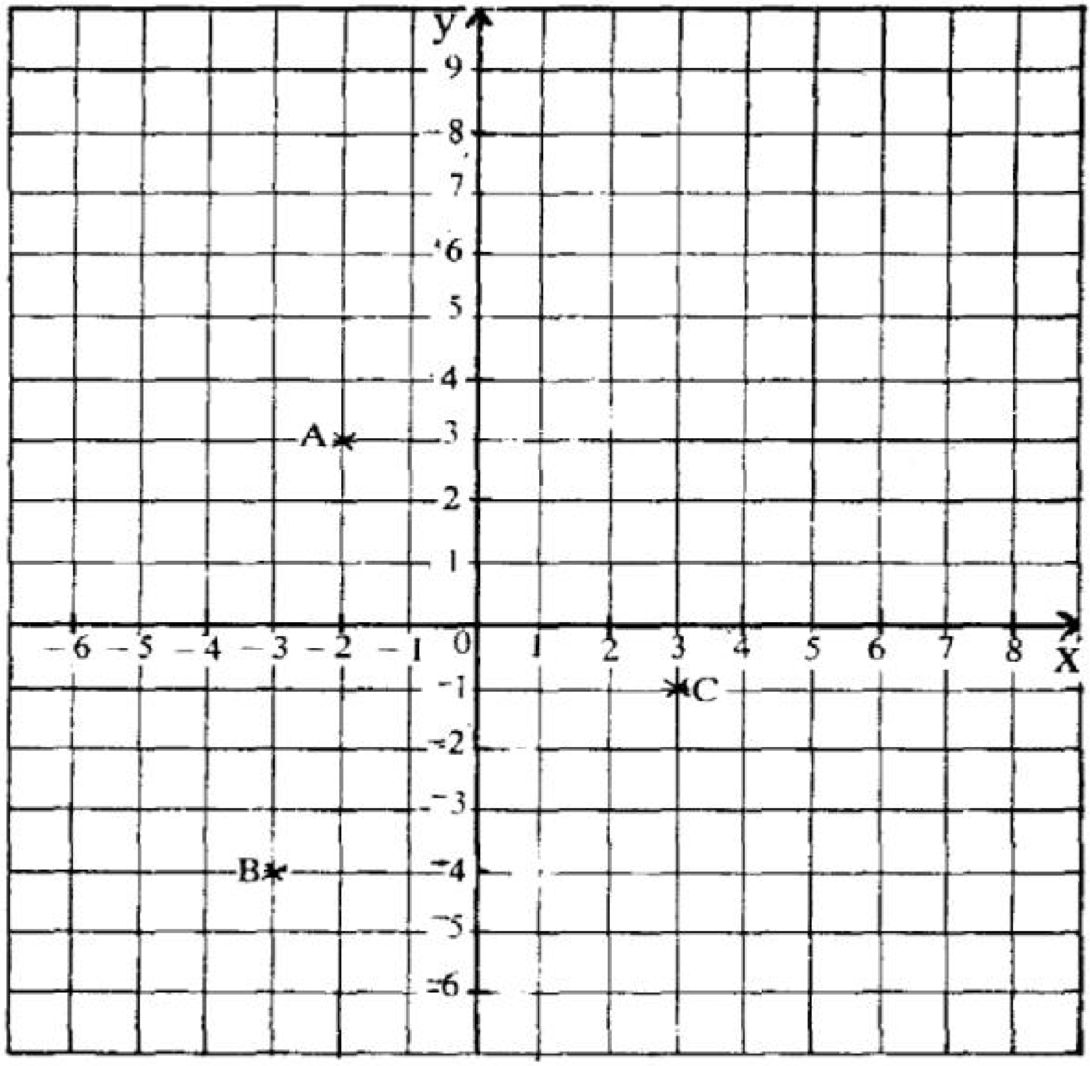
- Coordinates are the numbers in a point's name. For instance, the point (−3, 4) has an x-coordinate −3 and y-coordinate 4. The x-coordinate is the first coordinate; the y-coordinate is the second coordinate.

The two important rules to plot a point in the Cartesian plane are given below:

18. The first coordinate in the ordered pair (x) represents the left/right movement of a point from the origin.
19. The second coordinate in the ordered pair (y) represents the up/down movement of the point from the origin.

Example;

Plot the following points on a cartesian plane given the coordinates A(2, 3), B(3, 4) and C(3, 1).



EXERCISE

(1). Plot the coordinates (3, 5) and (5, 4) in the cartesian coordinate system

(2). Plot the following points in the cartesian plane (Use the scale: x-axis = 1 cm and y-axis = 1 cm)

x	-3	0	-1	4	2
y	7	-3.5	-3	4	-3

(3). Plot the following on a Cartesian coordinate plane:

a. (1, 2)

b. (3, 4)

c. (2, 1)

(4). Plot the point (4, 2) and identify which quadrant or axis it is located.

(5). Draw a pair of axes on a squared paper and plot the following points: L (4, 2), M (4, 2), N (2.72, 3.25)

(6). Plot the points of a triangle with vertices A(4,–1), B(4,–4), and C(−3,–4).

(7). Plot the following points on the graph and name the quadrant in which each point lies:

a. A (–7, –8)

b. B (–8, 7)

c. C (–5, 0)

d. D (1, 5)

(8). Plot the points (3, 4), (–3,–3), (–7, 6), and (0, –6) on the Cartesian plane and give their positions in quadrants/axes.

(9). In which quadrant or on which axis do each of the points (–2, 4), (3, –1), (–1, 0), (1, 2), and (–3, –5) lie? Verify your answer by locating them on the Cartesian plane.

(10). Plot the point (2, 3) on a cartesian plane.

Graphs of Straight Lines (y= h and x = k)

Graphs of straight lines(y =h and x =k)

- To **plot straight line graphs** we need to substitute values for x into the equation for the graph and work out the corresponding values for y.
- We often put these values in a table to make our work clearer. Once we have calculated the coordinates, we can plot these as a graph.

Example;

Y = 4

A horizontal line crossing y-axis at 4

X=-4

A vertical line crossing x-axis at -4

EXCERCISE

(1). Plot a graph of **y = -6**

(2). Using a suitable scale, come up with a graph of **x = +2**

(3). Draw a straight line graph **y = + 10**

(4). For each of the following pair of lines, draw their graphs

a. y = 5

b. y+3 = 0

(5).

a. y - 6 = 0

b. y = 3+0

(6). By choosing any two suitable points, draw a graph of **x = 0**

(7). Draw a graph of **y 4 = 0** using any two points

(8). Plot a graph of **y = +4** .

(9). Using -3 to 3 as the point, plot a graph of **y = +3**

(10). Draw the graph of the line **y-3 = 7**

Graphs of Straight Lines (y= mx +c)

Graphs of straight lines(y =h and x =k)

- To **plot straight line graphs** we need to substitute values for x into the equation for the graph and work out the corresponding values for y.
- We often put these values in a table to make our work clearer. Once we have calculated the coordinates, we can plot these as a graph.

Example;

Y = 4

A horizontal line crossing y-axis at 4

X=-4

A vertical line crossing x-axis at -4

EXCERCISE

(1). Plot a graph of **y = -6**

(2). Using a suitable scale, come up with a graph of **x = +2**

(3). Draw a straight line graph **y =+ 10**

(4). For each of the following pair of lines, draw their graphs

a. y = 5

b. y + 3 = 0

(5).

a. y - 6 = 0

b. y = 3 + 0

(6). By choosing any two suitable points, draw a graph of **x = 0**

(7). Draw a graph of **y 4 = 0** using any two points

(8). Plot a graph of **y = +4** .

(9). Using -3 to 3 as the point, plot a graph of **y = +3**

(10). Draw the graph of the line **y - 3 = 7**

Graphical Solution of Simultaneous Equations

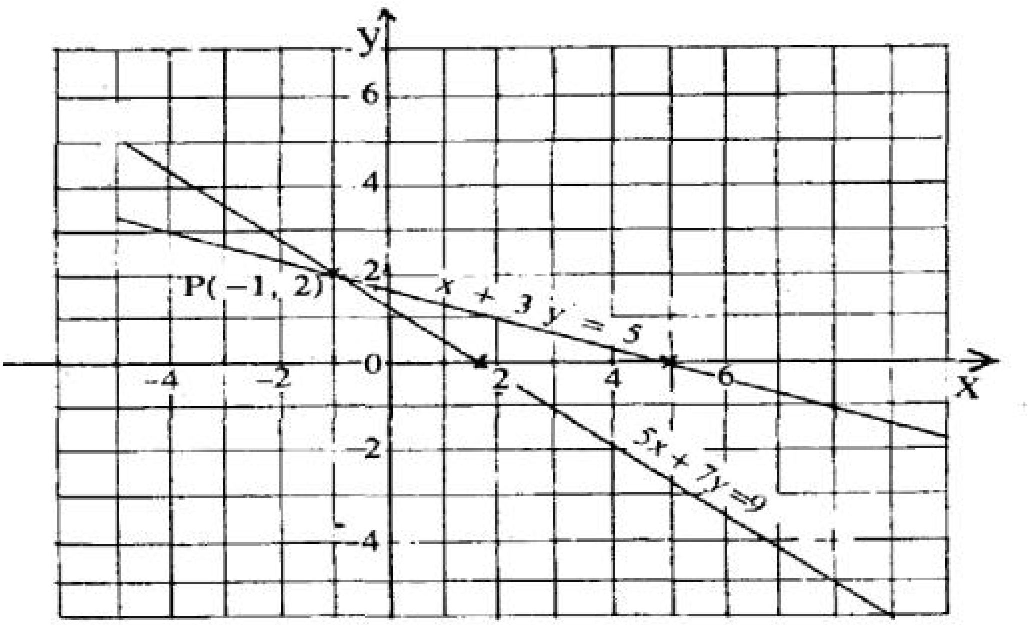
Graphical solution of simultaneous equations

- So far we have seen that equations of the form ax + by = c represents a straight line. When two such linear equations are graphically represented, their graphs may or may not intersect. The coordinates of the point of intersection represent the solution to the linear simultaneous equations.

Example;

In solving the simultaneous equations x + 3y = 5 and 5x + 7y = 9 graphically, the graph of two equations are drawn as

shown below;



The two lines intersect at P (-1, 2). The solution to the simultaneous equations is, therefore, x = -1 and y = 2

EXCERCISE

Solve each of the following pairs of simultaneous equations graphically;

(1).

a. $y = 3x - 1$

b. $2y + 2x = 3$

(2).

a. $2x - y = 3$

b. $7x - 2y =16$

(3).

a. $2x - y = 3$

b. $x + 2y = 14$

(4).

a. $5x +y = 7$

b. $3x + 2y = 0$

(5).

a. $y = 2x + x + 7 = 0$

b. $y = 2x - 1$

(6).

a. $3y - x - 4 = 0$

b. $2x - 5y + 7 = 0$

Using graphical method, solve the following:

(7).

a. $3x + 4y = 3.5$

b. $7x - 6y = 0.2$

(8).

a. $2y + 3x + 7 = 0$

b. $3y - x + 2 = 0$

(9).

a. $4x - y = 2$

b. $6x + 4y = 25$

(10).

a. $4x - 2y = 4$

b. $2x - 3y =0$

General Graphs

General graphs

- Graphs find a wide application in science and many other fields. It is therefore important to master the techniques of drawing graphs that convey information easily and accurately. Of these techniques, one of the most important is the choice of appropriate scales.

We illustrate this by considering the following situations:

(i). A man walks for four hours at an average speed of 5km/h. table (a) below shows the distance covered at a given times.

(ii). A motorist drives for four hours at an average speed of 80km/h. table (b) illustrates the situation.

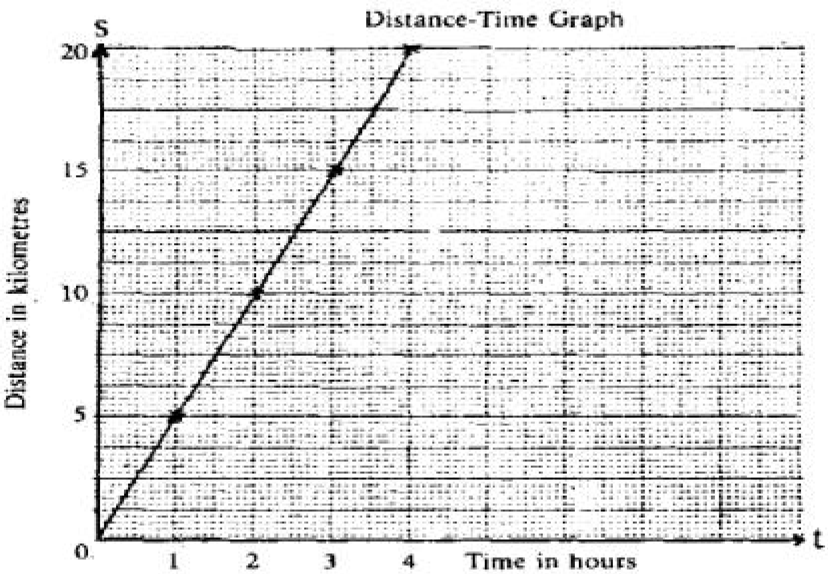
(a)

Time in hours (t)	1	2	3	4
Distance in km (S)	5	10	15	20

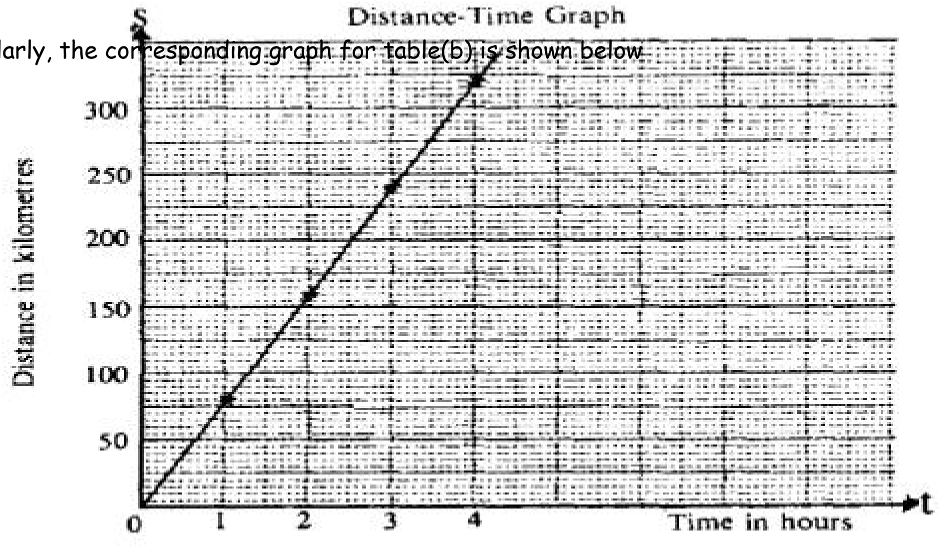
(b)

Time in hours (*t*)	1	2	3	4
Distance in km (*S*)	80	160	240	320

The corresponding graph for table(a)is drawn below



Similarly, the corresponding graph for table(b) is shown below



- In both graphs, the scale on the horizontal axes are the same.
- A good scale is one which uses most of the graph page and enable us to plot points and read off values easily and accurately. Avoid scales which:

(i). Give tiny graphs

(ii). Cannot accommodate all the data in the table

It also good practice to:

(i). Label the axes clearly

(ii). Give the title of the graph.

<u>EXCERCISE</u>

(1). A certain quality of gas is heated from 0°C and the volume measured at different temperatures. The table below gives the corresponding values:

Temperature (°C)	20	40	60	80	100
Volume (litres)	1.82	1.95	2.07	2.20	2.32

a. Draw a graph of volume against temperature using a suitable scale

b. Use your graph to find:

(i). The initial volume of gas

(ii). The volume of the gas when the temperature is 50°C and 64°C.

(iii) The temperature of the gas when the volume is 2.3 and 2litres.

(2). A man deposited a certain amount of money in a bank. The following table shows the amount of money due to him at the end of every year.

Time (years)	1	2	3	4	5
Amount (shillings)	45 000	50 000	55 000	60 000	65 000

a. Using a suitable scale, plot the graph of the amount of money in the bank against time

b. From your graph, estimate his initial deposit in the bank

c. Supposing at the end of $3\frac{1}{2}$ years he withdrew some amount of money such that the balance was sh. 40,000, how much did he withdraw?

d. If he had not withdrawn the money, what would be the amount in the bank after 66 months?

(3). If y = x2, make a table of values of y against values of x from x = -4 to x = 4. Draw a curve passing through the point. from the graph find:

33. (3.1)2
34. (2.9)2

(4). The surface area of an animal may be obtained from the mass of the animal. The following table gives the corresponding values of mass and surface area.

Mass in kg	50	100	150	200	250	300	350	400
Area in m^2	1.4	2.1	2.8	3.4	4.0	4.5	4.9	5.4

Draw the graph of the surface area against mass and use it to answer the following questions:

a. Find the surface area of an animal weighing;

(i). 155kg

(ii). 215kg

(iii). 370kg

b. A butcher A slaughters two animals weighing 155kg and 215kg. Another butcher B slaughters an animal weighing 370kg. which butcher a larger area of hides?

c. What is the mass of an animal whose surface area is one square meter?

STRAND 4: GEOMETRY
Scale Drawing

Introduction to Scale Drawing

The Scale

- Scale is a ratio that represents the relationship between the dimensions of a model and the corresponding dimensions on the actual figure or object.

The figure below shows the relative positions of Mombasa, Nairobi and Nakuru. The distance between Mombasa and Nairobi on a straight line is 450 km and that between Nairobi and Nakuru is 142 km.

(i). Measure in centimetres the distance between:

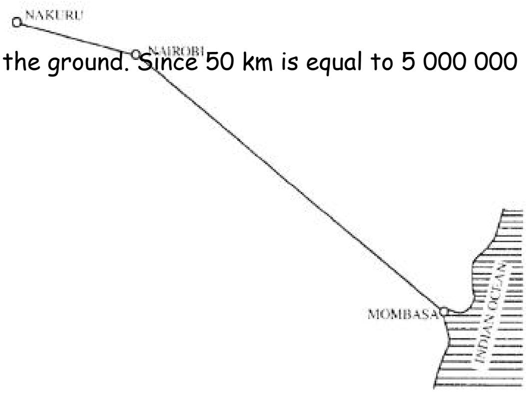
42. Mombasa and Nairobi.

43. Nairobi and Nakuru.

(ii). How many kilometres does 1 cm represent between:

20. Mombasa and Nairobi?

21. Nairobi and Nakuru?



You should notice that I cm on the map represents 50 km on the ground. Since 50 km is equal to 5 000 000 cm, this statement can be written in ratio form as 1: 5 000 000.

As a representative fraction (R.F.), 1: 5000 000 is written as $\frac{1}{5,000,000}$.

- The ratio of the distance on a map to the actual distance on the ground is called ***the scale of the map.***

Example;

(1). The scale of a map is given in a statement as '1 cm represents 4 km.' Convert this ratio form.

Solution

1 cm represents 4 x 100 000 cm

Therefore, the ratio is 1: 400 000 and the R.F. is $\frac{1}{400,000}$

(2). The scale of a map is given as 1: 250 000. Write this as a statement.

Solution

35. 250 000 means 1 cm on the map represents 250 000 cm on the ground. km, i.e., 1 cm represents 2.5 km.

Therefore, 1 cm represents $\frac{250,000}{100,000}$

EXCERCISE

(1). On a map, 1 cm represents 4 kilometres. Re-write as a ratio.

(2). What distance on the ground is represented by 3.7 cm on the map if 1 cm represents 4 kilometres?

(3). Two towns A and B are 42.8 km apart on the ground. What is this distance on the map given that 1 cm represents 1.5 kilometres?

(4). A map is drawn to a scale of 1:50 000. Write this scale as a statement connecting map distance to ground distance.

(5). What is the actual distance if the distance on the map is 12.7 cm given that 1 cm represents 2.5 kilometres?

(6). On a map, 1 cm represents 5 kilometres. A railway line measures 8.3 km. What is its length on the map?

(7). The scale of a map is given in a statement as '1 cm represents 4.5 km.' Convert this to a representative fraction (R.F.).

(8). The scale of a map is given as 1: 350 000. Write this as a statement.

(9). A map is drawn to a scale of 1:500 000. What is the actual distance if the distance on the map is 14.7 cm?

(10). What is the distance on the map of Two towns P and Q which are 62.8 km apart on the ground given that 1 cm represents 2.5 kilometres?

Scale Diagrams

Scale Diagrams.

- A scale drawing is an enlargement of an object.
- An enlargement changes the sizes of an object by multiplying each of the lengths by a scale factor to make it larger or smaller.
- It is usually stated as a ratio.

Note:

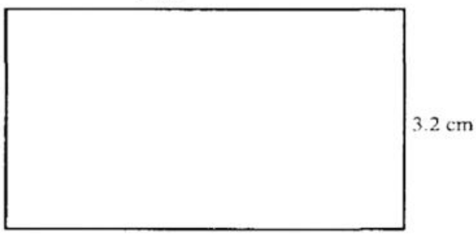
- One should be careful in choosing the right scale, so that the drawing fits on the paper without much detail being lost.

Example.

1. The length of a classroom is 10 metres and its width 6.4 metres. By scale drawing, represent this on a figure.

Solution

First look for a scale. Consider a scale of 1 cm to represent 2 m. Hence, the classroom will be 10/2 = 5 cm by 6.4/2



= 3.2 cm.

<u>Assignment</u>

(1). The length of a classroom is 10 metres and its width 6.4 metres. By scale drawing, represent this on a figure.

(2). A plot of land in form of a rectangle has dimensions 120 m by 180 m. Draw this on a paper.

(3). A rectangular field measures 40 m by 100 m. The length of the field on the map is 5 cm. What is the area of the field on the map?

(4). A rectangular field measures 40 m by 100 m. The length of the field on the map is 5 cm. Write the scale of the map as a representative fraction and hence the width of the field on the map?

(5). A plot of land in form of a rectangle measures 100 m by 80 m. Draw this on a paper.

(6). Two villages M and N are connected by a straight road 750 m long on a level ground. A third village L is 450 m from M and 650 m from N. Using a suitable scale, draw the diagram and find the shortest distance of L from the road.

(7). The length of a rectangular field on the map is 4 cm, If the rectangular field measures 50 m by 120 m write the scale of the map as a representative fraction and hence the width of the field on the map?

(8). Represent the dimensions of a plot of land in form of a rectangle measuring 150 m by 70 m. on a paper.

(9). Two Towns S and T are connected by a straight road 650 m long on a level ground. A third village R is 350 m from S and 550 m from T. Using a suitable scale, draw the diagram and find the shortest distance of R from the road.

(10). Using a suitable scale, draw the diagram of a rectangular plot of land measuring 6 km by 3 km.

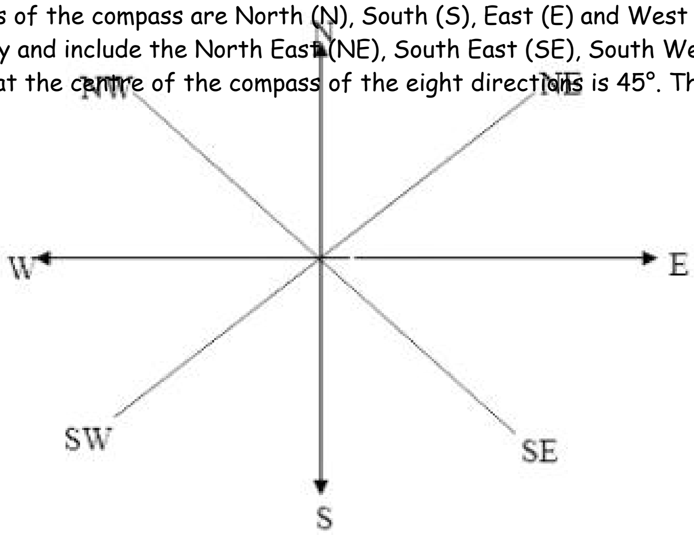
Compass Bearing

Bearing and Distance.

- **Bearing** is the angle measured in a clockwise or counterclockwise direction from the north direction or South direction to a given direction.
- **Distance** is the length between the two places. Bearing and distance can be used to locate the position of a given point from another reference point.

Points of the compass

- The figure below shows the eight points of the compass.
- The four main points of the compass are North (N), South (S), East (E) and West (W). The other four points shown are secondary and include the North East (NE), South East (SE), South West (SW) and North West (NW). Each angle formed at the centre of the compass of the eight directions is 45°. The angle between N



and E is 90°.

Compass Bearing

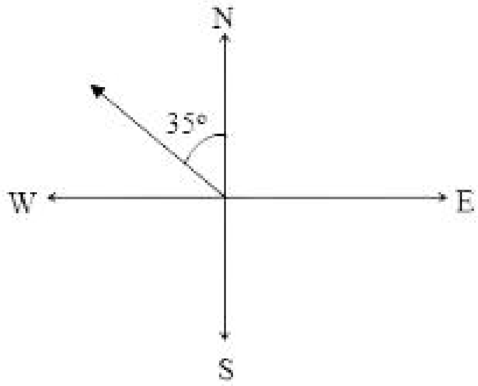
- It is measured in either clockwise direction or anti-clockwise direction from North or South and the angle is an acute angle. E.g. N 45° W, S 60° E

Example;

Draw a sketch to show the bearing marking the angle N35°W clearly.

Solution

N35°W means from N measure 35° toward the W or 35°W of N.



Assignment

(1). Three boys Isaac, Alex and Ken are standing in different parts of a football field. Isaac is 100 metres north of Alex and Ken is 120 metres east of Alex. Find the Compass bearing of Ken from Isaac.

(2). Kilo school is 12 kilometres from Sokomoko on a bearing of N 50°W. Tiba dispensary is 10 kilometres from Kilo on a bearing of S 60°E. Find the compass bearing of Sokomoko from Tiba.

(3). Survey posts R, Q and P are situated such that they form a triangle. If Q is on a bearing of S 60° W and 12 kilometres away and R is on a bearing of S 30°E and 8 kilometres away from P, find the compass bearing of Q from R.

(4). Kisumu and Nanyuki are situated in such a way that Nanyuki is on a bearing of N 75° E from Nakuru and Kisumu on a bearing of N 80° W from Nakuru. If Kisumu is 190 km and Nanyuki is 160 km from Nakuru,

Find:

a. The compass bearing of Kisumu from Nanyuki.

b. The distance of Kisumu from Nanyuki.

(5). From a meteorological weather station P on a plateau, a hill Q is 5 km on a bearing N 78° E and a railway station, R, is 1.5 km away on a bearing S20°W. Use scale drawing to find the compass bearing of Q from the railway station.

(6). A town P is 200 km West of Q. Town R is at a distance of 80 km on a bearing of N490E from P. Town S is due East of R and due North of Q. Determine the compass bearing of S from P.

(7). A route for safari rally has five sections AB, BC, CD, DE and EA. B is S 20° W km on a bearing N 50° E from A.C is 500km from B. The bearing of B from C is N 60° W. D is 400km on a bearing S 50° W from C. E is 250km on a bearing N 25° E from D. Using the scale 1cm for 50km draw the diagram representing the route for the rally. From the diagram determine:

a. The distance in km of A from E

b. The compass bearing of E from A.

(8). Manyatta village is 74 km North West of Nyangata village. Chamwe village is 42 km west of Nyangate. By using an appropriate scale drawing, find the compass bearing of Chamwe from Manyatta.

(9). Four towns R, T K and G are such that T is 84 km directly to the north of R, and K is on a distance of N 65° W from R at a distance of 60 km. G is on a bearing of N 20° W from K and a distance of 30 km. Using a scale of 1cm to represent 10km, make an accurate scale drawing to show the relative positions of the towns. Find the distance and the compass bearing of G from T.

(10). Using the scale: 1 cm represents 10 km, construct a diagram showing the positions of B, C, Q and D.
Determines the distance between B and C and the compass bearing of D from B.

True Bearing

Bearing and Distance.

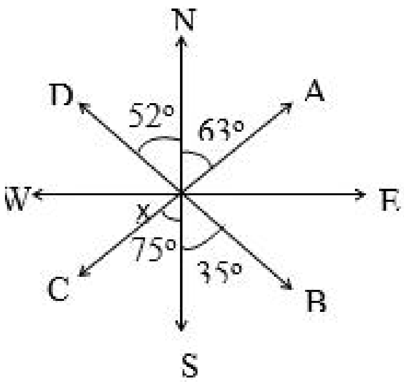
True Bearing.

- It is also known as ***Three-figure bearings***.
- True bearing is given as the number of degrees from north, measured in a clockwise direction.
- Three-digit are always used for given angles but for angles less than 100° extra zero(s) is always added in front of the digits e.g. 008° , 060°, 070° up to 099°.
- The true bearings of due north is given as 000°. Due South East as 135° and due North West as 315°, etc. In figure 22.7, the true bearing of Q from P is 138°.

Example.

Find the three-figure bearings of A, B, C, and D from X.

Solution



(1). The arrow N shows the direction N, NXA = 63°. the bearing of A from X is 063°

(2). NXB = 180 - 35 = 145°. The bearing of B from X is 145°

(3). NXC clockwise = 180 + 75 = 255°. The bearing of C from X is 255° 4. NXD clockwise = 360 - 52 = 308°. The bearing of D from X is 308°.

Assignment

(1). A coastguard at a port observes two steamships approaching the harbour. The first ship P appears on a bearing 100° and the second ship Q on a bearing 020°. If the guard estimates the distances of the ships to be 120 km and 80 km respectively, find:

(a) the distance between ships P and Q

(b) the bearing of Q from P.

(2). A prison guard on a watchtower sees a bridge 120 m away on a bearing of 230° and a bus stop 80 m away on a bearing of 090°. Use scale drawing to find:

36. The bearing of the bridge from the bus stop.

37. The distance between the bus stop and the bridge

(3). From a point P, the bearing of a house is 060°. From a point Q 100m due east of P, the bearing is 330°. Draw a labelled sketch to show the positions of P, Q and the house.

(4). Three boys Isaac, Alex and Ken are standing in different parts of a football field. Isaac is 100 metres north of Alex and Ken is 120 metres east of Alex. Find the True bearing of Ken from Isaac.

(5). Kilo school is 12 kilometres from Sokomoko on a bearing of 320°. Tiba dispensary is 10 kilometres from Kilo on a bearing of 120°. Find the True bearing of Sokomoko from Tiba.

(6). Survey posts R, Q and P are situated such that they form a triangle. If Q is on a bearing of 210° and 12 kilometres away and R is on a bearing of 150° and 8 kilometres away from P, find the true bearing of Q from R.

(7). Kisumu and Nanyuki are situated in such a way that Nanyuki is on a bearing of 075° from Nakuru and Kisumu on a bearing of 280° from Nakuru. If Kisumu is 190 km and Nanyuki is 160 km from Nakuru, find:

49. The bearing of Kisumu from Nanyuki.

50. The distance of Kisumu from Nanyuki.

(8). Town A is on a bearing 050° from town C. Town B is on a bearing 020° from C. If B is 500 km from C and A is 500 km from B, find by scale drawing:

51. The distance of A from C.

52. The bearing of B form A.

(9). A route for safari rally has five sections AB, BC, CD, DE and EA. B is 200 km on a bearing 050° from A.C is 500km from B. The bearing of B from C is 300° . D is 400km on a bearing 230° from C. E is 250km on a bearing 025° from D. Using the scale 1cm for 50km draw the diagram representing the route for the rally. From the diagram determine

a. The distance in km of A from E

b. The bearing of E from A

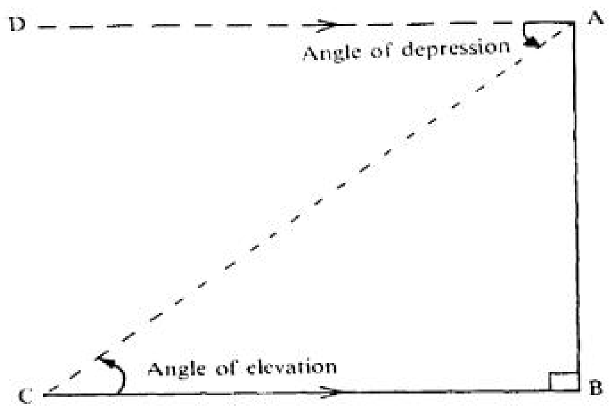
(10). A town P is 200 km West of Q. Town R is at a distance of 80 km on a bearing of 049° from P. Town S is due East of R and due North of Q. Determine the bearing of S from P.

Angle of Elevation and Depression

Angle of Elevation and the Angle of Depression.

- The angle of elevation is the angle between the horizontal line of sight and the line of sight up to an object. • The angle of depression is the angle between the horizontal line of sight and the line of sight down to an object.

- The angle of elevation is equal to the angle of depression.



- Angles of depression and elevation can be measured by use of an instrument called clinometer.
- A simple clinometer can be made from a cardboard in the shape of a protractor.

N/B:

- The angle of elevation increases as the observer moves towards the object and decreases as the observer moves away from the object.

Example.

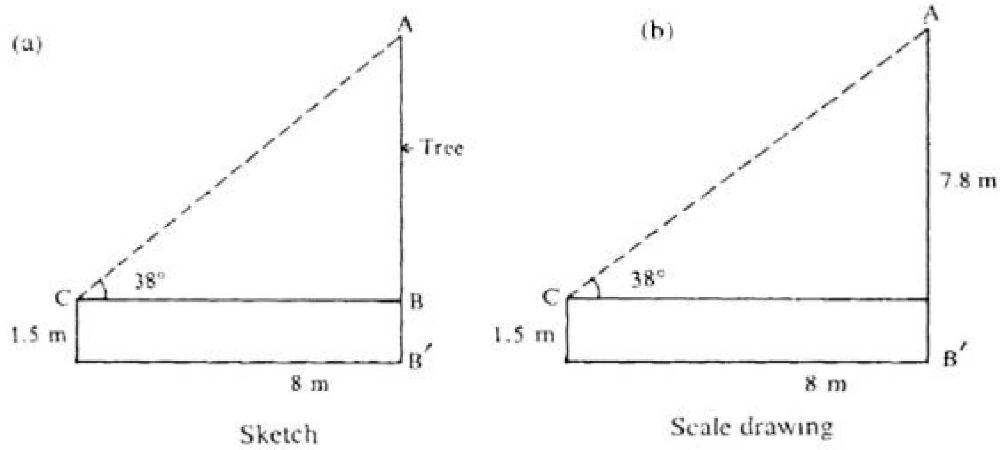
A boy 1.5 m tall and 8 m from a tree finds that the angle of elevation to the top of the tree is 38°. Find the height of the tree by scale drawing.

Solution

The figure below shows the sketch and scale drawing.

Using a scale of 1 cm to 2 m. The measurement of AB' on the scale is 3.9 cm.

(1). cm represents 2 m, therefore 3.9 cm represents (3.9 x 2) m = 7.8 m.



<u>Assignment</u>

(1). A boy 1.5 m tall and 8 m from a tree finds that the angle of elevation to the top of the tree is 38°. Find the height of the tree by scale drawing.

(2). The angle of elevation of the top of a flagpole from a point A, 14 metres away, is 36°. Use scale drawing to find the height of the flagpole.

(3). The angle of depression from the top of a cliff to a stationary boat is 48°. Find the horizontal distance by accurate drawing if the height of the cliff is 80 metres. Measure the angle of elevation of the top of the cliff from the boat. What to you notice?

(4). A building tower casts a shadow 33 metres away. The angle of elevation of the tower from the tip of the shadow is 21°. Find the height of the tower by scale drawing.

(5). The angle of elevation of a church tower from a point A, 50 metres away from the foot of the church, is 24°. Find the distance between A and B if the angle of elevation of the tower from B is 20°.

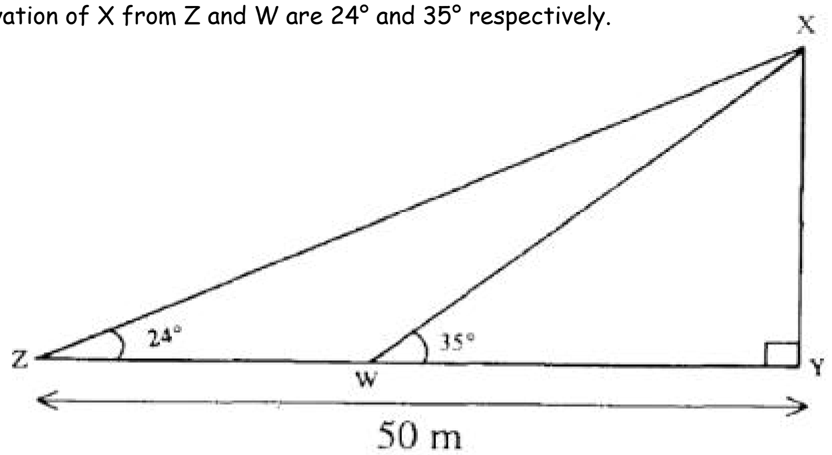
(6). From a viewing tower 15 metres above the ground, the angle of depression of an object on the ground is 30° and the angle of elevation of an aircraft vertically above the object is 42°. By choosing a suitable scale, find the height of the aircraft above the object.

(7). A soldier standing on top of a cliff 100 m high notices two enemy boats in line, whose angles of depression are 10 and 23°. Find, by scale drawing. the distance between the boats.

(8). The angles of elevation of an aircraft from two villages A and B 1 km apart are 67° and 53° respectively. Find the height of the aircraft in metres by scale drawing.

(9). The angle of elevation of a stationary hot air balloon 50 m above the ground from a man on the ground is 17°. The balloon moves vertically upwards so that the angle of elevation from the man is 30°. Find, by scale drawing, the distance the balloon moves. How far above the ground is the balloon?

(10). In figure below, Z is 50 m away from Y. Use scale drawing to find the distance from W to Y, given that the angle of elevation of X from Z and W are 24° and 35° respectively.



Survey

Simple Survey Techniques.

- Surveying an area of land involves taking field measurements of the area so that a map of the area can be drawn to scale. Pieces of land are usually surveyed in order to fix boundaries (land adjudication) of land for different owners, for town planning, road construction, water supplies, mineral development, etc.
- There are two simple methods used in surveying.

53. Triangulation.
54. Survey of an area by use of Compass Bearing and Distances.

(a). Triangulation.

- This is a method in which the area to be surveyed is divided into convenient geometrical figures, or is covered by a suitable geometrical figure.

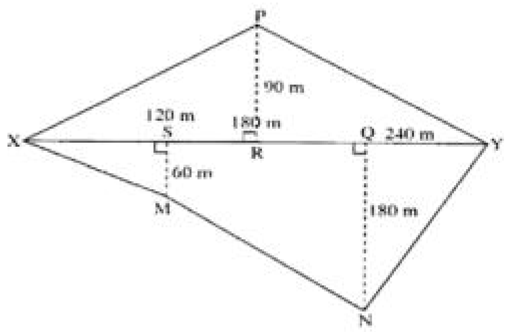
(b). Survey of an area by use of Compass Bearing and Distances.

- In this survey method, only bearings and distances from a chosen point are considered.

In this chapter, we will narrow down into Triangulation into details.

Example.

(1). A survey of a small island was drawn as shown below. Record the measurements shown in the figure in a tabular form in a field book.



Solution

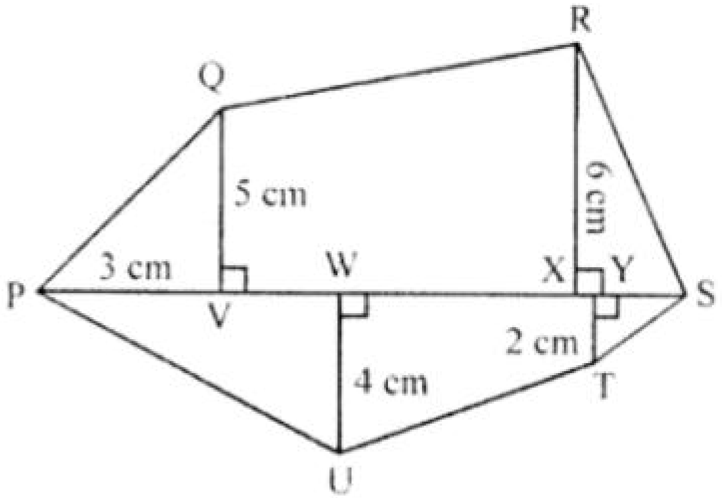
The line XY is called base lines of the survey. The lines perpendicular to the base lines and joining the points M, N, and P at the edge S, R and Q are called offsets.

The measurements are recorded in a field book in a tabular form as follows:

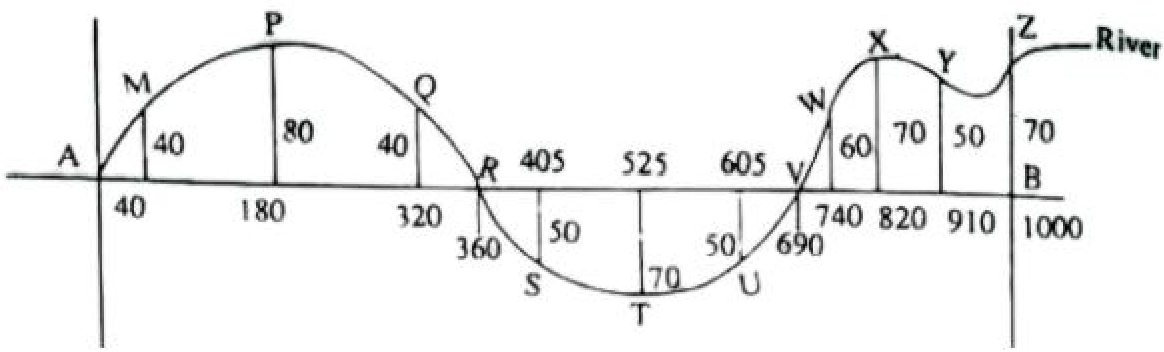
	Y	
	240	180 to N
To R90	180	
	120	60 to M
	X	

Assignment

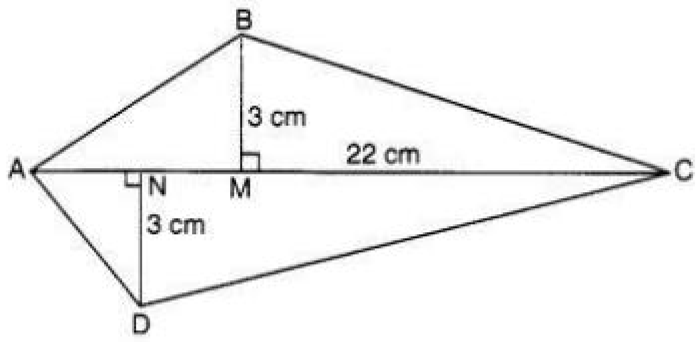
(1). A survey of a small island was made by using a triangle PQRSTU as shown in the sketch of figure below. Record the measurements shown in the figure in a tabular form in a field book.



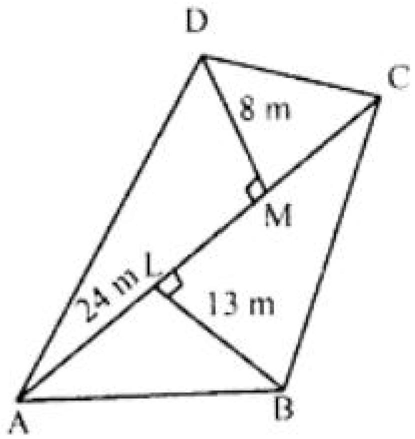
(2). A river can also be surveyed and its map made using base line AB as shown in the sketch of figure below. Record the measurements from the sketch in a field book.



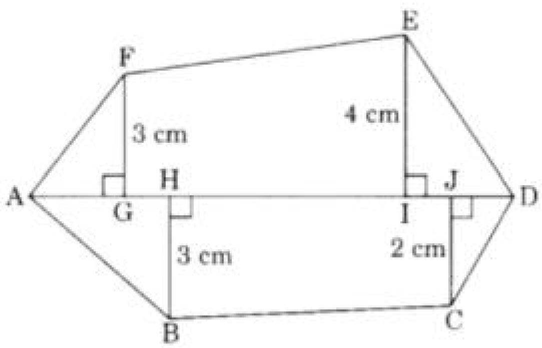
(3). A plot of land is draw as shown below. Record the measurements from the plot in a field book given that AN = NM = 2 cm. Use a scale of 1cm rep 25 m.



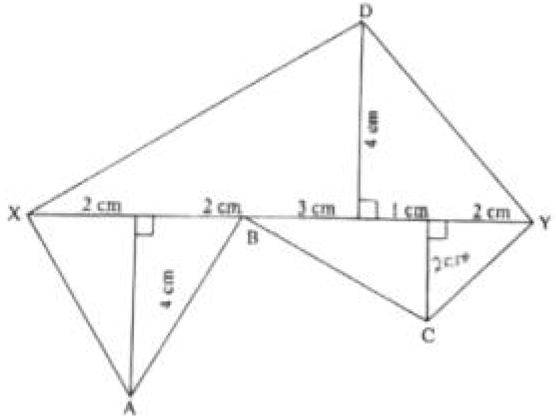
(4). A piece of land is drawn with a base line AC as shown below. Record the measurements from the plot in a field book given that LM = 12 m MC = 20 m.



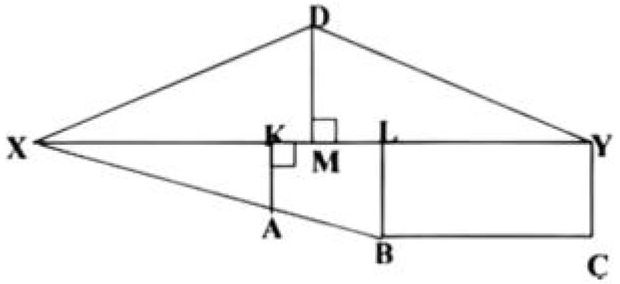
(5). A survey of a small island was drawn as shown below. Record the measurements shown in the figure in a tabular form in a field book given that baseline AD = 10 cm and AG = JD = 2.5 cm. Use a scale of 1cm rep 45 m.



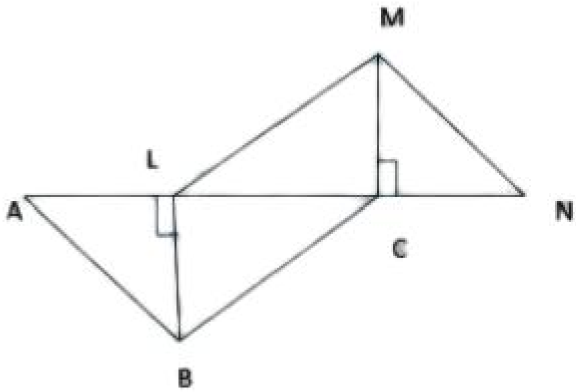
(6). Record the measurements shown in the figure in a tabular form in a field book for a survey of a small piece of land. Use a scale of 1cm rep 60m.



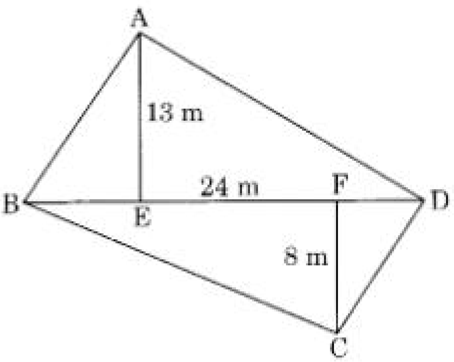
(7). The figure below represents a surveyor's sketch of a plot of land. Record the measurements from the sketch in a field book given that XY = 50m, XK = 20m, XM = 25m, XL = 35m, KA = 40m, MD = 38m and LB = YC = 60m.



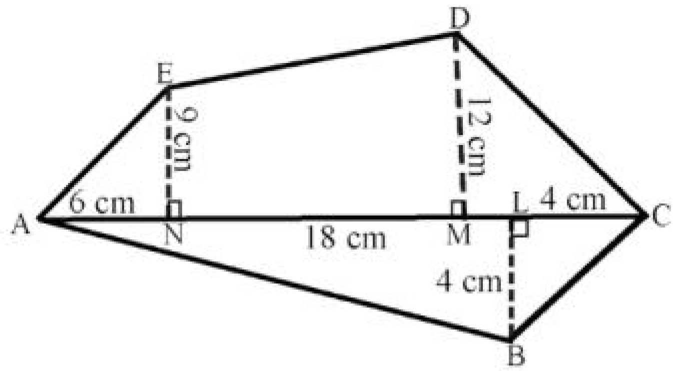
(8). A surveyor's sketch of a plot of land is as shown below. Record the measurements from the sketch in a field book given that AN = 10 cm, AL = 2 cm, LC = 2.5 cm, CN = 3.5 cm, LB = 4 cm, CM = 3.8 cm. use a scale of 1 cm rep 50 m.



(9). The figure below represents a surveyor's plot of land. Record the measurements from the plot in a field book.



(10). Using a scale of 1cm rep 20 m, record the measurements shown in the figure below in a field book for a survey of a small piece of land.



Survey B

Survey

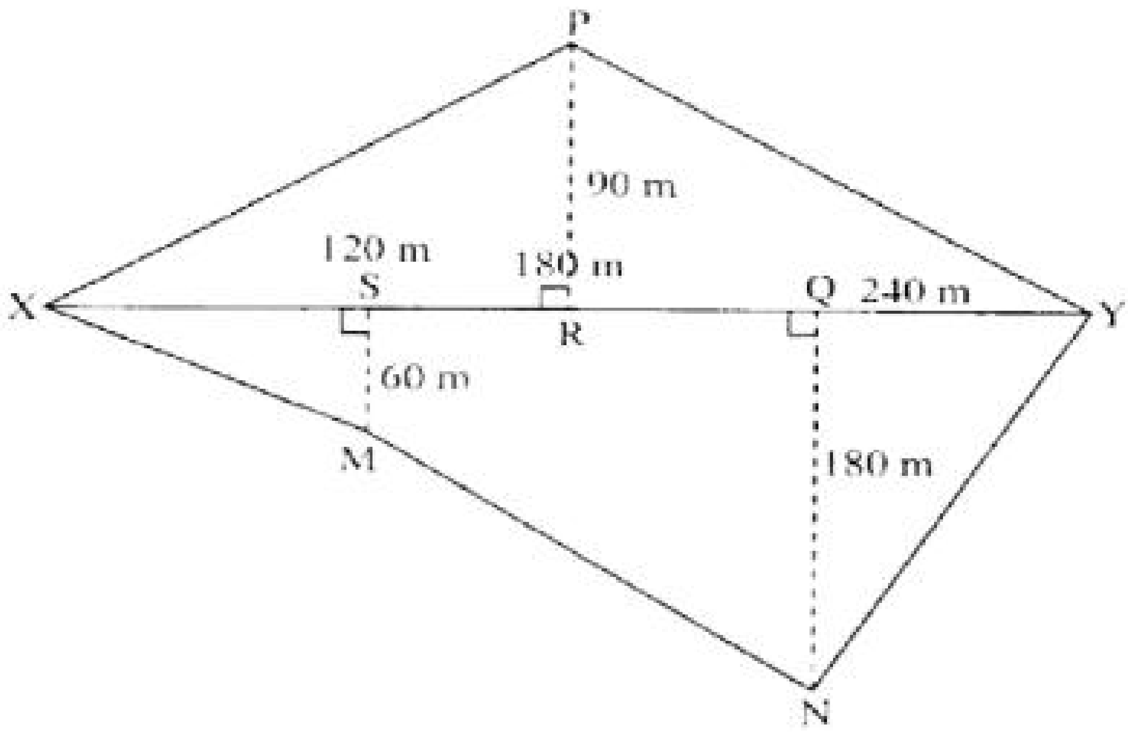
- Areas of pieces of land which have irregular shapes can be obtained by subdividing them into convenient geometrical shapes, e.g., triangles, rectangles or trapezia. This is done by the use of baseline and offsets of the area required.

Example.

(1). Find the area in hectares of a coffee field whose measurements are entered in a field book as follows. (Take XY = 360 m as the base line)

	Y	
	240	180 to N
To R90	180	
	120	60 to M
	X	

Solution



Area of:

- *Triangle XPR is* $^{1}/_{2} \times 180 \times 90\ m^2 = 8\ 100\ m^2$
- *Triangle PRY is* $^{1}/_{2} \times 180 \times 90\ m^2 = 8\ 100\ m^2$
- *Triangle XSM is* $^{1}/_{2} \times 120 \times 60\ m^2 = 3\ 600\ m^2$
- *Triangle QNY is* $^{1}/_{2} \times 120 \times 180\ m^2 = 10\ 800\ m^2$
- *Trapezium SQNM =* $^{1}/_{2}\ (QN+SM) \times SQ\ m^2$
- $^{1}/_{2}\ (180+60) \times 120\ m^2 = 14\ 400\ m^2$
- *Total area =* $45\ 000\ m^2$
- *Therefore, the area of the field is 4.5 ha.*

<u>Assignment</u>

(1). Find the area in hectares of a coffee field whose measurements are entered in a field book as follows. (Take XY = 400 m as the base line).

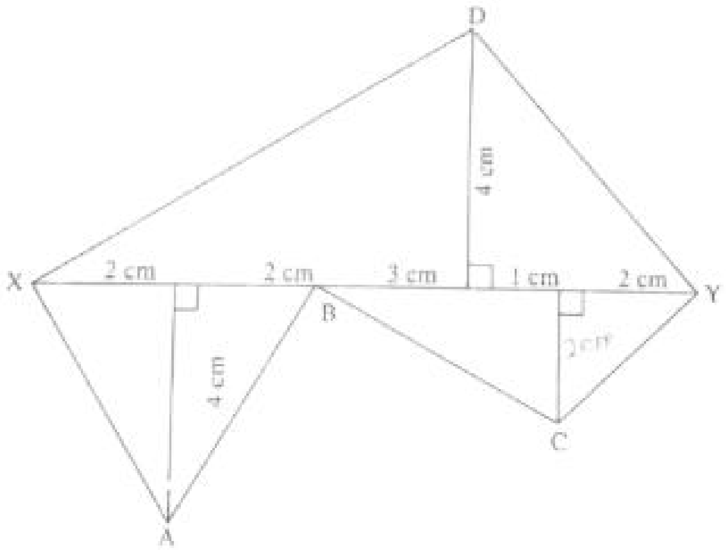
	Y	
	360	80 to Q
To R80	280	
To S160	200	
	80	200 to P
	X	

(2). A farm of land of measurements are shown in field book below. XY = 500 m. Find the area of the farm in hectares.

	Y	
To B100	450	
	300	20 to C
	120	70 to D
To A40	60	
	X	

(3). Use a scale of 1 cm to 40 m to draw the map of the coffee field.

(a). Find the area in hectares of a maize farm XABCYD in figure below which is drawn to a scale of 1 cm to 50 m.



(b). Taking XY as the base line and that the survey is from X to Y, enter the actual measurements of the farm in a field book.

(4). Measurements of a maize field using a base line XY were recorded as shown below. (Measurements are in metres)

Use a suitable scale to draw the map of the maize field hence find the area of the field in hectares.

		240	
	60	190	
		180	75 TO Q
		150	50 TO P
TO S	100	120	
		100	100 TO N
	30	50	
		20	20 TO M

(5). Find the area in hectares of farms whose measurements are shown in field book as in the tables below. AB = 600 m.

	B	
	550	120 to L
To K150	450	
	250	90 to M
To J60	40	
	A	

(6). A surveyor recorded the measurements of a small field in a field book using base lines AB = 75 cm, as shown below, draw the map of the field.

	B		C		A
R9	55	W5	80	Z17	70
Q7	42	V6	70	Y5	50
P15	30	U7	60	X6	25
A	A	S10	20		C
			B		

(7). Use a suitable scale to draw the map of the tea field hence find the area of the field in hectares. If a surveyor recorded the measurements of the tea field in a field book using base lines BC = 100 cm, as shown below.

	B		C		A
R9	55	W5	80	Z17	70
Q7	42	V6	70	Y5	50
P15	30	U7	60	X6	25
A	A	S10	20		C
			B		

(8). The table below gives a field book showing the results of a survey of a section of a piece of land between A and E. All measurements are in metres.

(a) Draw a sketch of the land.

	E	
D33	95	
	90	**F** 36
C21	70	
B 42	30	**G** 25
	25	**H** 40
	A	

(9). A field was surveyed and its measurements recorded in a field book as shown below.

(a) Using a scale of 1cm to represent 10m, draw a map of the field.

	F	
	100	
E 40	80	
	60	D 50
C 40	40	
	20	B 30
	A	

(b) Calculate the area of the field in hectares.

(10). A tea farm in Kakamega forest was surveyed and the results were recorded in the surveyors note book as shown below. The measurements are in meters. Using a scale of 1: 25, draw the map of the plot and hence calculate the area of the plot in Hectares.

	250	**Y**
	240	D70
C80	170	
	70	B60
A60	50	
X	**0**	

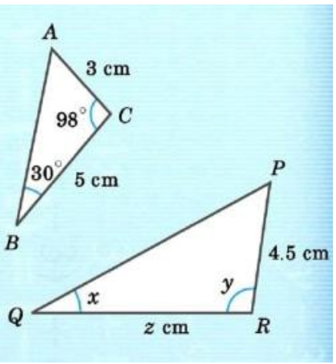
Similarity and Enlargement
Introduction

Similar Figures

- Two or more figures are said to be similar if:
- The ratio of the corresponding sides is constant.
- The corresponding angle are similar

Example 1

In the figures below, given that △ ABC ~ △ PQR, find the unknowns x, y and z.



Solution

BA corresponds to QP each of them has opposite angle y and 980 . Hence y is equal to 980 BC corresponds

to QR and AC corresponds to PR.

BA/QR = BC/QR = AC/PR

AC/PR = BC/QR

3 /4.5 = 5 /z

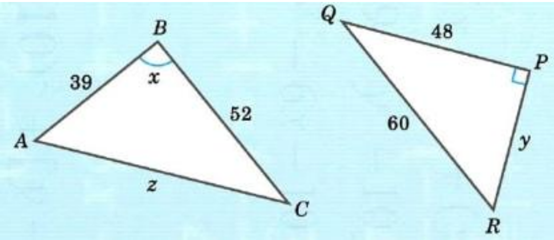
z = 7.5 cm

Note:

- Two figures can have the ratio of corresponding sides equal but fail to be similar if the corresponding angles are not the same.
- Two triangles are similar if either their all their corresponding angles are equal or the ratio of their corresponding sides is constant

Example:

In the figure, △ ABC is similar to △ RPQ. Find the values of the unknowns.



Solution

Since △ ABC~ △ RPQ,

∠B=∠P

44. x = 90°

Also,

AB/RP = BC/PQ

39/y = 52/48

(48 × 39)

52

23. y = 36

Also,

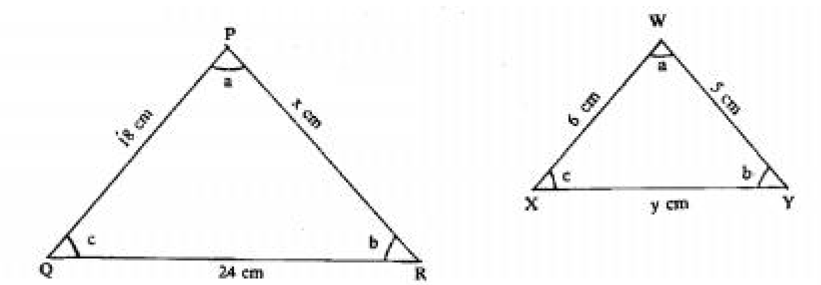
AC/RQ = BC/PQ

z /60 = 52/48

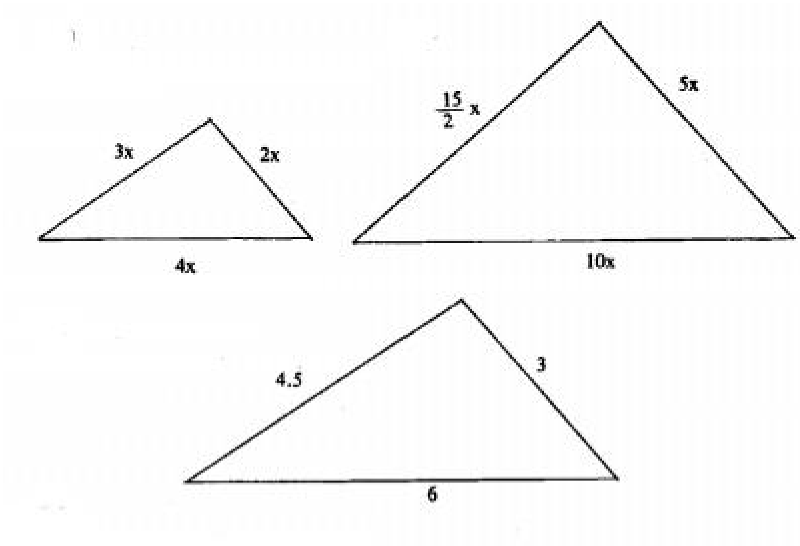
38. z = 65

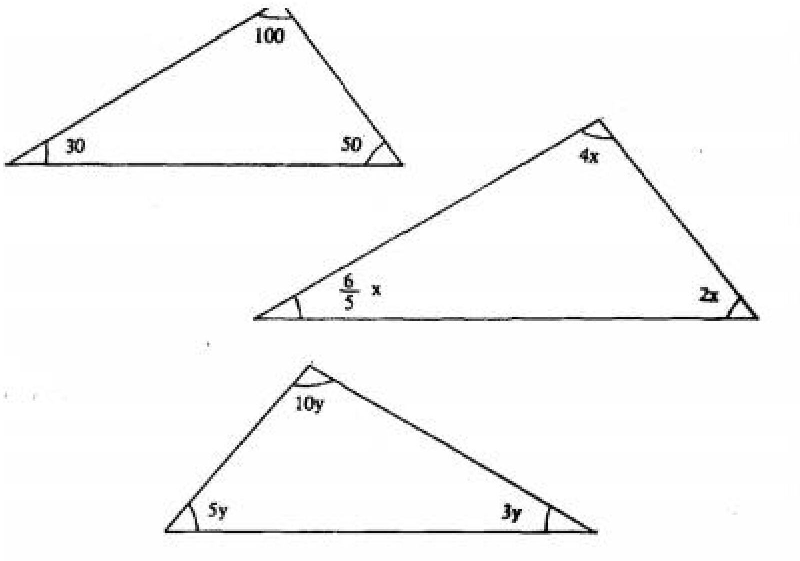
QUESTIONS

(1). In figure 7.2, the triangle PQR and WXY are similar. calculate the length of PR and XY.



Identify the similar triangles.

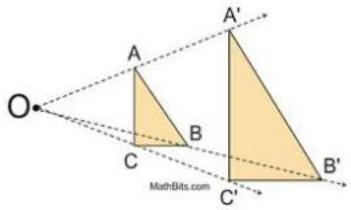




Enlargement

Enlargement

- Enlargement, sometimes called **scaling**, is a kind of transformation that changes the size of an object.
- The image created is **similar** to the object. Despite the name enlargement, it includes making objects smaller.
- For every enlargement, a **scale factor** must be specified. The scale factor is how many times larger than the object the image is.
- *Length of side in image = length of side in object × scale factor*
- For any enlargement, there must be a point called the **center of enlargement**.
- *Distance from center of enlargement to point on image = Distance from Centre of enlargement to point on object X scale factor*
- The Centre of enlargement can be anywhere, but it has to exist.



- This process of obtaining triangle A□ B □C□ from triangle A B C is called enlargement.
- Triangle ABC is the object and triangles A□ B □C □Its image under enlargement scale factor 2.

Hence;

OA□/OA = OB□/OB = OC□/OC = 2.

The ratio is called **scale factor of enlargement**. The scale factor is called **linear scale factor**

By measurement OA =1 .5 cm, OB = 3 cm and OC = 2.9 cm. To get A□, the image of A, we proceed

as follows

OA =1 .5 cm

OA□/OA = 2 (scale factor 2)

OA□=1.5×2

= 3 cm

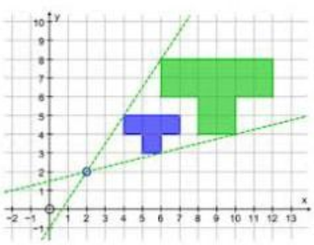
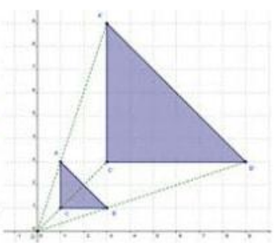
Also OB□/OB = 2

39. 3×2
40. 6 cm

Note: Lines joining object points to their corresponding image points meet at the Centre of enlargement.

<u>Center of Enlargement</u>

- To find center of enlargement join object points to their corresponding image points and extend the lines, where they meet gives you the Centre of enlargement. Or Draw straight lines from each point on the image, through its corresponding point on the object, and continuing for a little further.
- The point where all the lines cross is the **Centre of enlargement.**

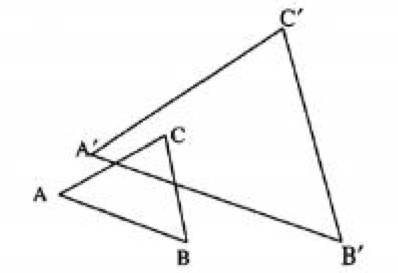
<u>QUESTIONS</u>

(1). Construct any triangle ABC. Take a point O outside the triangle. With O as the centre of enlargement and scale factor of 3. construct the image of ABC under the enlargement.

(2). Triangle A'B'C' is the image of triangle ABC under an enlargement

a. Locate the centre of the enlargement

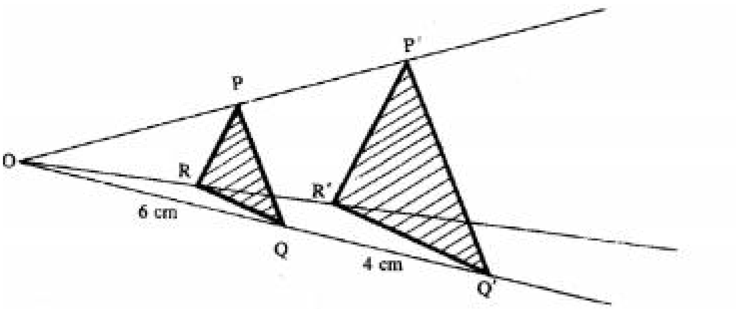
b. Find the scale factor of the enlargement.



(3). In figure 7.18,ΔP'Q'R' under an enlargement, center O.

a. If OQ = 6 cm and QQ' = 4 cm, find the scale factor of the enlargement.

b. If PQ = 4 cm, calculate the length of P'Q'



(4). In figure 7.19, rectangle A'B'C'D' is the image of rectangle ABCD under an enlargement with centre at O. OA = 12 cm, OA' = 4 cm, AB = 6cm and A'D' = 3cm

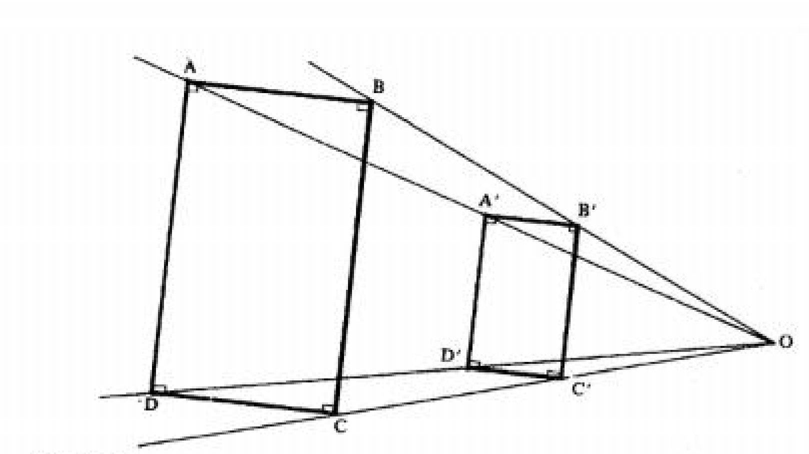


Fig. 7.19

Calculate:

a. the linear scale factor

b. the length of A'B'

c. the length of BC

Given that p(3. 4), Q (4, 4), R (6, $), S97, 1) and TS (5,0) are the vertices of an object, Find the vertices of the image after an enlargement with the centre at (0, 0) and scale factor

55. 1/2

56. 3

Negative Scale Factor

Negative Scale Factor

In figure 7.25, P'Q'R' and P'' Q'' R'' are the images of PQR under an enlargement, center O. Both images are twice as large as PQR.

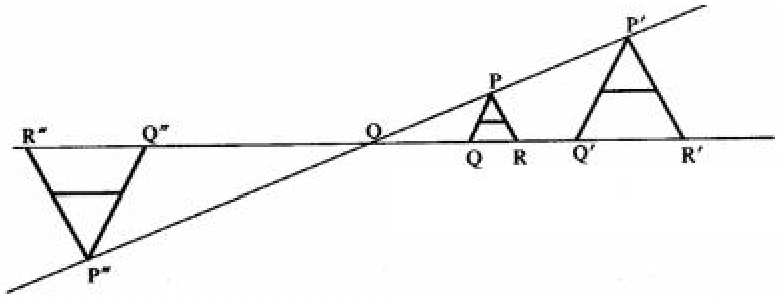


Fig. 7. 25

P'Q'R' is the image of PQR under an enlargement centre O, scale factor 2. How can we describe P'Q'R', the image of PQR under an enlargement with O as the centre?

Note:

i. PQR and its image P'Q'R' are on opposite sides of the centre of enlargement.

57. The image is inverted.

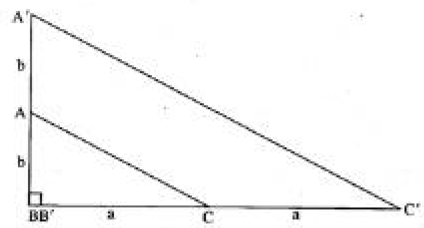
When this happens, the scale factor is said to be negative. In this case, the scale factor is -2.

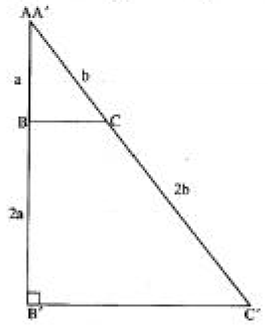
Note;

- To locate the image of the object under an enlargement with a negative scale factor. the same procedure as for the positive scale factor is followed. However, the object and the image fall on opposite sides of the centre of enlargement.

Questions

(1). Scale the centre of enlargement and the linear scale factor in each of the following figures if Δ A'B'C' is the image of ΔABC.





(2). Points A(4, 2), B(9, 2), C(7, -2) and D(2, -2) are the vertices of a parallelogram. Taking the origin as the centre of enlargement, find the image when the scale factor is;

58. -4
59. -2
60. $-\frac{1}{4}$

Area Scale Factor

Area scale factor

- Is a ratio in the form e: f or e/f. This ratio describes how many times to enlarge. Or reduce the area of two dimensional figure. Area scale factor can be calculated using

$$= \frac{\text{New Area}}{\text{Original Area}}$$

- Area scale factor = (linear scale factor)^2

Example;

A triangle whose area is 12 cm^2 is given an enlargement with linear scale factor.3

Find the area of the image.

Solution

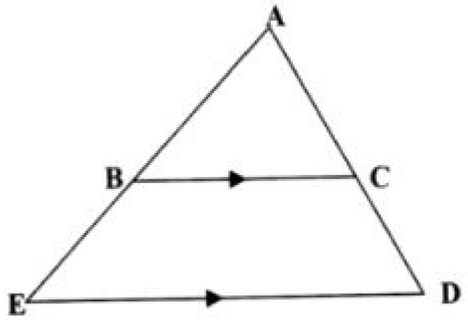
Linear scale factor is 3.
Area scale factor is 32 = 9

$$\frac{\text{Area of image}}{12} = 9$$

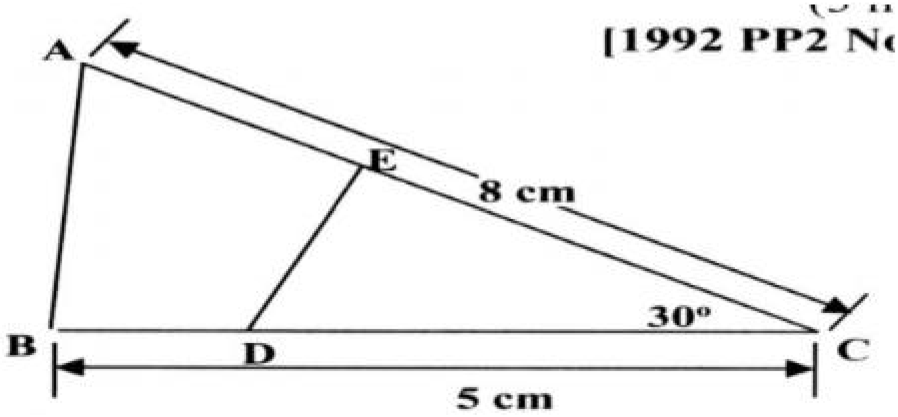
Therefore, area if image is 12 × 9 = 108cm^2

Questions

(1). In the figure above, triangle ABC is similar to triangle AED and BC // ED. Given that the ratio AB: AE = 2:5, find the ratio of the area of triangle ABC to that of the trapezium BCDE.

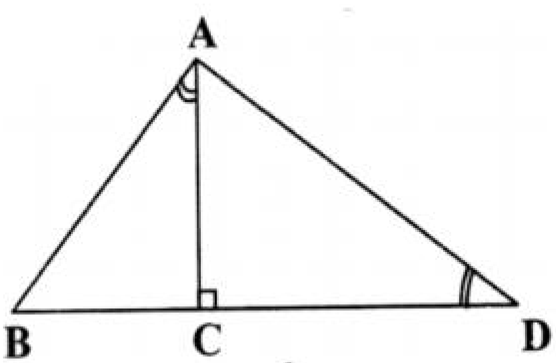


(2). In the triangle ABC below AC = 8 cm, BC = 5 cm and angle BCA = 30°. point D divides BC in the ratio 1:4 and point E divides AC in the ratio 2:3. Find the area of the quadrilateral ABDE.



(3). In the figure below, angles BAC and ADC are equal. Angle ACD is a right angle.

The ratio of the sides AC: BC = 4:3



Given that the area of triangle ABC is 24 cm², find the area of triangle ACD.

Volume Scale Factor

Volume scale factor

- Is the ratio that describes how many times to enlarge or reduce the volume of a three dimensional figure.
  Volume scale factor can be calculated using.

New Volume / Original Volume

Volume scale factor = (linear scale factor)3

Example

The base radii of two similar cones are 6 cm and 8 cm. If the volume of the smaller cone is $324cm^3$, Find the volume of the larger one.

Solution

The ratio of the radii is 8 cm/6 cm = 4/3

linear factor is 4/3.

Therefore, the volume scale factor is $\left(\frac{4}{3}\right)^3$ = 64/27.

Thus, volume of the larger cone 64/27 x 324 = 768 cm^3

QUESTIONS

(1). A football tube in the form of a sphere is inflated so that its radius increases in the ratio of 4:3. Find the ratio in which the volume is increased

(2). A container of height 30cm has a capacity of 1.5 litres. What is the height of a similar container of capacity 3.0 m^3?

(3). The ratio of the lengths of the corresponding sides of two similar rectangular water tanks is 3:5. The volume of the smaller tank is 8.1 m^3. Calculate the volume of the larger tank.

(4). Pieces of soap are packed in a cuboid container measuring 36 cm by 24 cm by 18 cm. Each piece of soap is similar to the container. If the linear scale factor between the container and the soap is 1/6, find the volume of each piece of soap

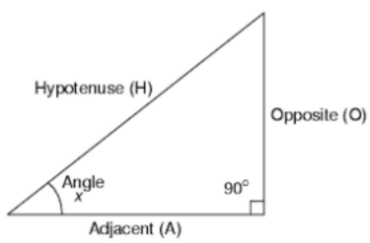
(5). The volumes of two similar solid cylinders are 4752 cm^3 and 1408 cm^3. If the area of the curved surface of the smaller cylinder is 352 cm^2, find the
area of the curved surface of the larger cylinder

TRIGONOMETRY
Tangent of an Acute Angle

Introduction

Tangent of Acute Angle

- The constant ratio between the vertical distance/horizontal distance is called the tangent. It's abbreviated as **tan**



QUESTIONS

(1). When the angle of elevation of the sun is 580, a vertical pole casts a shadow of length 5m on a horizontal ground.

Find the height of the pole.

(2). The angle of elevation of the top of a cliff from point P is 450. From a point Q which is 10m from P towards the foot of the cliff, the angle of elevation is 480.

Calculate the height of the cliff.

(3). A flag 10m long is fixed on top of a tower. From a point on horizontal

ground, the angles of elevation of the top and bottom of the flag post are

400 and 330 respectively.

Calculate

a. The height of the tower (6mks)

b. The shortest distance from the point on the ground to the top of the flag post (2mks).

Table of Tangents

- Scale drawing is one method of obtaining the tangent of an angle.
- Special tables have been prepared and can be used to obtain tangents of acute angles (see tables of natural tangents in your mathematical tables). the technique of reading tables of tangents is similar to that of reading tables of logarithms or square roots.

i. In the tables of tangents, the angles are expressed as decimals and degrees. or in degrees and minutes.

ii. One degree is equal to 60' (60 minutes). Thus. 30' = 0.50° , 54'' = 0.9° and 6' = 0.1 .

Example;

Find the tangent of each of the following angles from the tables:

45. 42°

46. 42.75°

47. 42° 47'

Solution;

From the tables:

a. tan 42° = 0.9004

b. Using decimal tables, tan 42.4 = 0.9228. from the difference column under 5, we read 0.0016.

Therefore, tan 42.47° = 0.9228 + 0.0016

= 0.9244

c. Using degrees and minutes tables, tan 42° 42' = 0.9228 and from difference column under 5, we read 0.0027.

Therefore, tan 42° 47' = 0.9228 + 0.0027

= 0.9255

<u>**Questions**</u>

(1). Express each of the following in degrees and minutes:

22. 15.3°
23. 25.75°
24. 30 ½°
25. 34 $^3/_4$°

(2). Read from the tables the tangent of:

41. 70.53°

42. 18.73°

43. 55° 53'

d. 63° 12'

e. 81° 08'

f. 10° 30'

g. 75° 58'

h. 89° 54'

i. 29° 34'

j. 87° 50'

k. 78° 08°

l. 48° 42'

44. 84°

45. 43° 51'

46. 57.17°

(3). Find from the tables the angle whose tangent is:

61. 0.3317

62. 0.6255

63. 1.6391

64. 0.44444

65. 0.0122

66. 0.8799

67. 0.1867

68. 0.5903

69. 5.1006

70. 1.0000

71. 0.2839

72. 2.0011

73. 3.6703

74. 0.7400

75. 40.92

Sine of an Acute Angle

<u>Sine of an Angle</u>

- The ratio of the side of angle x to the hypotenuse side is called the sine.

$$\text{Sin}\varnothing = \frac{\text{opposite side}}{\text{hypotenuse}}$$

Example

In figure 9.17, AB = 5 cm, CB = 12 cm and < ABC = 90° . Calculate:

a. sin x

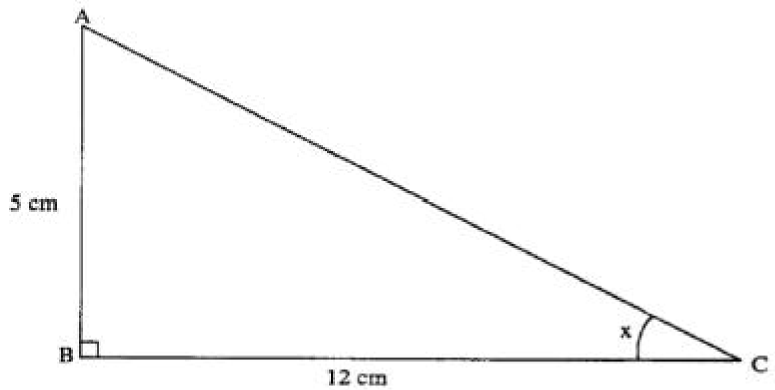


Fig. 9.17

Solution

a. sin x = $\frac{\text{opp}}{\text{hyp}} = \frac{AB}{AC} = \frac{5}{AC}$

But $AC^2 = 12^2 + 5^2$

=144 + 25

= 169

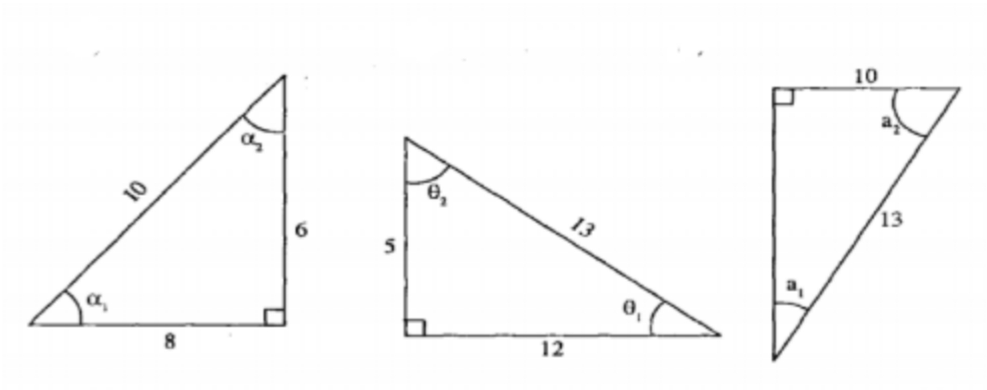
Therefore, AC = 13 cm

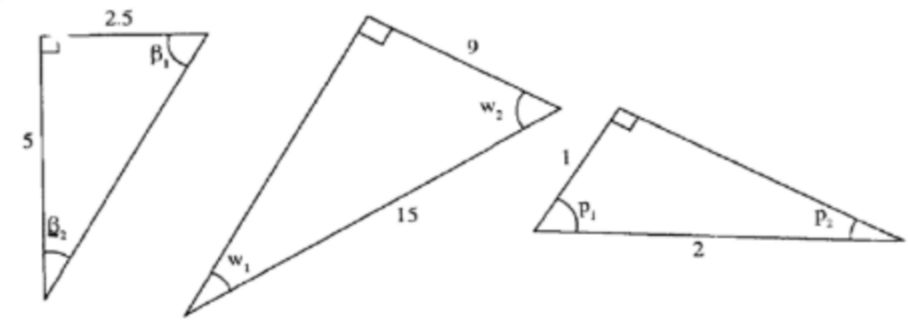
76. sin x = $^5/_{13}$

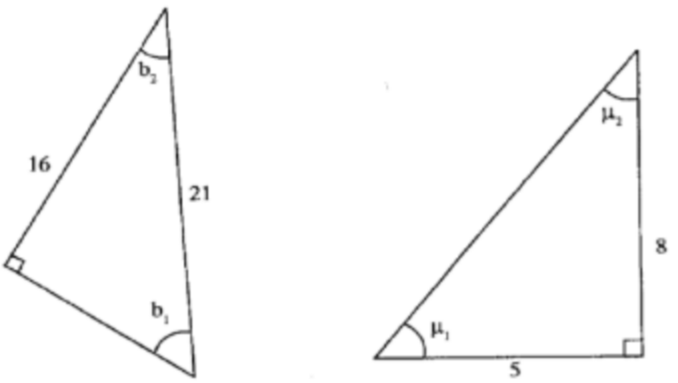
= 0.3846

<u>Questions</u>

(1). Find the cosine and sine of each in the following marked angle. 9 units are in centimeters.





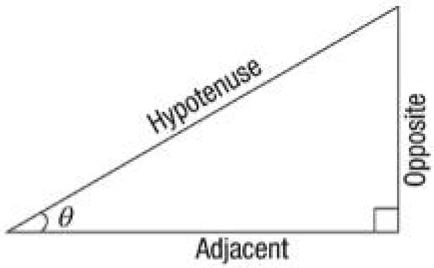


Cosine of an Acute Angle

Cosine of an Angle

- The ratio of the side adjacent to the angle and hypotenuse

$$\text{Cosine}\varnothing = \frac{\text{Adjacent}}{\text{hypotenuse}}$$



$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} \qquad \csc\theta = \frac{\text{hypotenuse}}{\text{opposite}}$$

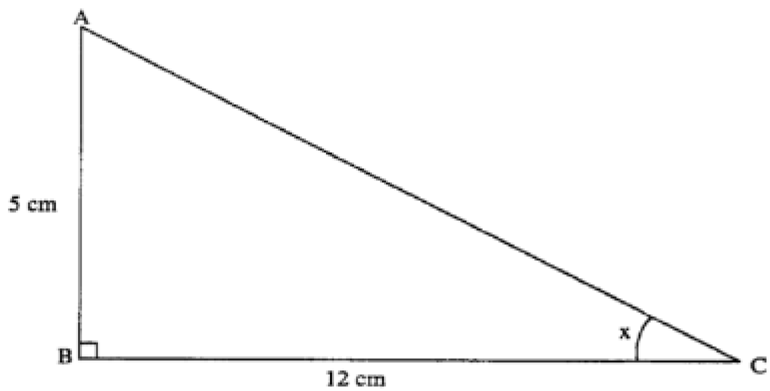
$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} \qquad \sec\theta = \frac{\text{hypotenuse}}{\text{adjacent}}$$

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}} \qquad \cot\theta = \frac{\text{adjacent}}{\text{opposite}}$$

Example

In figure 9.17, AB = 5 cm, CB = 12 and < ABC = 90°. **Calculate:**

cos x.



$$= \frac{\text{adj}}{\text{hyp}}$$

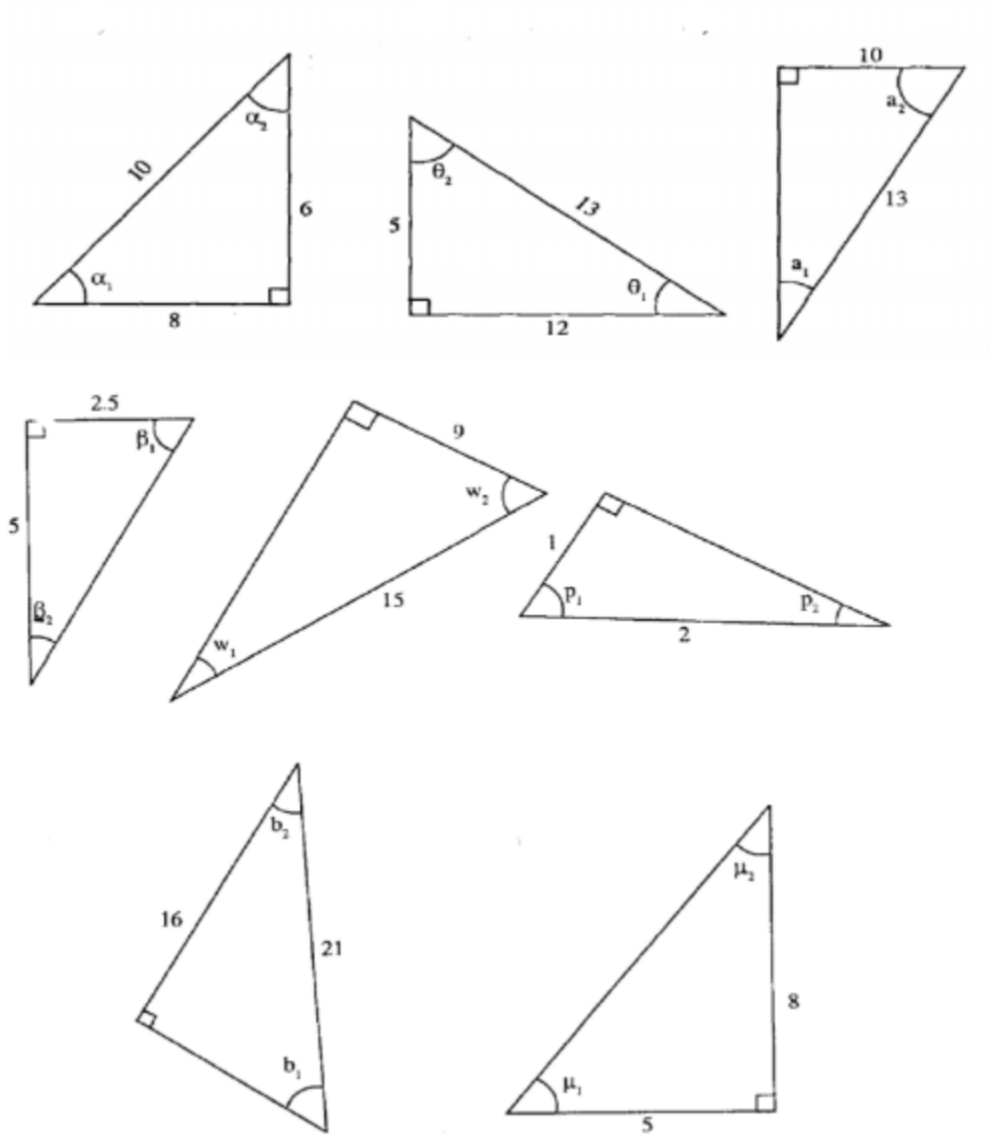
$$= \frac{CB}{AC}$$

$$= \frac{12}{13}$$

b. cos x $= 0.9231$

<u>Questions</u>

(1). Find the cosine and sine of each in the following marked angle. (units are in centimeters)



Tables of Sine

- The sine and cosine tables are read and used in the same way as the tangent tables. As the angles increase from 0° to 9°

i. The values of their sines increase from 0 to 1

ii. The values of their cosines decrease from 1 to 0.

- Therefore, the values in the difference columns of cosine tables have to be subtracted and those in the difference columns of the sine tables have to be added.

Example;

Read the sine and cosine values of the following angles from the tables:

32. 47.3°

33. 69.55°

Solution

a.

sin 47.3° = 0.7349

cos 47.3° = 0.6782

b.

sin 69.55° = 0.9370

cos 69.55° = 0.3494

<u>Questions</u>

(1). Find from the tables the angle whose sine is:

(xiii) 0.3367

(xiv) 0.5871

(xv) 0.0523

(xvi) 0.8500

(xvii) 0.1822

(xviii) 0.9834

(xix) 0.5012

(xx) 0.2518

(2). Read from the tables the sine of:

14. 31.46°

15. 77° 34'

16. 52° 9'

17. 66° 31'

18. 6.76°

19. 40.13°

20. 26° 47'

21. 13.07°

Tables of Cosine

- The sine and cosine tables are read and used in the same way as the tangent tables. As the angles increase from 0° to 90°

i. the values of their sines increase from 0° to 90° .

ii. the values of their cosines decrease from 1 to 0.

- Therefore, the values in the difference columns of cosines tables have to be subtracted and those in the difference columns of the sine tables have to be added.

Example;

Read the sine and cosine values of the following angles from the tables:

7. 47.3°

8. 69.55°

Solution

a.

sin 47.3° = 0.7349

cos 47.3°= 0.6782

b.

sin 69.55° = 0.9370

cos 69.55° = 0.3494

Questions

(1). Find from the tables the angle whose cosine is:

73. 0.1643

74. 0.7196

75. 0.9970

76. 0.8660

77. 0.4009

78. 0.9481

79. 0

80. 0.7371

(2). Read from the tables the cosine of:

a. 79° 42'

11.24.23°

12. 5° 37'

13. 60°

14. 88°.59'

15. 55.97°

16. 33.33°

17. 17° 52'

Sine and Cosine of Complementary Angles

Sine and Cosines of Complementary Angles

- For any two complementary angles x and y, sin x = cos y; cos x = sin y e.g. sin 600 = cos 300 , Sin 300 = cos 600 , sin 700 = cos 200 ,

Example

Find acute angles α and β if Sin α = cos 330

Solution

sin α = cos 33

Therefore α + 33 = 90

α=57

Find acute angles $\propto$ and 3β if:

17. sin$\propto$ = cos 33°

18. cosβ = sin3β

Solution

114. sin$\propto$ = cos 33

Therefore, $\propto$ + 33 = 90

$\propto$ = 90° - 33°

= 57°

17. $\cos \beta = \sin 3\beta$

Therefore, $\beta + 3\beta = 90°$

$4\beta = 90°$ $\beta = 22\ ^{1}/_{2}°$

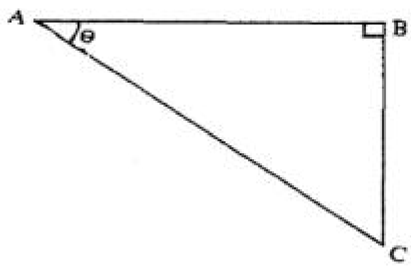
Questions

(1). If A and B are complementary angles and sin A = $^{4}/_{5}$, find cos B

(2). In figure 9.32, AB = 16 cm and AC = 20cm. Find:

a sin θ

b cos θ



(3). X and B are complementary angles. If sign B = 0.9975. Find X

(4). A and B are complementary angles. If A = $\frac{1}{2}$B Find:

a. sin A.

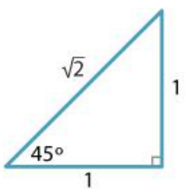
b. cos A

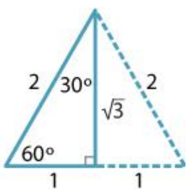
(5). Find the acute angle X given that cos x° = sin 2x°

Tangent, Cosine and Sine of 45 degrees

Trigonometric Ratios of Special Angles: 30° , 45° , 60° .

- These trigonometric ratios can be deducted by the use of isosceles right – angled triangle and equilateral triangles as follows.





Tangent, Cosine and Sine of 45°

- The triangle should have a base and a height of one unit each, giving hypotenuse of √2.

Cos 45° = 1 /√2

Sin 45° = 1 /√2

Tan 45° = 1

QUESTIONS

(1). Simplify the following without using tables (use trigonometric ratios):

Note: $\sqrt{a} \times \sqrt{a} = a$

a. sin 30° cos 30°

b. 4 cos 45° sin 60°

c. 3 cos 30° + cos 60°

d. tan 45° + cos 45° sin 45°

e. sin 60° cos 30° + sin 30° cos 30°

f. $\cos^2 60° + \sin^2 60°$ **(sin θ × sin θ is written as $\sin^2$ θ)**

(2). Given sin (90 – a) = ½ , find without using trigonometric tables the value of cos a (2mks)

(3). If θ = 24/25, find without using tables or calculator, the value of

tan θ – cos θ

cos θ + sin θ

(4). At point A, David observed the top of a tall building at an angle of 30°. After walking for 100 meters towards the foot of the building he stopped at point B where he observed it again at an angle of 60°. Find the height of the building

(5). Find the value of θ, given that ½ sinθ = 0.35 for 0o ≤ θ ≤ 360°

(6). A man walks from point A towards the foot of a tall building 240 m away. After covering 180m, he observes that the angle of elevation of the top of the building is 45°. Determine the angle of elevation of the top of the building from A

(7). Solve for x in 2 Cos 2 x 0 = 0.6000; 00≤ x ≤ 3600 .

(8). Wangechi whose eye level is 182cm tall observed the angle of elevation to the top of her house to be 32° from her eye level at point A. She walks 20 m towards the house on a straight line to a point B at which point she observes the angle of elevation to the top of the building to the 40°.

Calculate, correct to 2 decimal places the ;

a. Distance of A from the house

b. The height of the house

(9). Given that cos $A = 5/13$ and angle A is acute, find the value of:-

$2 \tan A + 3 \sin A$

(10). Given that tan 5° = 3 + √5, without using tables or a calculator, determine tan 25°, leaving your answer in the form a + b√c

(11). Given that tan x = 5 /12, find the value of the following without using mathematical tables or calculator:

a. Cos x

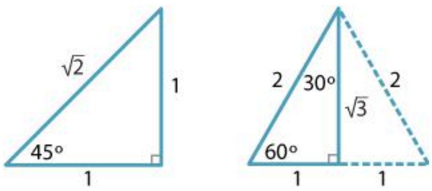
b. Sin2 (90–x)

(12). If tan θ = 8 /15, find the value of Sinθ – Cosθ without using a calculator or table Cosθ + Sinθ

Tangent , Cosine and Sine of 30 and 60 Degrees

<u>Trigonometric Ratios of Special Angles: 30°, 45°, 60°.</u>

- These trigonometric ratios can be deducted by the use of isosceles right – angled triangle and equilateral triangles as follows.



Tangent, Cosine and Sine of 30° and 60° .

- The equilateral triangle has a sides of 2 units each

Sin 30° = 1 /2

Cos 30° = √3/2

Tan 30° = 1 /√3

Sin 60° = √3/2

Cos 60° = 1 /2

Tan 60° = √3/1 = √3

<u>QUESTIONS</u>

(1). Simplify the following without using tablets (use trigonometric ratios):

Note: $\sqrt{a} \times \sqrt{a} = a$

a. sin 30° cos 30°

b. 4 cos 45° sin 60°

c. 3 cos 30° + cos 60°

d. tan 45° + cos 45° sin 45°

e. sin 60° cos 30° + sin 30° cos 60°

f. $\cos^2 60° + \sin^2 60°$ (sin θ x sin θ is written as $\sin^2\theta$)

(2). Find the height of an equilateral triangle of side x cm. Use the triangle to show that $\sin^2 60° + \cos^2 60° = 1$ (without using tables).

(3). Given sin (90 – a) = ½ , find without using trigonometric tables the value of cos a (2mks)

(4). If θ = 24/25, find without using tables or calculator, the value of

tan θ – cos θ

cos θ + sin θ

(5). At point A, David observed the top of a tall building at an angle of 30°. After walking for 100 meters towards the foot of the building he stopped at point B where he observed it again at an angle of 60°. Find the height of the building.

(6). Find the value of θ, given that ½ sinθ= 0.35 for 0o ≤ θ ≤ 360°

(7). A man walks from point A towards the foot of a tall building 240 m away. After covering 180 m, he observes that the angle of elevation of the top of the building is 45°. Determine the angle of
elevation of the top of the building from A

(8). Solve for x in 2Cos 2 x 0 = 0.6000; 00≤ x ≤ 3600 .

(9). Wangechi whose eye level is 182 cm tall observed the angle of elevation to the top of her house to be 32° from her eye level at point A. She walks 20 m towards the house on a straight line to a point B at which point she observes the angle of elevation to the top of the building to the 40°.

Calculate, correct to 2 decimal places the ;

a. Distance of A from the house

b. The height of the house

(8). Given that cos A = 5 /13 and angle A is acute, find the value of:-

2 tan A + 3 sin A

(9). Given that tan 5° = 3 + √5, without using tables or a calculator, determine tan 25°, leaving your answer in the form a + b√c

(10). Given that tan x = 5 /12, find the value of the following without using mathematical tables or calculator:

a. Cos x

b. Sin2 (90–x)

(11). If $\tan \theta = 8/15$, find the value of $\frac{\text{Sin}\theta - \text{Cos}\theta}{\text{Cos}\theta + \text{Sin}\theta}$ without using a calculator or table

Logarithm of Tangent, Cosine and Sine

Logarithms of Tangents, Sines and Cosines

- In this section, we extend the use of logarithms in computations involving trigonometry.

For example,

To evaluate 234 sin 36°, we can use the following method: from tables, sin 36° = 0.5878

Therefore, 234 sin 36° = 234 × 0.5878

No	Log
234	2.3692
0.5878	$\bar{1}.7692$ +
1.375×10^2	2.1384

Therefore; 234 sin 36 = 137.5

Alternatively, log sin 36° can be read directly from the tables of logarithms of sines.

No	Log
234	2.3692
sin 36°	$\bar{1}.7692$ +
1.375×10^2	2.1384

Therefore, 234 sin 36° = 137.5.

Similarly, values of logo(cos x) and log(tan x) can be read from their respective tables.

Questions

(1). Evaluate;

a. 8.52 tan 42.2°

b. 7.9 sin 79°

c. $\dfrac{7\cos 50.2^0}{9.5 \sin 60^0}$

d. $\dfrac{7\cos 50.2^0}{9.5 \sin 60^0}$

(2). In a triangle PQR, <PQR = <PQR = 58° and QR = 5.2 cm. Calculate the length of PQ.

(3). In a triangle PQR, OR = 5.2 cm and PQ = PR = 8.2 cm. Calculate:

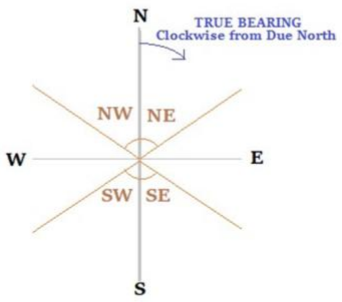
a. <PQR and <QPR

b. The area of ΔPQR.

Bearing

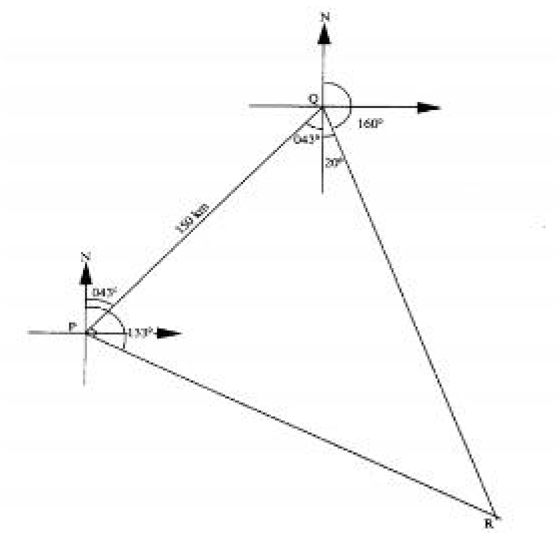
Bearings

- Bearings are used in navigation. A bearing is an angle measurement used to describe precisely the direction of one location from a given reference point.
- Three-figure bearings, also called true bearings, use angles from 000°to 360°to show the amount of turning measured clockwise from north 000°. Note that the angles are always written with three digits.



Example;

Three towns P, Q and R are such that Q is 150 km from P on a bearing of 043° (see figure 9.36). The bearing of R from P is 133° and the bearing of R from Q is 160°. Calculate the distance of R from P, Q from R and the bearing of P from R.



Solution;

In the figure, ΔPQR is right-angled at P <PQR = 63°,

Hence, tan 63° = PR/150

150 tan 63° = PR

No	Log
150	2.1761
tan 63^0	0.2928 +
2.944×10^2	2.4689

Therefore, PR = 294.4 km

In the right-angled ∆PQR, QR is the hypotenuse;

cos 63° = 150/QR

QR = 150/cos 63°

No	Log
150	2.1761
cos 63^0	$\bar{1}.6570$ –
3.305×10^2	2.5191

Therefore, QR = 330.5 km

<PRQ = 180° - (90 + 63)°

= 27°

<PRT = 90° - (20 + 27)°

= 43°

Therefore, the bearing of p from R = 270° + 43° = 313°

QUESTIONS

(1). A man walks directly from point A towards the foot of a tall building 240m away. After covering 180m, he observes that the angle of the top of the building is 450. Determine the angle of elevation of the top of the building from A.

(2). There are two signposts A and B on the edge of the road. A is 400 m to the west of b. A tree is on a bearing of 0600 from A and a bearing of 3300 from B Calculate the shortest distance of the tree from the edge of the road.

(3). A point A is directly below a window. Another point B is 15 m from A and at the same horizontal level. From B angle of elevation of the top of the

bottom of the window is 300 and the angle of elevation of the top of the window is 350

Calculate the vertical distance.

a. From A to the top of the window

b. From A to the bottom of the window

c. From the bottom to top of the window

(4). An electric pylon is 30m high. A point S on top of the pylon is vertically above another point on the ground . Points A and B are on the same horizontal ground as R. Point A is due south of the pylon and the angle of elevation of S from A is 260. Point B is due west of the pylon and the angle of elevation of S from B is 320.

a. Distance from A and B

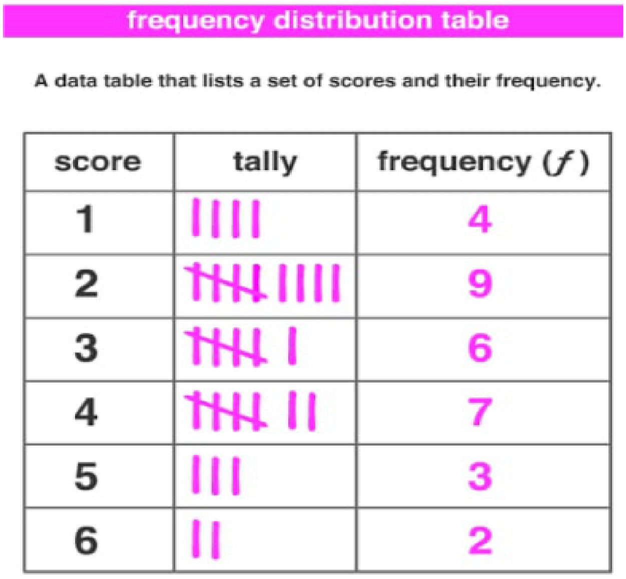
DATA INTERPRETATION

Introduction

- This is the branch of mathematics that deals with the collection, organization, representation and interpretation of data. Data is the basic information.

Frequency Distribution Table

- A data table that lists a set of scores and their frequency



frequency distribution table

A data table that lists a set of scores and their frequency.

score	tally	frequency (f)
1	||||	4
2	~~||||~~ ||||	9
3	~~||||~~ |	6
4	~~||||~~ ||	7
5	|||	3
6	||	2

Tally

- In tallying each stroke represent a quantity.

Frequency

Mean

This is usually referred to as arithmetic mean, and is the average value for the data

-

Data value	Tally	Frequency	Frequency × Data value
2	III	3	6
3	II	2	6
4	~~IIII~~	5	20
5	III	3	15
6	IIII	4	24
7	~~IIII~~ I	6	42
8	III	3	24
9	IIII	4	36
	SUM =	30	173

The mean ($\bar{x}$)

$= \dfrac{\text{total marks scored}}{\text{total number of students}}$

48. $\dfrac{\sum fx}{\sum f}$

$= \dfrac{173}{30}$

$= 5.767$

Mode

- This is the most frequent item or value in a distribution or data. In the above table its 7 which is the most frequent.

Median

- To get the median arrange the items in order of size. If there are N items and N is an odd number, the item occupying $({}^{n+1}/_{2})^{th}$
- If N is even, the average of the items occupying ${}^{n}/_{2}$

Grouped Data

- Then difference between the smallest and the biggest values in a set of data is called the range. The data can be grouped into a convenient number of groups called classes. 30 – 40 are called class boundaries.

- The class with the highest frequency is called the modal class. In this case its $50 \leq m < 60$, the class width or interval is obtained by getting the difference between the class limits. In this case, 30 – 40 = 10, to get the mid-point you divide it by 2 and add it to the lower class limit.

Mass (m) kg	Midpoint	Frequency	Midpoint × Frequency
$30 \leq m < 40$	35	7	245
$40 \leq m < 50$	45	6	270
$50 \leq m < 60$	55	8	440
$60 \leq m < 70$	65	4	260
	Totals:	25	1,215

The mean mass in the table above is $\sum f = 25$, $\sum fx = 1215$ Mean

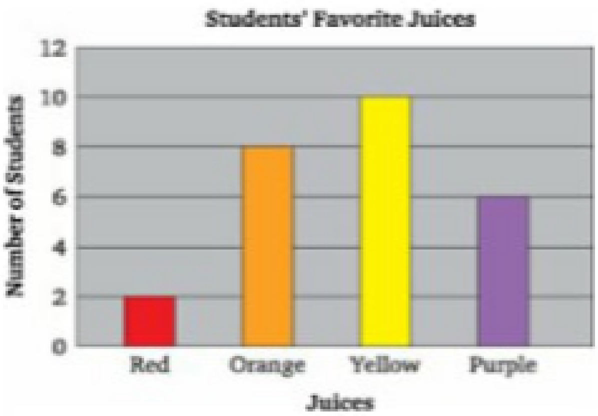
${}^{1215}/_{25} = 48.6$

Representation of Statistical Data

- The main purpose of representation of statistical data is to make collected data more easily understood.
- Methods of representation of data include.

Bar Graph

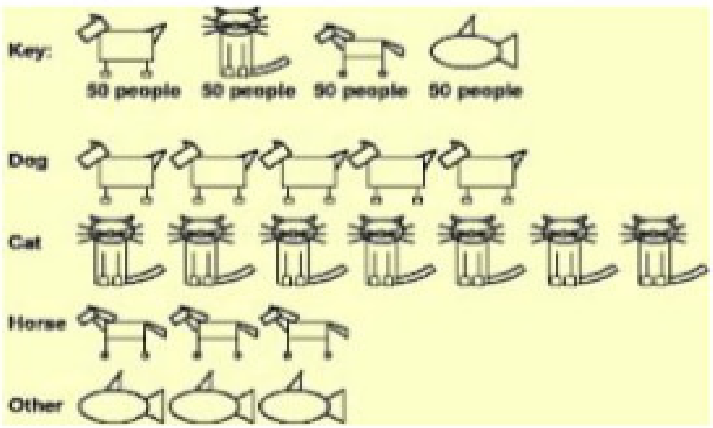
- Consist of a number of spaced rectangles which generally have major axes vertical. Bars are uniform width.
- The axes must be labelled and scales indicated



- The students' favorite juices are as follows
  Red 2
  Orange 8
  Yellow 10
  Purple 6

Pictograms

- In a pictogram, data is represented using pictures.
- Consider the following data



- The data shows the number of people who love the following animals Dogs 250, Cats 350, Horses 150, other 150

Pie Chart

- A pie chart is divided into various sectors .Each sector represent a certain quantity of the item being considered the size of the sector is proportional to the quantity being measured .consider the export of US to the following countries. Canada $ 1 3390, Mexico $ 81 36, Japan $5824, France $2110.

- This information can be represented in a pie chart as follows
  Canada angle

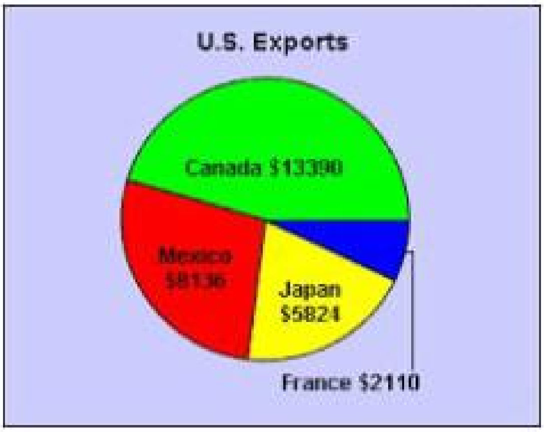
24. $\frac{\text{amount of export}}{\text{total population}} \times 360$

$\frac{13390}{29460} \times 360 = 163.62^{o}$

Mexico $= \frac{8136}{29460} \times 360 = 99.42^{o}$

Japan $\frac{5824}{29460} \times 360 = 71.16^{o}$

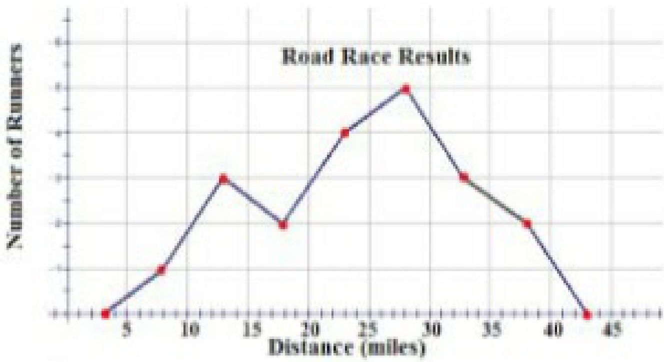
France $\frac{2110}{29460} \times 360 = 25.78^{o}$



Line Graph

Data represented using lines

-

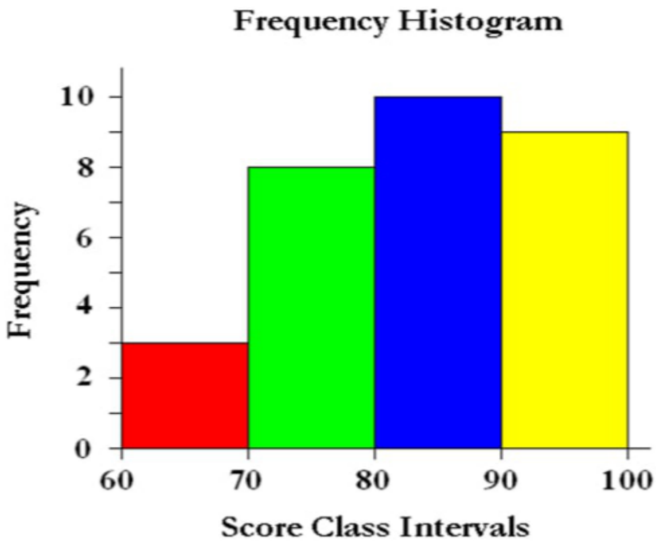


Histograms

- Frequency in each class is represented by a rectangular bar whose area is proportional to the frequency, where the bars are of the same width and the height of the rectangle is proportional to the frequency .

Note;

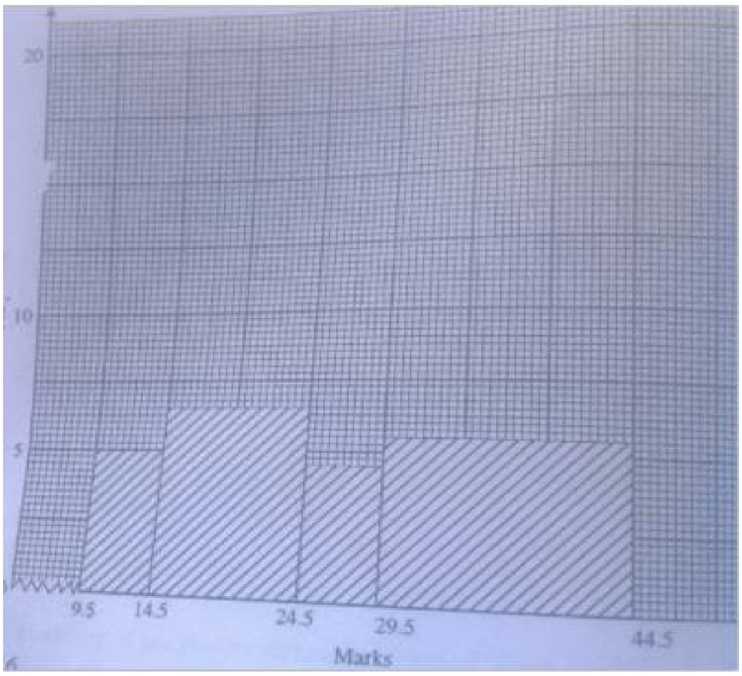
- The bars are joined together.
- The class boundaries mark the boundaries of the rectangular bars in the histogram



- Histograms can also be drawn when the class interval is not the same

The below information can be represented in a histogram as below

Marks	10-14	15- 24	25-29	30-44
No.of students	5	16	4	15

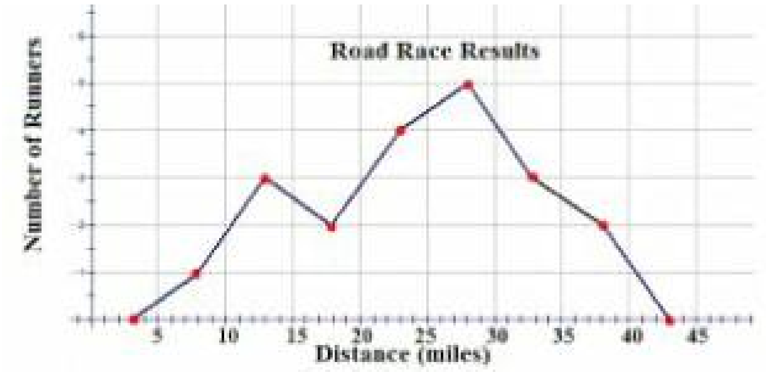


Note;

- When the class is doubled the frequency is halved

Frequency Polygon

- It is obtained by plotting the frequency against mid points.



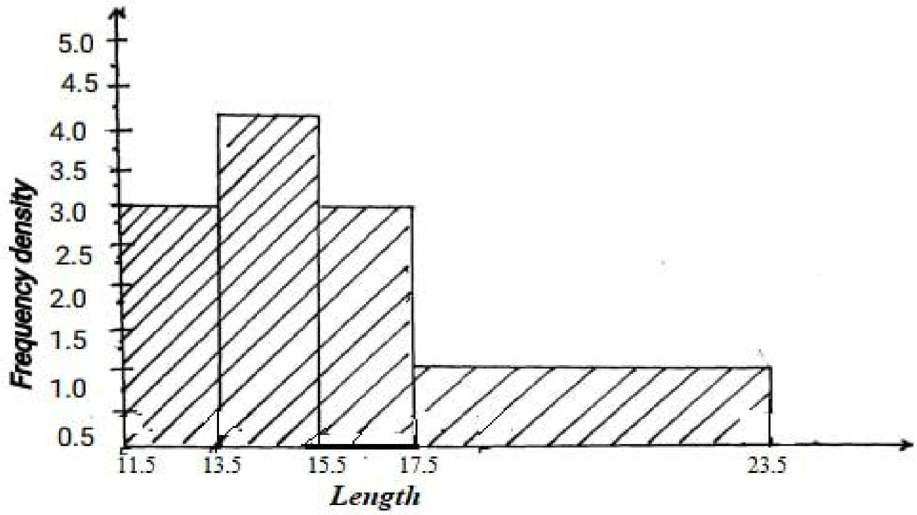
Questions

1. The height of 36 students in a class was recorded to the nearest centimeters as follows. 148 159 163 158 166 155 155 179 158 155 171 172 156 161 160 165 157 165 175 173 172 178 159 168 160 167 147 168 172 157 165 154 170 157 162 173

Make a grouped table with 145.5 as lower class limit and class width of 5. (4mks)

Use your table in (a) to draw a histogram to represent the data

2.



Use the histogram above to complete the frequency table below:

Length	Frequency
11 .5 ≤ x ≤ 13.5	
13.5 ≤ x ≤ 15.5	
15.5 ≤ x ≤ 17.5	
17.5 ≤ x ≤ 23.5	

3. Kambui spent her salary as follows:

Food	40%
Transport	10%
Education	20%
Clothing	20%
Rent	10%

Draw a pie chart to represent the above information

4. The examination marks in a mathematics test for 60 students were as follows;-

60	54	34	83	52	74	61	27	65	22
70	71	47	60	63	59	58	46	39	35
69	42	53	74	92	27	39	41	49	54
25	51	71	59	68	73	90	88	93	85
46	82	58	85	61	69	24	40	88	34
30	26	1 7	1 5	80	90	65	55	69	89

Class	Tally	Frequency	Upper class limit
10-29			
30-39			
40-69			
70-74			
75-89			
90-99			

From the table;

a. State the modal class

b. On the grid provided , draw a histogram to represent the above information

5.The marks scored by 200 from 4 students of a school were recorded as in the table below.

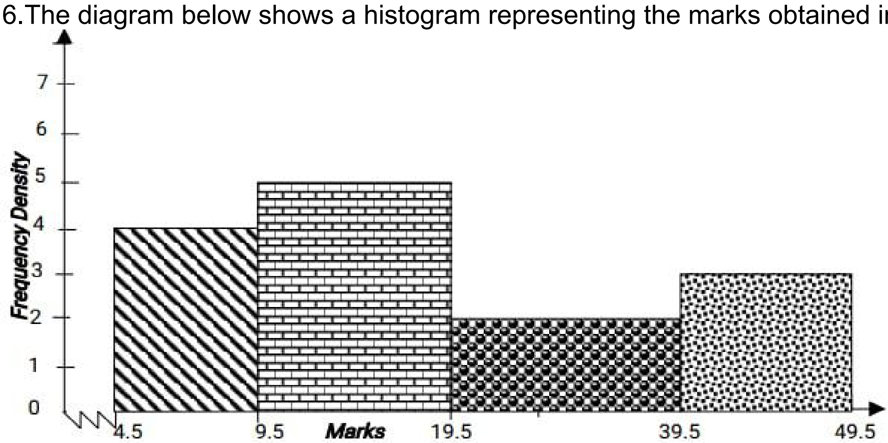
Marks	41–50	51–55	56–65	66–70	71–85
Frequency	21	62	55	50	1 2

a. On the graph paper provided, draw a histogram to represent this information.

b. On the same diagram, construct a frequency polygon.

c. Use your histogram to estimate the modal mark.

6.The diagram below shows a histogram representing the marks obtained in a certain test:-



a. If the frequency of the first class is 20, prepare a frequency distribution table for the data b. State the modal class

3. Estimate:

The mean mark

The median mark

Probability
Introduction

- The likelihood of an occurrence of an event or the numerical measure of chance is called probability.

Experimental Probability

- This is where probability is determined by experience or experiment. What is done or observed is the experiment. Each toss is called a trial and the result of a trial is the outcome.
- The experimental probability of a result is given by <u>(the number of favorable outcomes)</u>
  (the total number of trials)

Example

A boy had a fair die with faces marked 1 to6 .He threw this die up 50 times and each time he recorded the number on the top face. The result of his experiment is shown below.

face	1	2	3	4	5	6
Number of shown up	11	6	7	9	9	8

What is the experimental provability of getting?

49. 1
50.6

Solution

26. P(Event) = <u>the number of favorable outcomes</u>
    the total number of trials

P(1)= $^{11}/50$

b. P(4)= $^{9}/50$

Example

From the past records, out of the ten matches a school football team has played, it has won seven.How many possible games might the school win in thirty matches?

Solution

P (winning in one math) = $^{7}/_{10}$.

Therefore the number of possible wins in thirty matches = $^{7}/_{10}$ x 30 = 21 matches

Range of Probability Measure

- If P(A) is the probability of an event A happening and P(A') is the probability of an event A not happening, Then P(A')= 1 – P(A) and P(A') + P(A)= 1
- Probability are expressed as fractions, decimals or percentages.

Probability Space

- A list of all possible outcomes is **probability space** or **sample space**.
- The coin is such that the head or tail have equal chances of occurring.
- The events head or tail are said to be equally likely or equiprobable.

Theoretical Probability

- This can be calculated without necessarily using any past experience or doing any experiment. • The probability of an event happening = <u>number of favorable outcomes</u>

total number of outcomes

Example

A basket contains 5 red balls, 4 green balls and 3 blue balls. If a ball is picked at random from the basket, find:

47. The probability of picking a blue ball
48. The probability of not picking a red ball

Solution

49. Total number of balls is 1 2
    The number of blue balls is 3
    therefore, P (a blue ball)
    $=^{3}/_{12}$
50. The number of balls which are not red is 7.

    Therefore P (not a red ball)= $^{7}/_{12}$

Example

A bag contains 6 black balls and some brown ones. If a ball is picked at random the probability that it is black is 0.25.

Find the number of brown balls.

Solution

Let the number of balls be x

Then the probability that a black ball is picked at random is $^{6}/_{x}$

Therefore $^{6}/_{x}$ = 0.25

$x = 24$

The total number of balls is 24

Then the number of brown balls is $24 - 6 = 18$

Note:

- When all possible outcomes are countable, they are said to be **discrete**.

Types of Probability

Combined Events

- These are probability of two or more events occurring

Mutually Exclusive Events

- Occurrence of one excludes the occurrence of the other or the occurrence of one event depends on the occurrence of the other.
- If A and B are two mutually exclusive events, then (A or B) = P (A) + P (B). For example when a coin is tossed the result will either be a head or a tail.

Example

If a coin is tossed ;

P(head) + P(tail)

$=^1/_2+^1/_2=1$

Note;

- If [OR] is used then we add

Independent Events

- Two events A and B are independent if the occurrence of A does not influence the occurrence of B and vice versa.
- If A and B are two independent events, the probability of them occurring together is the product of their individual probabilities .That is;
  P (A and B) = P (A) x P(B)

Note;

- When we use [AND] we multiply ,this is the multiplication law of probability.

Example

A coin is tosses twice. What is the probability of getting a tail in both tosses?

Solution

The outcome of the 2nd toss is independ of the outcome of the first .

Therefore;

P (T and T) = P(T) X P(T)

77. $^1/_2 \times ^1/_2 = ^1/_4$

Example

A boy throws fair coin and a regular tetrahedron with its four faces marked 1,2, 3 and 4. Find the probability that he gets a 3 on the tetrahedron and a head on the coin.

Solution

These are independent events.

P (H) = $^1/_2$, P(3) = $^1/_4$

Therefore;

P (H and 3) = P (H) x P (3)

78. $\frac{1}{2} \times \frac{1}{4}$
79. $^1/_8$

Example

A bag contains 8 black balls and 5 white ones. If two balls are drawn from the bag, one at a time,find the probability of drawing a black ball and a white ball.

80. Without replacement
81. With replacement

Solution

34. There are only two ways we can get a black and a white ball: either drawing a white then a black,or drawing a black then a white. We need to find the two probabilities;

   P(W followed by B) = P (W and B)

   $^8/_{13} \times {}^5/_{12} = {}^{10}/_{39}$

35. P(B followed by W) = P (B and W)

   $^5/_{13} \times {}^8/_{12} = {}^{10}/_{39}$

   Note;

   - The two events are mutually exclusive, therefore.

   P (W followed by B) or (B followed by W)= P(W followed by B) + P (B followed by W)

   = P (W and B) + P(B and W)

   $= {}^{40}/_{156} + {}^{40}/_{156} = {}^{20}/_{39}$

   Since we are replacing, the number of balls remains 13.

   Therefore;

   P (W and B) = $^5/_{13} \times {}^8/_{13} = {}^{40}/_{169}$

   P (B and W) = $^8/_{13} \times {}^5/_{13} = {}^{40}/_{169}$

   Therefore;

   P [(W and B) or (B and W)] = P (W and B) + P (B and W)

   $= {}^{40}/_{169} + {}^{40}/_{169} = {}^{80}/_{169}$

Example

Kamau ,Njoroge and Kariuki are practicing archery .The probability of Kamau hitting the target is 2/5,that of Njoroge hitting the target is $\frac{1}{4}$ and that of Kariuki hitting the target is $^3/_7$, Find the probability that in one attempt;

(xxi) Only one hits the target

(xxii) All three hit the target

(xxiii) None of them hits the target

(xxiv) Two hit the target

(xxv) At least one hits the target

Solution

a. P(only one hits the target)

=P (only Kamau hits and other two miss) = $^{2}/_{5} \times {^{3}/_{5}} \times {^{4}/_{7}}$

= $^{6}/_{35}$

P (only Njoroge hits and other two miss) = $^{1}/_{4} \times {^{3}/_{5}} \times {^{4}/_{7}}$

= $^{3}/_{35}$

P (only Kariuki hits and other two miss) = $^{3}/_{7} \times {^{3}/_{5}} \times \frac{3}{4}$

= $^{27}/_{140}$

P (only one hits) = P (Kamau hits or Njoroge hits or Kariuki hits)

$^{6}/_{35} + {}^{3}/_{35} + {}^{27}/_{140}$

$^{9}/_{20}$

22. P (all three hit) = $^{2}/_{5} \times {}^{1}/_{4} \times {}^{3}/_{7}$

$^{3}/_{70}$

23. P (none hits) = $^{3}/_{5} \times {}^{3}/_{4} \times {}^{4}/_{7}$

$^{9}/_{35}$

24.P (two hit the target) is the probability of ;

Kamau and Njoroge hit the target and Kariuki misses = $^{2}/_{5} \times {}^{3}/_{7} \times {}^{4}/_{7}$

Njoroge and Kariuki hit the target and Kamau misses = $^{1}/_{4} \times {}^{3}/_{7} \times {}^{3}/_{5}$

Or

Kamau and Kariuki hit the target and Njoroge misses = $^{2}/_{5} \times {}^{3}/_{7} \times {}^{3}/_{4}$

Therefore P (two hit target) = $({}^{2}/_{5} \times {}^{1}/_{4} \times {}^{4}/_{7}) + ({}^{1}/_{4} \times {}^{3}/_{7} \times {}^{3}/_{5}) + ({}^{2}/_{5} \times {}^{3}/_{7} \times {}^{3}/_{4})$

$^{8}/_{140} + {}^{9}/_{140} + {}^{18}/_{140}$

$\frac{1}{4}$

25. P (at least one hits the target) = 1 – P (none hits the target)

$1 - {}^{9}/_{35}$

$^{26}/_{35}$

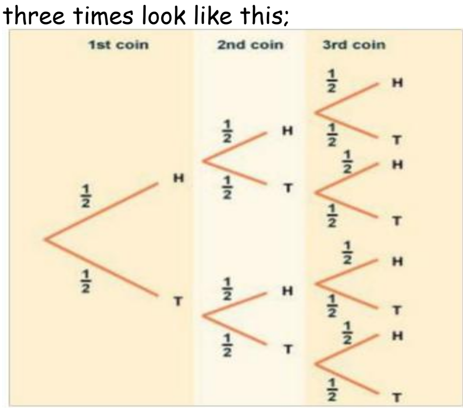
9. $^{26}/_{35}$

Note;

- P (one hits the target) is different from P (at least one hits the target)

Tree Diagram

- Tree diagrams allows us to see all the possible outcomes of an event and calculate their probality.
- Each branch in a tree diagram represents a possible outcome .A tree diagram which represent a coin being tossed three times look like this;



- From the tree diagram, we can see that there are eight possible outcomes. To find out the probability of a particular outcome, we need to look at all the available paths (set of branches).
- The sum of the probabilities for any set of branches is always 1 .
- Also note that in a tree diagram to find a probability of an outcome we multiply along the branches and add vertically.
  - The probability of three heads is:

$$P(HHH)={}^{1}/_{2}\times{}^{1}/_{2}\times{}^{1}/_{2}={}^{1}/_{8}$$

P (2 Heads and a Tail) = P (H H T) + P (H T H) + P (T H H)

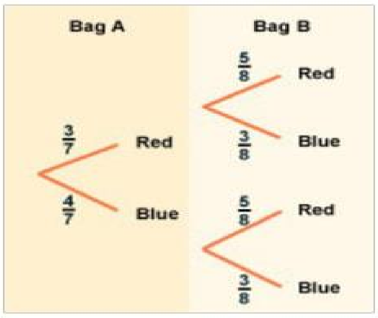
81. $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$

82. $\frac{1}{8} + \frac{1}{8} + \frac{1}{8}$

83. $\frac{3}{8}$

Example

Bag A contains three red marbles and four blue marbles.Bag B contains 5 red marbles and three blue marbles. A marble is taken from each bag in turn.



18. What is the probability of getting a blue bead followed by a red
19. What is the probability of getting a bead of each color

Solution

a. Multiply the probabilities together

P(blue and red) = $^4/_7 \times {}^5/_8 = {}^{20}/_{56}$

$= {}^5/_{14}$

b. P(blue and red or red and blue) = P(blue and red) + P (red and blue)

$= {}^4/_7 \times {}^5/_8 + {}^3/_7 \times {}^3/_8$

$= {}^{20}/_{56} + {}^9/_{56}$

$= {}^{29}/_{56}$

Example

The probability that Omweri goes to Nakuru is $\frac{1}{4}$ .If he goes to Nakuru, the probability that he will see flamingo is 19. If he does not go to Nakuru, the probability that he will see flamingo is $^1/_3$. Find the probability that;

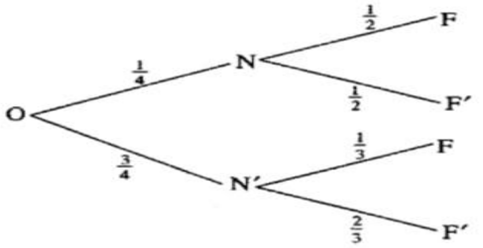
Omweri will go to Nakuru and see a flamingo.

Omweri will not go to Nakuru yet he will see a flamingo

Omweri will see a flamingo

Solution

Let N stand for going to Nakuru ,N' stand for not going to Nakuru, F stand for seeing a flamingo and F' stand for not seeing a flamingo.



115. P (He goes to Nakuru and sees a flamingo) = P(N and F)

P(N) × P(F)

$\frac{1}{4} \times \frac{1}{2}$

$^1/_8$

116. P(He does not go to Nakuru and yet sees a flamingo) =P(N') × P(F)

P (N' and F)

$^3/_4 \times ^1/_3$

$\frac{1}{4}$

117. P (He sees a flamingo) = P(N and F) or P (N' and F)

P (N and F) + P (N' and F)

18. $^1/_8 + ^1/_4$

19. $^3/_8$

Questions

19. The probabilities that a husband and wife will be alive 25 years from now are 0.7 and 0.9 respectively. Find the probability that in 25 years time,

    Both will be alive

    Neither will be alive

    One will be alive

    At least one will be alive

20. A bag contains blue, green and red pens of the same type in the ratio 8:2:5 respectively. A pen is picked at random without replacement and its colour noted

    Determine the probability that the first pen picked is

        Blue

        Either green or red

    Using a tree diagram, determine the probability that

        The first two pens picked are both green

        Only one of the first two pens picked is red.

21. A science club is made up of boys and girls. The club has 3 officials. Using a tree diagram or otherwise find the probability that:

    The club officials are all boys

    Two of the officials are girls

22. Two baskets A and B each contain a mixture of oranges and limes, all of the same size. Basket A contains 26 oranges and 13 limes. Basket B contains 18 oranges and 15 limes. A child selected a basket at random and picked a fruit at a random from it.

    Illustrate this information by a probabilities tree diagram

    Find the probability that the fruit picked was an orange.

23. In a class there are 22 girls and boys. The probability of a girl completing the secondary education course is 3 whereas that of a boy is 2/3

    A student is picked at random from class. Find the possibility that,

        The student picked is a boy and will complete the course

        The student picked will complete the course

    Two students are picked at random. Find the possibility that they are a boy and a girl and that both will not complete the course.

24. Three representatives are to be selected randomly from a group of 7 girls and 8 boys. Calculate the probability of selecting two girls and one boy.

25. A poultry farmer vaccinated 540 of his 720 chickens against a disease. Two months later, 5% of the vaccinated and 80% of the unvaccinated chicken, contracted the disease. Calculate the probability that a chicken chosen random contacted the disease.

26. The probability of three darts players Akinyi, Kamau, and Juma hitting the bulls eye are 0.2, 0.3 and 1 .5 respectively.

    Draw a probability tree diagram to show the possible outcomes

    Find the probability that:

        All hit the bull's eye

Only one of them hit the bull's eye

At most one missed the bull's eye

9.

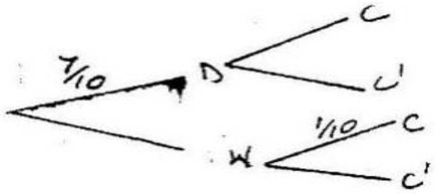
169. An unbiased coin with two faces, head (H) and tail (T), is tossed three times, list all the possible outcomes.

Hence determine the probability of getting:

At least two heads

Only one tail

21. During a certain motor rally it is predicted that the weather will be either dry (D) or wet (W). The probability that the weather will be dry is estimated to be 7/10. The probability for a driver to complete (C) the rally during the dry weather is estimated to be 5/6. The probability for a driver to complete the rally during wet weather is estimated to be 1/10. Complete the probability tree diagram given below.



What is the probability that:

The driver completes the rally?

The weather was wet and the driver did not complete the rally?

2 There are three cars A, B and C in a race. A is twice as likely to win as B while B is twice as likely to win as c. Find the probability that.

a. A wins the race
b. Either B or C wins the race.

3 In the year 2003, the population of a certain district was 1.8 million. Thirty per cent of the population was in the age group 15 – 40 years. In the same year, 1 20,000 people in the district visited the Voluntary Counseling and Testing (VCT) centre for an HIV test. If a person was selected at random from the district in this year. Find the probability that the person visited a VCT centre and was in the age group 1 5 – 40 years.

12.

a Two integers x and y are selected at random from the integers 1 to 8. If the same integer may be selected twice, find the probability that

i $|x - y| = 2$
ii $|x - y|$ is 5 or more
iii $x>y$

b A die is biased so that when tossed, the probability of a number r showing up, is given by p(r)= Kr where K is a constant and r = 1 , 2,3,4,5 and 6 (the number on the faces of the die

i Find the value of K
ii If the die is tossed twice, calculate the probability that the total score is 11

(%) Two bags A and B contain identical balls except for the colours. Bag A contains 4 red balls and 2 yellow balls. Bag B contains 2 red balls and 3 yellow balls.

If a ball is drawn at random from each bag, find the probability that both balls are of the same colour.

If two balls are drawn at random from each bag, one at a time without replacement, find the probability that:

The two balls drawn from bag A or bag B are red

All the four balls drawn are red

(%) During inter – school competitions, football and volleyball teams from Mokagu high school took part. The probability that their football and volleyball teams would win were 3/8 and 4/7 respectively. Find the probability that

Both their football and volleyball teams

At least one of their teams won

(%) A science club is made up of 5 boys and 7 girls. The club has 3 officials. Using a tree diagram or otherwise find the probability that:

The club officials are all boys

Two of the officials are girls

(%) Chicks on Onyango's farm were noted to have either brown feathers brown or black tail feathers. Of those with black feathers $^2/_3$ were female while 2/5 of those with brown feathers were male. Otieno bought two chicks from Onyango. One had black tail feathers while the other had brown find the probability that Otieno's chicks were not of the same gender was

(%) Three representatives are to be selected randomly from a group of 7 girls and 8 boys. Calculate the probability of selecting two girls and one boy

(%) The probability that a man wins a game is $\frac{3}{4}$. He plays the game until he wins. Determine the probability that he wins in the fifth round.

10. The probability that Kamau will be selected for his school's basketball team is $\frac{1}{4}$. If he is selected for the basketball team. Then the probability that he will be selected for football is 1/3 if he is not selected for basketball then the probability that he is selected for football is 4/5. What is the probability that Kamau is selected for at least one of the two games?

11. Two baskets A and B each contains a mixture of oranges and lemons. Baskets A contains 26 oranges and 13 lemons. Baskets B contains 1 8 oranges and 15 lemons. A child selected a basket at random and picked at random a fruit from it. Determine the probability that the fruit picked an orange.